

PRAGMATIC METHOD OF DEDUCING A WAKE FUNCTION FOR A GENERAL 3D STRUCTURE

G. Skripka*, CERN, 1211 Geneva, Switzerland
R. Nagaoka, Synchrotron SOLEIL, 91190 Saint-Aubin, France

Abstract

A key quantity in simulating collective beam instabilities is the wake potential of a bunch of particles whose charge distribution is continuously evolving in time. However, obtaining such wake potential is only possible if a wake excited by a single particle in the surrounding environment is known. A practical self-consistent approach was developed to obtain an effective wake function from a numerical wake potential computed for a finite length bunch. The wake potential is processed to a numerical impedance which is decomposed into a set of well-known analytical wake functions. The decomposed impedance is then transformed back into time domain and, thus, converted into an effective wake function which is by nature physical and most consistent with the numerical wake potential. Though the method is limited by the initial numerical impedance data and the choice of impedance decomposition, the retrieved wake function can be used in instability simulations with a bunch whose length is comparable to that used in the electromagnetic field solver. We show that the method can be applied to a general 3D structure, which allows finding effective wake functions of realistic vacuum chambers.

INTRODUCTION

One of the main sources of wakefields in particle accelerators is the deviation of a vacuum chamber from a smooth pipe, namely variation of the aperture along the machine. These fields, the so called geometric wakefields, are known to impose limitations on machine performance in terms of beam-induced heating of vacuum components, single and multibunch collective beam instabilities [1, 2]. Deducing them is, therefore, of crucial importance for the study of the above issues. For a limited number of simple geometries the analytical or semi-analytical forms of wake functions are known [3–5]. However, in most cases, realistic vacuum components are much more complicated and the corresponding wake functions must be found using other methods. A well-established method in obtaining a wakefield in a general 3D structure is the use of Electromagnetic (EM) field solvers, such as GdfidL [6], ECHO [7, 8], CST Particle Studio [9], which simulate the passage of a bunch in a given structure in time domain. The numerical computation of wake potentials as described above is becoming a common step in the machine design of an accelerator worldwide [10–13] as they help identify in advance beam-intensity-dependent issues caused by the geometry of a vacuum component. Despite the great capabilities of these solvers, they are limited to calculations of a wake potential of a finite length bunch.

On the other hand, a key quantity in simulating collective beam instabilities is the wake potential of a bunch whose charge distribution is continuously evolving in time. This in turn signifies that the knowledge of a wake function, i.e. the wake of a point-charge, is essential in order to calculate the former. One typical, rather crude approach is to approximate the impedance of the machine with a single broadband resonator with guessed values for the involved parameters and convert it to time domain. Another approach is to approximate it by the numerical wake potential of a very short bunch. However, it would require a significant amount of computation time and the choice of the length of a test bunch remains a critical issue. Besides, the direct use of wake potentials derived numerically may associate non-physical behavior of the latter due to numerical errors [14]. Provided that a set of numerical data from EM solver is available for the realistic 3D vacuum structures around a ring it is reasonable to employ them to deduce the wake function instead of introducing a simplified impedance model without full justification.

We propose here a method for obtaining an effective point-charge wake of a general 3D structure from numerical data provided by an EM solver. We aim to deduce a wake function which is physical and most consistent with the numerical wake potential, to be used in instability simulations with a bunch whose length is comparable to that used in the 3D EM solver. We will describe more specifically our method below for the longitudinal plane. The transverse case follows essentially in the same way.

METHOD AND APPLICATION

Method Basics

As was already mentioned earlier, our underlying objective is to make the best use of 3D EM wakefields numerically obtained with an EM solver in simulations of beam instability. Usually in the instability simulation, particles receive a kick due to the wakefield excited by the bunch. In longitudinal plane this kick would correspond to the relative energy deviation of the particle in the bunch $\Delta\delta$:

$$\Delta\delta = \frac{q}{E/e} V_{\parallel}(s; \rho). \quad (1)$$

here, q is the bunch charge, E is the nominal beam energy, e is the electron charge, and $V_{\parallel}(s; \rho)$ is the wake potential at the relative longitudinal position s of the particle with respect to the synchronous position. The wake potential $V_{\parallel}(s; \rho)$ is given as a convolution of the wakefield $W_{\parallel}(s)$ of a point charge and the bunch distribution function $\rho(s)$:

* galina.skripka@cern.ch

Content from this work may be used under the terms of the CC BY 3.0 licence (© 2018). Any distribution of this work must maintain attribution to the author(s), title of the work, publisher, and DOI.

$$V_{\parallel}(s; \rho) = q \int_0^{\infty} W_{\parallel}(s') \rho(s - s') ds' \quad (2)$$

in which $W(s)$ is zero for $s < 0$ due to causality. While the wake potential is computed by an EM field solver for a given 3D structure, it is only valid for the bunch profile used in the simulations. To remove this dependence on the bunch profile we choose to work with impedance, related to wake potential as follows

$$\int_{-\infty}^{\infty} V_{\parallel}(s; \rho) e^{i\omega s/c} ds/c = qc Z_{\parallel}(\omega) \bar{\rho}(\omega), \quad (3)$$

where ω is the angular frequency, c is the speed of light. $Z_{\parallel}(\omega)$ is the coupling impedance which is the FT of $W_{\parallel}(s)$, and $\bar{\rho}(\omega)$ is the bunch spectrum which is FT of the $\rho(s)$. Mathematically, an inverse FT of $Z_{\parallel}(\omega)$ gives the $W_{\parallel}(s)$. However, there are several reasons why this would not be true under the described procedure:

- wakefields simulated by the EM solver are only those excited by a rigid beam of finite length
- fields are computed numerically solving the Maxwell's equations by discretization of the space with a finite number of grid points

Hence, the more correct nomenclature for the impedance calculated as described above would be $Z_{\parallel}^{\rho}(\omega)$ where ρ indicates dependence on the bunch profile $\rho(s)$.

Here, through a least square fit we decompose a numerically obtained $Z_{\parallel}^{\rho}(\omega)$ into a series of known analytical impedance functions:

$$Z_{\parallel}^{\rho}(\omega) = \sum_k a_k^{\rho} f_k^{\rho}(\omega) \quad (4)$$

where a_k^{ρ} are the decomposition coefficients and $f_k^{\rho}(\omega)$ are the analytical impedance functions. For a geometric impedance these functions $f_k^{\rho}(\omega)$ include for example inductive, capacitive, resistive and resonator impedances for which the corresponding wake functions $g_k^{\rho}(s)$ are well-known and can be found in [2, 3]. The advantage of the above decomposition is that the resultant function becomes "causal", and that the corresponding "effective" wake function is easily accessible:

$$W_{\parallel}^{\rho}(s) = \sum_k a_k^{\rho} g_k^{\rho}(s). \quad (5)$$

In a simulation code, whether in time or frequency domain, it suffices to just once read in the data on the decomposition and construct $W_{\parallel}^{\rho}(s)$ or $Z_{\parallel}^{\rho}(\omega)$. Another useful feature in fitting the impedance is that it allows us to have a better physical interpretation of the impedance characteristics of a given structure, such as the resonances involved, the magnitude of inductive impedance at low frequencies and resistive impedance at high frequencies. We note that for a purely geometric EM field computation that does not take the electric conductivity of the chamber material into account, the

width Q and the height R of a resonance are determined by the length of the integration performed on the s variable. Since the physical quantities such as a loss factor or an instability threshold should depend on the area of the resonance R/Q , the fit should be made on the latter. It is also important to note that the quality of $Z_{\parallel}^{\rho}(\omega)$ calculated by an EM field solver depends critically on the bunch length and the mesh size used. One must therefore bear in mind the limit in frequency only up to which the impedance decomposition makes sense, especially when a fit made at low frequencies becomes incompatible at high frequencies.

As was stated earlier, the obtained effective wake function $W_{\parallel}^{\rho}(s)$ can be used to reconstruct $V_{\parallel}^{\rho}(s; \rho)$ through Eq. 2 and compare it with the original $V_{\parallel}^{EM}(s; \rho)$ calculated by the EM field solver. Besides, $W_{\parallel}^{\rho}(s)$ can also be used to reconstruct a wake potential of a bunch having a different bunch length from the original value and be compared with the output of an EM field solver. Namely, compare reconstructed $V_{\parallel}^{\rho_1}(s; \rho_2)$ with $V_{\parallel}^{EM}(s; \rho_2)$. To improve further the quality of the fit this procedure can be iterated several times. The weight of the fit may be differentiated in the range of s : for example the fit may be concentrated only in the short range seen by the bunch itself if used for the subsequent simulation of single bunch effects.

Wake of a Tapered Bellow

We chose to work with the code GdfidL [6] as it is capable of performing the parallel computation of EM fields in general 3D structures. An example structure is a combination of RF shielded bellows and tapered transitions between round and octagonal cross-sections of a beam-pipe, hereafter, referred to as Structure A (Fig. 1). This structure combines several components typical for many circular machines.

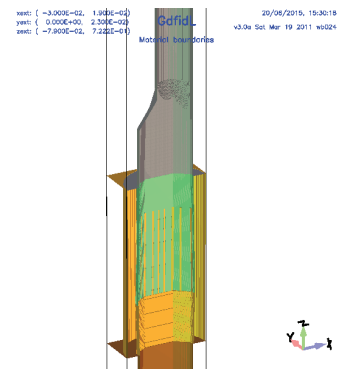


Figure 1: Half of Structure A: taper transition from a round to octagonal beam pipe and a RF-shielded bellow.

The computation of a wake potential was done using a test-bunch of $\sigma = 10$ mm. The longitudinal impedance in this case can be well described by broad- and narrow-band resonators, purely inductive and resistive impedances and the results of the fit are shown in Fig. 2

We proceed with wake potential reconstruction to check if the improvement of the fit is necessary or the present

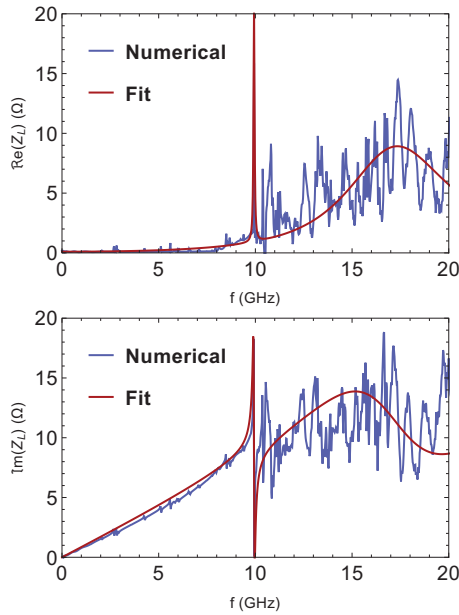


Figure 2: Longitudinal impedance of Structure A: data obtained from the wake potential calculated with a 10 mm bunch using GdfidL (blue) and resonator fit (red).

version is sufficient for reconstruction. The wake potential is reconstructed as follows

$$V_{\parallel}(s) = \int_0^s \rho(s') \sum_{i=1}^N W_{\parallel}^i(s-s') ds' + \int_0^s \rho(s') W_{\parallel}^{off}(s-s') ds'. \quad (6)$$

where the W_{\parallel}^i is the wake function of a i -th resonator and the W_{\parallel}^{off} is the wake-function arising from resistive and inductive components used in the fit.

Figure 3 shows the comparison of wake potentials reconstructed from Eq. 6 and the one calculated with GdfidL. We observe that a rather coarse fit of impedance data shown in Fig. 2 gives a wake function adequate to reconstruct the short range wakefield acting on the bunch itself.

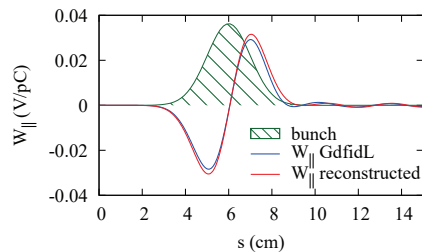


Figure 3: Longitudinal wake potential of Structure A with $\sigma = 10$ mm: wake potentials calculated with GdfidL (blue) and reconstructed from impedance fit (red).

Additional GdfidL computation of Structure A with bunch lengths of $\sigma = 8, 6$ and 4 mm were done and the computed wake potentials were compared to the reconstructed ones

from the same impedance fit (Fig. 4). To quantitatively describe the goodness of obtained results the comparison of loss factors k_{\parallel} is given in Table 1. The loss factors calculated with GdfidL and reconstructed wake potentials are within 10% of each other.

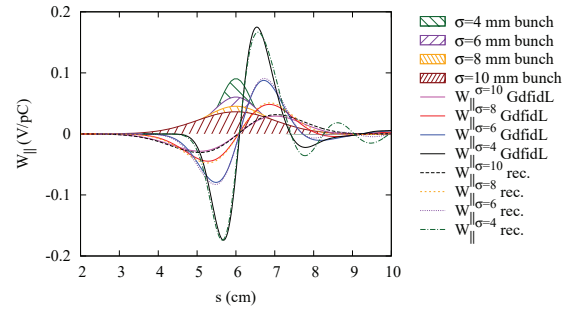


Figure 4: Longitudinal wake potential of Structure A with $\sigma = 10, 8, 6$ and 4 mm: GdfidL data and wake potentials reconstructed from the resonator fit in Fig. 2.

Table 1: Loss Factors for Wake Potentials Calculated with GdfidL and Reconstructed Using the Fit in Fig. 2

Test bunch σ_L	mm	10	8	6	4
k_{\parallel} (GdfidL)	mV/pC	1.1	2.2	6.6	28.6
k_{\parallel} (reconstructed)	mV/pC	1.1	2.4	7.1	28.2

CONCLUSION AND OUTLOOK

The wakefields are an essential part of the beam instability studies, and deducing them for complex 3D geometries is a complicated task, requiring the use of EM solvers with a lot of CPU time. We presented a pragmatic self-consistent method for obtaining an effective wake function for a general 3D structure based on numerical impedance, calculated for realistic vacuum components. The wake functions obtained from decomposition of numerical impedance are known functions that are causal and satisfy Maxwell's equations thus, excluding possible numerical artifacts. Though the form of the deduced wake function is subject to the choice of the quality of the impedance decomposition made by the user, one can improve the precision through several iterations of "impedance fit - wake potential reconstruction" in a self-consistent manner. We have shown that the method works for a standard component and one could repeat the procedure for each component of the ring to obtain the total effective wake function for the whole machine which can be used for subsequent simulations of beam instabilities with a bunch whose length is comparable to that used in the 3D EM solver.

ACKNOWLEDGEMENT

We thank Thomas Günzel for carrying out most of the computations with GdfidL for the MAX IV 3 GeV ring. Authors also thank Pedro F. Tavares for helpful discussions. We are grateful to all of the SOLEIL and MAX IV staff for their support.

REFERENCES

- [1] J. L. Laclare, "Bunched Beam Coherent Instabilities", CAS - CERN Accelerator School : Accelerator Physics, Oxford, UK, pp.264-326, CERN-1987-003-V-1, 16 - 27 Sep 1985.
- [2] A. W. Chao, "Physics of collective beam instabilities in high-energy accelerators", New York, USA: Wiley, 1993, p. 371 (ISBN-9780471551843).
- [3] K. Y. Ng and K. Bane, "Explicit expressions of impedances and wake functions", SLAC-PUB-15078, FERMILAB-FN-0901-APC, 2010.
- [4] K. L. F. Bane, A. Mosnier, A. Novokhatsky and K. Yokoya, "Calculations of the short range longitudinal wakefields in the NLC linac", in *Proc. ICAP '98*, Monterrey, CA, USA, SLAC-PUB-7862).
- [5] K. L. F. Bane, "Short range dipole wakefields in accelerating structures for the NLC", SLAC-PUB-9663, LCC-0116, 2003.
- [6] W. Bruns, The GdfidL Electromagnetic Field Simulator, <http://www.gdfidl.de>
- [7] I. Zagorodnov, "Wakefield Code ECHO", Second Topical Workshop on Instabilities, Impedance and Collective Effects, 2016.
- [8] I. Zagorodnov and T. Weiland, "TE/TM field solver for particle beam simulations without numerical Cherenkov radiation", *Phys. Rev. ST Accel. Beams*, vol. 8, p. 042001, 2005, doi: 10.1103/PhysRevSTAB.8.042001
- [9] CST Computer Simulation Technology AG. <http://www.cst.com>
- [10] T. F. Günzel and F. Perez, "Impedance and Instabilities for the ALBA Storage Ring", in *Proc. EPAC'08*, Genoa, Italy, 2008, paper TUPP037.
- [11] R. Nagaoka, "Numerical Evaluation of Geometric Impedance for SOLEIL", in *Proc. EPAC'04*, Lucerne, Switzerland, 2004, paper WEPLT081.
- [12] T. F. Günzel, "Longitudinal Impedance of the MAX IV 3 GeV Ring", ALBA Report No. AAD-SR-IZ-AN-0177, 2009.
- [13] R. Nagaoka, "Impedance Computation and Measurement in Modern Storage Rings", in *Proc. EPAC'08*, Genoa, Italy, paper FRXBGM01.
- [14] I. Zagorodnov, "Indirect methods for wake potential integration", *Phys. Rev. ST Accel. Beams*, vol. 9, p.102002, 2006, doi: 10.1103/PhysRevSTAB.9.102002