ISBN: 978-3-95450-184-7 doi:10.18429/JACoW-IPAC2018-WEYGBE2

APPLICATIONS OF CAUSTIC METHODS TO LONGITUDINAL PHASE SPACE MANIPULATION

T. K. Charles*, School of Physics, University of Melbourne, Melbourne, Victoria, Australia
also at CERN, Geneva, Switzerland
David Douglas, JLab, Newport News, Virginia,
Peter Williams, STFC/DL/ASTeC, Daresbury, Warrington, Cheshire, UK

Abstract

Longitudinal phase space management is a key feature of recirculating machines. Careful consideration of the longitudinal matching is required not only in order to ensure a high peak current, low energy spread bunch is delivered to the FEL but also to support the deceleration and energy recovery of the spent beam. In a similar manner, longitudinal phase space manipulation can be utilised for pulse shaping in bunch compression, to minimise the influence of CSR-induced emittance growth. In this paper, we present a method for longitudinal phase space matching based upon the avoidance of electron trajectory caustics. Through considering the conditions under which caustics will form, we generate exclusion plots identifying the viable parameter space at numerous positions through beam acceleration and energy recovery. The result is a method for selecting the linear momentum compaction and the higher-order momentum compaction to satisfy the non-caustic condition whilst achieving the bunch compression or lengthening as required.

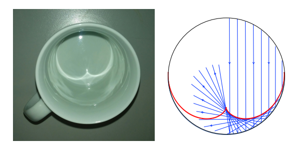
INTRODUCTION

Caustics are commonly recognized and well understood in light optics and electron microscopy. This phenomenon has recently been identified in accelerator physics as the driving term behind current modulations observed in a numerous accelerator physics applications [1].

Under certain conditions, neighboring electron trajectories merge to form caustics, often resulting in characteristic current spikes which are accompanied by folds in the longitudinal phase space distribution.

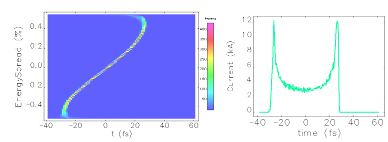
In optics, a commonly cited example of caustics is an ordinary coffee cup in a well–lit room, where bright lines can be seen, forming a cardioid shape (see Fig. 1a). Figure 1b shows how the reflected rays of light coalesce to form an envelope of rays, highlighted by the bold red line which maps out the caustic. At the location of the caustic the intensity of the light is greatly enhanced.

Another example of caustics drawn from optics is the dancing network of light appearing at the bottom of swimming pools on a sunny day [2, 3]. Due to the unintended nature of this caustic focusing, it is often referred to as 'natural focusing' [3]. These examples are analogous to current spikes appearing in many accelerator physics applications, such as the current horns shown in Fig. 2. Instead of rays of light reflecting or refracting, relativistic particle trajectories can be focused or defocused to form an envelope of trajectories associated with current peaks.



(a) (b) .

Figure 1: Optical caustics. (a) photo of caustic lines appearing in a coffee cup. (b) illustration of light rays forming the caustic (red line) [1].



(a) (b)

Figure 2: Folds in the longitudinal phase space distribution (left) and the projected charge density (right), for a bunch after passing through a bunch compressor chicane [4].

CAUSTICS – KEY EQUATIONS

The longitudinal position of the caustics with respect to the center of the bunch, $\tilde{z}$, for a given set of control parameters, R_{56}, T_{566}, and U_{5666} (i.e. the first-, second-, and third order longitudinal dispersion),was derived in [1] as,

$$\tilde{z}(z_i) = z_i - \frac{\delta(z_i)}{\delta'(z_i)} - T_{566}\delta^2(z_i) - 2U_{5666}\delta^3(z_i) \tag{1a}$$

$$\tilde{R_{56}}(z_i) = \frac{-1}{\delta'(z_i)} - 2T_{566}\delta(z_i) - 3U_{5666}\delta^2(z_i). \tag{1b}$$

The boundaries between the regions of parameter space where caustics will and will not form can be calculated using the following expression [5],

$$\begin{aligned} f(R_{56}, &T_{566}, U_{5666}, h_1, h_2, h_3; z_\mathrm{m}) = \\ &1 + h_1 R_{56} + 2h_2 R_{56} z_\mathrm{m} + 3h_3 R_{56} z_\mathrm{m}^2 \\ &+ 2T_{566} h_1^2 z_\mathrm{m} + z_\mathrm{m}^2 (6T_{566} h_1 h_2 + 3h_1^3 U_{5666}), \end{aligned} \tag{2}$$

where z_m are the maximum or minimum values of the initial bunch, h_1, h_2, and h_3 are the first–, second– and third–order chirps respectively. Where $f(R_{56}, T_{566}, U_{5666}, h_1, h_2, h_3\,; z_\mathrm{m}) = 0$, defines the boundaries between the regions of where

* tessa.charles@cern.ch

ISBN: 978-3-95450-184-7 doi:10.18429/JACoW-IPAC2018-WEYGBE2

zero and one caustics are expected, and regions of where one and two caustics are expected. The mathematical description of caustics lies within the broader field of catastrophe theory [2], and so we could also describe Eq. (2) by stating that where $f(R_{56}, T_{566}, U_{5666}, h_1, h_2, h_3 ; z_{\mathrm{m}}) = 0$, defines the boundaries between regions of catastrophes of codimension one or two forming.

APPLICATION TO BUNCH COMPRESSION

One clear example of electron trajectory caustics are the current horns that can appear in bunch compression [1]. These current horns often present themselves in strong bunch compression, such as that used in FEL linacs, where the current horns can cause CSR-induced emittance growth and degrade FEL performance [6].

Consideration of Eq. (2) reveals that there may be conditions under which the caustic current horns cannot form. As mentioned earlier, caustics fit within the field of catastrohpe theory and as such it is not surprising that only slight deviations in the chirp away from linear can result in these strong peaks forming. However also evident in Eq. (1) is the role that T_{566} and U_{5666} play in the formation of the current horns. In fact through deliberate manipulation of T_{566} and U_{5666} conditions can be meet which prevent the current horns from forming. An examples of current horn suppression can be found in references [4, 5]

APPLICATION TO RECIRCULATING MACHINES

Longitudinal phase space management is a key feature of recirculating machines. Careful consideration of the longitudinal matching is required not only in order to ensure a high peak current, low energy spread bunch is delivered to the FEL but also to support the deceleration and energy recovery of the spent beam [7, 8]. Development of this approach and it's applicability to recirculating systems was motivation by the UK-XFEL design project [9], however the discussion below approach is more widely applicable to any system that requires longitudinally bright beams (e.g. FELs and Compton sources). This approach is particularly beneficial for providing a framework for recirculating systems where long sequences of longitudinal phase space manipulations are necessarily required.

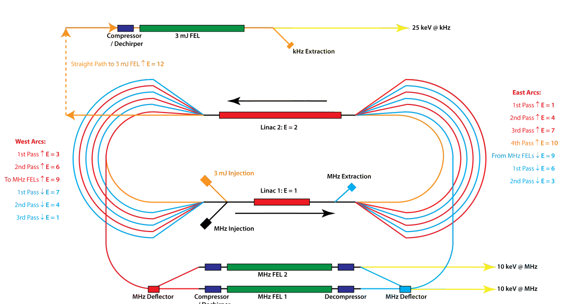


Figure 3: Layout of the UK- FEL design considered.

The UK-XFEL design considered here, involves a multi-pass, recirculating, superconducting CW linac with energy recovery, based on a GERBAL (Generic Energy Recovered Bisected Asymmetric Linac) layout (see Fig. 3). Applying the caustic analysis to this multi-pass recirculating machine, can place restrictions on the viable R_{56}, T_{566}, U_{5666} parameter space, for longitudinal phase space manipulation that ensures a high quality beam is delivered to the user facility.

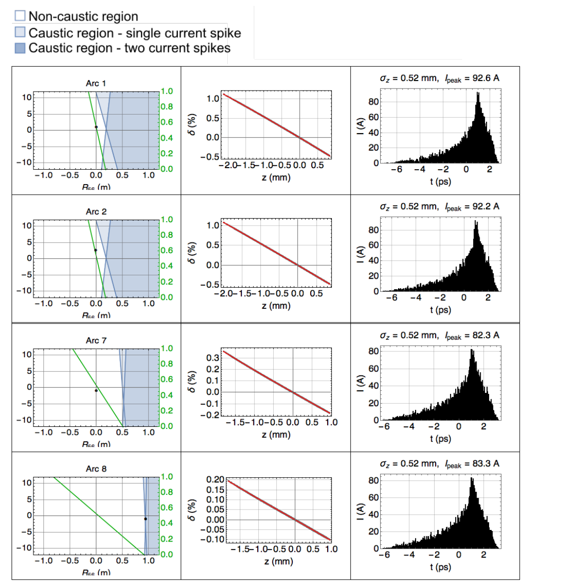


Figure 4: Accelerating arcs. Left column: caustic boundaries and bunch length variation with R_{56}. Middle column: longitudinal phase space distribution of the bunch entering the arc with polynomial fit (red). Right column: corresponding histogram of charge density of distribution.

Longitudinal phase space manipulation is required to: minimise CSR through avoiding current spikes, allow for sufficient bunch compression, reduce final energy spread, and allow for optimum energy recovery.

Removal of the second-order non-linearites of the longitudinal phase space through optical elements at ERL facilities has been demonstrated in [10, 11]. Here we present a method for determining the optics parameters that (as with the previous bunch compressor example), will prevent current spikes from forming, and allow for correction of the non-linear (in z) terms, and ensure a one-to-one mapping of the longitudinal phase space distributions from the injector to the undulator and then again from the undulator to the dump.

Figure 4 shows the caustic boundary condition expressions [Eq. (2)] for 4 of the 8 arcs. The arcs not shown are globally and locally isochronous to at least third-order. Figure 5 shows the equivalent plots for deceleration and energy recovery. The left most column of Figs. 4 and 5 shows the caustic exclusion plots where the white section indicates the non-caustic region of parameter space.

Included in each exclusion plot is the (R_{56}, T_{566}) working point (black point), that will influences the phase space distribution entering the next arc. Also included in both Figs. 4

ISBN: 978-3-95450-184-7 doi:10.18429/JACoW-IPAC2018-WEYGBE2

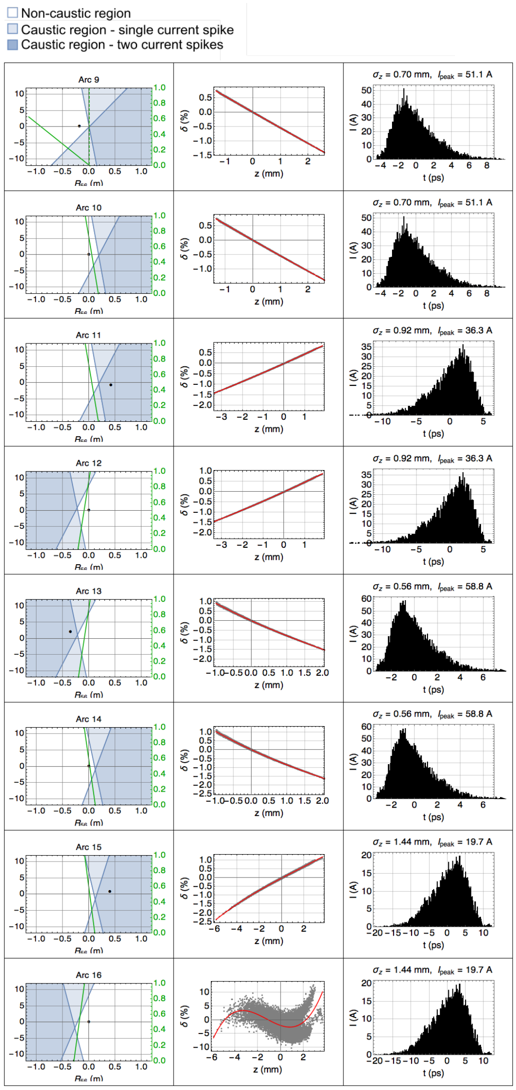


Figure 5: Deccelerating arcs. Left column: caustic boundaries and bunch length variation with R_{56}. Middle column: longitudinal phase space distribution of the bunch entering the arc with polynomial fit (red). Right column: corresponding histogram of charge density of distribution.

and 5 is the RMS bunch length. This is useful for during both acceleration (where bunch compression is necessary) and deceleration (where energy compression is necessary). The middle column is the phase space distribution entering the arc, and in the right column its associated current profile. Note that the current peaks visible in the right-hand column are not caustic in nature. If the working point were in the caustic region, these peaks would be sharper and larger.

In Fig. 5, a number of the arcs show the working point within the caustic region. Here the bunch has been over-compressed, producing caustic current spikes during the transition. Further studies are underway to determine if these parasitic crossing can be avoided whilst still maintaining the RF load balance and still ensure all particles within the bunch have energies of less than 12 MeV at the dump.

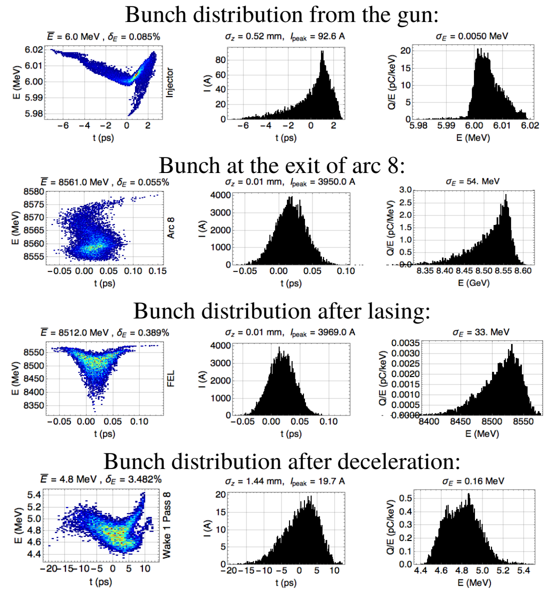


Figure 6: left column: Long. phase space distribution of the bunch after lasing, middle column: corresponding current profile, and right column: corresponding energy spectrum.

The bunch compression that is required to ensure a large current in at the FEL, is saved until the final arc – arc 8. This ensures that the bunch is short and susceptible to CSR in only one of the arcs. As can be seen in the last row of Fig. 4, the working point is chosen for a minimal bunch length that still resides in the non-caustic region. Correction of the second-order linearities are performed at lower energy in arcs 1 and 2 where the beam is less rigid and the longitudinal phase space more malleable.

After lasing, the longitudinal phase space distribution has three tails – two from the injector and one introduced by the lased portion of the bunch where the energy is sheared (see Fig. 6). Despite the shot-to-shot variability of mean energy and energy spread the distribution is mapped in a one-to-one manner onto the dump provided there is sufficient buffer between the working point and caustic boundaries.

OTHER POTENTIAL APPLICATIONS

Other potential applications of this caustic approach include: the unfolding the hooks that often appear in the longitudinal phase space generated in DC guns [12], investigating halo as 'natural focusing', and microbunching which can also be described as a caustic phenomenon [13].

CONCLUSION

Applications of caustic methods to longitudinal phase space manipulation was discussed.

ACKNOWLEDGMENTS

The authors would like to thank and acknowledge Julian McKenzie for supplying the gun distribution. T.K. Charles acknowledges the support of the veski Victoria Fellowship.

ISBN: 978-3-95450-184-7 doi:10.18429/JACoW-IPAC2018-WEYGBE2

REFERENCES

[1] T. K. Charles, D. M. Paganin, and R. T. Dowd, "Caustic-based approach to understanding bunching dynamics and current spike formation in particle bunches," *Phys. Rev. AB*, vol. 19, no. 104402, p. 104402, 2016.

[2] M. V. Berry and C. Upstill, "Catastrophe Optics: Morphologies of Caustics and Their Diffraction Patterns," *Prog. Opt.*, vol. 18, pp. 257–346, 1980.

[3] J. F. Nye, *Natural Focusing and Fine Structure of Light: Caustics and Wave Dislocations*. Taylor & Francis, Philadelphia, 1999.

[4] T. K. Charles, D. M. Paganin, M. J. Boland, and R. T. Dowd, "Singularities of particle trajectory caustics and beam shaping in bunch compressors," *World Sci. Proc. Nonlinear Dyn. Collect. Work.*, 2018.

[5] T. K. Charles, D. M. Paganin, A. Latina, M. J. Boland, and R. T. Dowd, "Current-horn suppression for reduced coherent-synchrotron-radiation- induced emittance growth in strong bunch compression," *Phys. Rev. Accel. Beams*, vol. 20, no. 030705, p. 030705, 2017.

[6] Y. Ding, K. L. F. Bane, W. Colocho, F. Decker, P. Emma, J. Frisch, M. W. Guetg, Z. Huang, R. Iverson, J. Krzywinski, H. Loos, A. Lutman, T. J. Maxwell, H. Nuhn, D. Ratner, J. Turner, J. Welch, and F. Zhou, "Beam shaping to improve the free-electron laser performance at the Linac Coherent Light Source," *Phys. Rev. AB*, vol. 19, no. 100703, 2016.

[7] D. Douglas and E. Al., "Use of Multipass Recirculation and Energy Recovery In CW SRF X-FEL Driver Accelerators," *in Proc. FEL'10, Malmo, Sweden*, no. TUOA4.

[8] C. Tennant and D. Douglas, "Design Concept for a Compact ERL to Drive a VUV/Soft X-Ray FEL," *Proc. PAC'11, New York, NY, Mar.-Apr. 2011*, no. THP187, pp. 2468–2470, 2011.

[9] P. Williams, "A Staged, Multi-User X-Ray Free Electron Laser and Nuclear Physics Facility based on a Multi-Pass Recirculating Superconducting CW Linac," *Futur. Light Source*, 2018.

[10] P. Piot, D. R. Douglas, and G. A. Krafft, "Longitudinal phase space manipulation in energy recovering linac-driven free-electron lasers," *Phys. Rev. ST AB*, vol. 6, no. 3, p. 030702, 2003.

[11] G. H. Hoffstaetter and Y. H. Lau, "Compensation of wakefield-driven energy spread in energy recovery linacs," *Phys. Rev. Spec. Top. - Accel. Beams*, vol. 11, no. 7, pp. 1–9, 2008.

[12] Y. M. Saveliev, F. Jackson, J. K. Jones, and J. W. McKenzie, "Electron bunch structure in energy recovery linac with high-voltage dc photoelectron gun," *Phys. Rev. AB*, vol. 19, no. 094002, p. 094002, 2016.

[13] T. K. Charles, D. M. Paganin, M. J. Boland, and R. T. Dowd, "Microbunching Instability As a Caustic Phenomenon," pp. 18–21, 2017.