The Infrared Fixed Point of the Top Quark Mass and its Implications within the MSSM*

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Abstract

We analyse the general features of the Higgs and supersymmetric particle spectrum associated with the infrared fixed point solution of the top quark mass in the Minimal Supersymmetric Standard Model. We consider the constraints on the mass parameters, which are derived from the condition of a proper radiative electroweak symmetry breaking in the low and moderate $\tan\beta$ regime. In the case of universal soft supersymmetry breaking parameters at the high energy scale, the radiative $SU(2)_L \times U(1)_Y$ breaking, together with the top quark Yukawa fixed point structure imply that, for any given value of the top quark mass, the Higgs and supersymmetric particle spectrum is fully determined as a function of only two supersymmetry breaking parameters. This result is of great interest since the infrared fixed point solution appears as a prediction in many different theoretical frameworks. In particular, in the context of the MSSM with unification of the gauge and bottom–tau Yukawa couplings, for small and moderate values of $\tan\beta$ the value of the top quark mass is very close to its infrared fixed point value. We show that, for the interesting range of top quark mass values $M_t \simeq 175 \pm 10$ GeV, both a light chargino and a light stop may be present in the spectrum. In addition, for a given top quark mass, the infrared fixed point solution of the top quark Yukawa coupling minimizes the value of the lightest CP-even Higgs mass m_h. The resulting upper bounds on m_h read $m_h \leq 90$ (105) (120) GeV for $M_t \leq 165$ (175) (185) GeV.

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1 Introduction

In the present evidence of a heavy top quark, it is of interest to study in greater detail the phenomenological implications of the infrared fixed point predictions for the top quark mass. The low energy fixed point structure of the Renormalization Group (RG) equation of the top quark Yukawa coupling determines the value of the top quark mass independently of the precise symmetry conditions at the high energy scale. This *quasi* infrared fixed point behaviour of the RG solution is present in the Standard Model (SM) [1] as well as in the Minimal Supersymmetric Standard Model (MSSM) [2], [3], and it is associated with large values of the top quark Yukawa coupling, which, however, remain in the range of validity of perturbation theory. Within the MSSM, for a range of high energy values of the top quark Yukawa coupling, such that it can reach its perturbative limit at some scale $M_X = 10^{14}$–10^{19} GeV, the value of the physical top quark mass is focused to be

$$M_t = 190\text{–}210 \text{ GeV } \sin\beta \tag{1}$$

where $\tan\beta = v_2/v_1$ is the ratio of the two Higgs vacuum expectation values. The above variation in M_t is due to a variation in the value of the strong gauge coupling, $\alpha_3(M_Z) = 0.11$–0.13.

The infrared fixed point structure is independent of the supersymmetry breaking scheme under consideration. On the contrary, since the Yukawa couplings – especially if they are strong – affect the running of the mass parameters of the theory, once the infrared fixed point structure is present, it will play a decisive role in the resulting (s)particle spectrum of the theory. In particular, in the low and moderate $\tan\beta$ regime, in which the effects of the bottom and tau Yukawa couplings are negligible, it is possible to determine the evolution of the soft supersymmetry breaking mass parameters of the model as a function of their boundary conditions at high energy scales and the ratio of the top quark Yukawa coupling h_t to its quasi infrared fixed point value h_f [4]–[7], giving definite predictions in the limit $h_t \to h_f$ [8].

In minimal supergravity grand unified models, the soft supersymmetry breaking mass parameters proceed from common given values at the high energy scale. In addition, to assure a proper breakdown of the electroweak symmetry, one needs to impose conditions on the low energy mass parameters appearing in the scalar potential. This yields interesting correlations among the free high energy mass parameters of the theory, which then translate into interesting predictions for the Supersymmetric (SUSY) spectrum [8]–[15].

In the above, we have emphasized the infrared fixed point structure, which determines the value of the top quark mass as a function of $\tan\beta$. There is a small dependence of the infrared fixed point prediction on the supersymmetric spectrum, which, however, comes mainly through the dependence on the spectrum of the running of the strong gauge coupling.

Moreover, considering the MSSM with unification of gauge couplings at a grand unification scale M_{GUT} [16], the value of the strong gauge coupling is determined as a function of the electroweak gauge couplings while its dependence on the SUSY spectrum can be characterized by a single effective threshold scale T_{SUSY} [17]-[18]. Thus, the stronger dependence of the infrared fixed point prediction on the SUSY spectrum can be parametrized through T_{SUSY}. (There is also an independent effect coming from supersymmetric threshold corrections to the Yukawa coupling, which, for supersymmetric particle masses smaller than 1 TeV or of this order, may change the top quark mass predictions in a few GeV, but without changing the physical picture [19]).

The infrared fixed point structure of the top quark mass is interesting in itself, due to the many interesting properties associated with its behaviour. As we shall show below, it gives a highly predictive framework for the Higgs and supersymmetric particle spectrum. Moreover, it has recently been observed in the literature that the condition of bottom–tau Yukawa coupling unification in minimal supersymmetric grand unified theories requires large values of the top quark Yukawa coupling at the unification scale [17]-[18], [20]-[23]. Most appealing, in the low and moderate $\tan\beta$ regime, for values of the gauge couplings compatible with recent predictions from LEP and for the experimentally allowed values of the bottom mass, the conditions of gauge and bottom–tau Yukawa coupling unification predict values of the top quark mass within 10% of its infrared fixed point results [17],[24].

In this talk we shall consider approximate analytical solutions to the one–loop RG equations of the low energy parameters, showing their dependence on the high energy soft supersymmetry breaking mass parameters and the top quark Yukawa coupling and analysing the implications of the infrared fixed point solution in the low and moderate $\tan\beta$ regime. We shall then incorporate the radiative electroweak symmetry breaking condition, to derive approximate analytical correlations among the free, independent high energy parameters of the theory. The analytical results are extremely useful in understanding the properties derived from the full numerical study, in which a two–loop RG evolution of the gauge and Yukawa couplings is considered. In the numerical analysis the evolution of the Higgs and supersymmetric mass parameters are considered at the one–loop level, and the one–loop radiative corrections to the Higgs quartic couplings are taken into account. We shall then concentrate on the infrared fixed point predictions for the Higgs and SUSY spectrum as a function of given values for the top quark mass. We shall also present an analysis of the results obtained in the context of gauge and bottom–tau Yukawa coupling unification, to show the proximity of the top quark mass predictions obtained in this framework to the infrared fixed point top quark mass values as a function of $\tan\beta$. We summarize our results in the last section.

2 Infrared Fixed Point and the Evolution of the Mass Parameters

In the Minimal Supersymmetric Standard Model, with unification of gauge couplings at some high energy scale $M_{GUT} \simeq 10^{16}$ GeV, the infrared fixed point structure of the top quark Yukawa coupling may be easily analysed, in the low and moderate $\tan\beta$ regime, $1 \leq \tan\beta < 10$, considering its analytical one–loop RG solution. As we said before, for such values of $\tan\beta$ the effects of the bottom and tau Yukawa couplings are negligible. For large values of $\tan\beta$, instead, the bottom Yukawa coupling becomes large and, in general, a numerical study of the coupled equations for the couplings becomes necessary even at the one–loop level. There are, however, particular cases for which, for sizeable effects from the bottom and top Yukawa couplings, approximate analytical expressions may still be obtained.

In terms of $Y_t = h_t^2/4\pi$, the one–loop solution in the small and moderate $\tan\beta$ region reads [4],[6]:

$$Y_t(t) = \frac{2\pi Y_t(0)E(t)}{2\pi + 3Y_t(0)F(t)}, \tag{2}$$

with E and F functions of the gauge couplings,

$$E = (1 + \beta_3 t)^{16/3b_3}(1 + \beta_2 t)^{3/b_2}(1 + \beta_1 t)^{13/9b_1}, \qquad F = \int_0^t E(t')dt', \tag{3}$$

where $\beta_i = \alpha_i(0)b_i/4\pi$, b_i is the beta function coefficient of the gauge coupling α_i, and $t = 2\log(M_{GUT}/Q)$ [1]. As we mentioned above, the fixed point solution, $h_f(t)$, is obtained for values of the top quark Yukawa coupling that become large at the grand unification scale, that is, approximately,

$$Y_f(t) \simeq \frac{2\pi E(t)}{3F(t)}, \tag{4}$$

where $Y_f = h_f^2/4\pi$. For values of the grand unification scale $M_{GUT} \simeq 10^{16}$ GeV, the fixed point value, Eq. (4), is given by $Y_f \simeq (8/9)\alpha_3(M_Z)$. Indeed, since $F(Q = M_Z) \simeq 300$, the infrared fixed point solution is rapidly reached for a wide range of values of $Y_t(0) \simeq 0.1$–1. This behaviour is shown in Fig. 1, in which the value of the running top quark mass, $m_t(t) = h_t(t)v_2 = h_t(t)v\sin\beta$, with $v^2 = v_1^2 + v_2^2$, is plotted as a function of the energy scale, for a moderate value of $\tan\beta = 5$. For a wide range of high energy values, the value of $h_t(m_t)$ tends to h_f, implying that the running top quark mass tends to its infrared fixed point value,

$$m_t^{IR}(t) = h_f(t)\ v\ \sin\beta = m_t^{IRmax}(t)\ \sin\beta, \tag{5}$$

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[1]The corresponding solution for the bottom and tau Yukawa couplings in this regime are: $Y_b(t) = Y_b(0)E(t)'/[1+(3/2\pi)Y_t(0)F(t)]^{1/6}$ and $Y_\tau(t) = Y_\tau(0)\tilde{E}(t)$, where E' may be obtained from E by changing the exponent coefficient 13/9 by 7/9, and $\tilde{E}(t)$ can be obtained from $E(t)$ by changing the exponent coefficients 16/3, and 13/9 by 0 and 3, respectively. These expressions are useful only when b–τ Yukawa coupling unification is to be considered.

where for $\alpha_3(M_Z) = 0.11$–0.13, m_t^{IRmax} is approximately given by

$$m_t^{IRmax}(M_t) \simeq 196 \text{ GeV } [1 + 2(\alpha_3(M_Z) - 0.12)]. \tag{6}$$

One should remember that there is a significant quantitative difference between the running top quark mass and the physical one M_t, defined as the location of the pole in its two–point function. The main source of difference comes from the QCD corrections, which at the two–loop level are given by

$$M_t = m_t(M_t) \left[1 + 4\alpha_3(M_t)/3\pi + 11(\alpha_3(M_t)/\pi)^2 \right]. \tag{7}$$

In Fig. 1 we present the result of a two–loop RG analysis, showing the stability of the infrared fixed point under higher order loop contributions.

Fig. 1. Running top quark Yukawa coupling evolution, normalized in order to get the running top quark mass at low energies, $h_t v_2$, for different boundary conditions at an energy scale $Q \simeq 10^{16}$ GeV.

Moreover, using Eq. (4) it follows that:

$$\frac{6Y_t(0)F(t)}{4\pi} = \frac{Y_t(t)/Y_f(t)}{1 - Y_t(t)/Y_f(t)} \ , \tag{8}$$

with Y_t/Y_f the ratio of Yukawa couplings at low energies. The value of the top quark Yukawa coupling at M_{GUT}, $Y_t(0)$, appears in the RG solutions of the soft SUSY breaking parameters, and the above equation permits to express it as a function of the gauge couplings (through F) and the ratio Y_t/Y_f.

A similar analytical study can be done for the large $\tan\beta$ regime when the bottom and top Yukawa couplings are equal at the unification scale. Neglecting in a first approximation the effects of the tau Yukawa coupling and identifying the right-bottom and right-top

hypercharges, the solution for $Y = Y_t \simeq Y_b$ reads,

$$Y(t) = \frac{4\pi Y(0) E(t)}{4\pi + 7 Y(0) F(t)} \ . \tag{9}$$

Then, if the Yukawa coupling is large at the grand unification scale, at energies of the order of the top quark mass it will develop an infrared fixed point value approximately given by

$$Y_f(t)^{(Y_t = Y_b)} \simeq \frac{4\pi E(t)}{7 F(t)} \simeq \frac{6}{7} Y_f(t)^{(low \tan\beta)}. \tag{10}$$

Relaxing the unification condition of the bottom and top Yukawa couplings, but still neglecting in a first approximation the effects of the tau Yukawa coupling and identifying the hypercharges, then, a general approximate analytical expression for Y_b and Y_t may be considered:

$$Y_{t,b}(t) = Y_{t,b}(0) E(t) / \left[1 + (3/2\pi) F(t) (Y_t(0) + Y_b(0)) \right]^{1/6} \left[1 + (3/2\pi) F(t) Y_{t,b}(0) \right]^{5/6} \ ; \tag{11}$$

this goes to the correct limits for $Y_t \gg Y_b$ as well as for $Y_b \gg Y_t$, while for the case $Y_b \simeq Y_t$, it gives the result for the top and bottom quark masses with an error of the order of 2%. If both Yukawa couplings are large at the grand unification scale, unlike the two previous cases, their infrared fixed point expressions depend on the relative values of their boundary conditions at the high energy scale. In fact, the ratio of the top to bottom Yukawa couplings at the infrared fixed point depends on the ratio of their boundary conditions as follows,

$$\frac{Y_t^f}{Y_b^f} = \left(\frac{Y_t^f(0)}{Y_b^f(0)} \right)^{1/6} \ . \tag{12}$$

Using the above relation to replace the dependence of the general solutions, Eq. (11), on the boundary conditions of the Yukawa couplings, we obtain an infrared fixed point contour in the Y_t–Y_b plane,

$$\left[\left(Y_t^f \right)^6 + \left(Y_b^f \right)^6 \right]^{1/6} = \frac{2\pi E}{3 F} \ . \tag{13}$$

In general, in the large $\tan\beta$ region the bottom quark Yukawa coupling becomes strong and plays an important role in the RG analysis. There are also possible large radiative corrections to the bottom quark mass coming from loops of supersymmetric particles, which are strongly dependent on the particular spectrum and are extremely important in the analysis if unification of bottom and tau Yukawa couplings is to be considered. Moreover, in some of the minimal models of grand unification, large $\tan\beta$ values are in conflict with proton decay constraints [25]. In the special case of tau–bottom–top Yukawa coupling unification, the infrared fixed point solution for the top quark mass is not achievable unless a relaxation in the high energy boundary conditions of the mass parameters of the theory is arranged, and it is necessarily associated with a heavy supersymmetric spectrum. The large $\tan\beta$

regime will be analysed in detail at this workshop, in the presentations of U. Sarid [26] and C. Wagner [27]. In the following we shall concentrate on the low and moderate $\tan\beta$ region, which involves interesting phenomenological implications.

We shall now consider that the breakdown of supersymmetry comes through the addition of all possible soft supersymmetry breaking terms. In the framework of minimal supergravity one considers universal soft supersymmetry breaking parameters at the grand unification scale. This includes common soft supersymmetry breaking mass terms m_0 and $M_{1/2}$ for the scalar and gaugino sectors of the theory, respectively, and a common value A_0 (B_0) for all trilinear (bilinear) couplings A_i (B) appearing in the full scalar potential, which are proportional to the trilinear (bilinear) terms in the superpotential. In addition, the supersymmetric Higgs mass parameter μ appearing in the superpotential takes a value μ_0 at the grand unification scale M_{GUT}. Knowing the values of the mass parameters at the unification scale, their low energy values may be specified by their renormalization group evolution [4]–[7], which contains also a dependence on the gauge and Yukawa couplings. In the limit of small $\tan\beta$, $\tan\beta < 10$, the following approximate analytical solutions are obtained [8],

$$m_L^2 \;=\; m_0^2 + 0.52 M_{1/2}^2 \;, \qquad\qquad m_E^2 = m_0^2 + 0.15 M_{1/2}^2 \;,$$

$$m_{Q(1,2)}^2 \;=\; m_0^2 + 7.2 M_{1/2}^2 \;, \qquad\qquad m_{U(1,2)}^2 \simeq m_D^2 \simeq m_0^2 + 6.7 M_{1/2}^2 \;,$$

$$m_Q^2 \;=\; 7.2 M_{1/2}^2 + m_0^2 + \frac{\Delta m^2}{3} \;, \qquad m_U^2 = 6.7 M_{1/2}^2 + m_0^2 + 2\frac{\Delta m^2}{3} \;, \qquad (14)$$

where E, D and U are the right–handed leptons, down–squarks and up–squarks, respectively, L and $Q = (T\;B)^T$ are the lepton and top–bottom left–handed doublets, and m_η^2, with $\eta = E, D, U, L, Q$ are the corresponding soft supersymmetry breaking mass parameters. The subindices (1,2) are to distinguish the first and second generations from the third one, whose mass parameters receive the top quark Yukawa coupling contribution to their renormalization group evolution, singled out in the Δm^2 term:

$$\Delta m^2 \;=\; -\frac{3m_0^2}{2}\frac{Y_t}{Y_f} + 2.3 A_0 M_{1/2}\frac{Y_t}{Y_f}\left(1 - \frac{Y_t}{Y_f}\right)$$

$$-\; \frac{A_0^2}{2}\frac{Y_t}{Y_f}\left(1 - \frac{Y_t}{Y_f}\right) + M_{1/2}^2\left[-7\frac{Y_t}{Y_f} + 3\left(\frac{Y_t}{Y_f}\right)^2\right] \;. \qquad (15)$$

For the Higgs sector, the mass parameters involved are

$$m_{H_1}^2 = m_0^2 + 0.52 M_{1/2}^2 \qquad \text{and} \qquad m_{H_2}^2 = m_{H_1}^2 + \Delta m^2 \;, \qquad (16)$$

which are the soft supersymmetry breaking parts of the mass parameters m_1^2 and m_2^2 appearing in the Higgs scalar potential (see section 3). Moreover, the renormalization group

evolution for the supersymmetric mass parameter μ reads,

$$\mu^2 = 2\mu_0^2 \left(1 - \frac{Y_t}{Y_f}\right)^{1/2}, \tag{17}$$

while the running of the soft supersymmetry breaking bilinear and trilinear couplings gives,

$$B = B_0 - \frac{A_0}{2} \frac{Y_t}{Y_f} + M_{1/2}\left(1.2\frac{Y_t}{Y_f} - 0.6\right) \tag{18}$$

$$A = A_0 \left(1 - \frac{Y_t}{Y_f}\right) - M_{1/2}\left(4.2 - 2.1\frac{Y_t}{Y_f}\right), \tag{19}$$

respectively. Equation (17) shows that the RG evolution of μ, which is a supersymmetry preserving parameter, does not involve any dependence on the soft supersymmetry breaking parameters.

The coefficients characterizing the dependence of the mass parameters on the universal gaugino mass $M_{1/2}$ depend on the exact value of the gauge couplings. In the above, we have taken the values of the coefficients that are obtained for $\alpha_3(M_Z) \simeq 0.12$. The above analytical solutions are sufficiently accurate for the purpose of understanding the properties of the mass parameters in the limit $Y_t \to Y_f$. We shall then confront the results of our analytical study with those obtained from the numerical two–loop analysis.

3 Mass Parameter Correlations from Radiative Electroweak Symmetry Breaking

The solutions for the mass parameters may be strongly constrained by experimental and theoretical restrictions. The experimental contraints come from the present lower bounds on the supersymmetric particle masses. Concerning the theoretical constraints, many of them impose bounds on the allowed space for the soft supersymmetry breaking parameters in model dependent ways to various degrees. The conditions of stability of the effective potential and a proper breaking of the $SU(2)_L \times U(1)_Y$ symmetry are, instead, basic necessary requirements.

The Higgs potential of the Minimal Supersymmetric Standard Model may be written as [3], [28]–[30]

$$
\begin{aligned}
V_{eff} &= m_1^2 H_1^\dagger H_1 + m_2^2 H_2^\dagger H_2 - m_3^2(H_1^T i\tau_2 H_2 + \text{h.c.}) \\
&+ \frac{\lambda_1}{2}\left(H_1^\dagger H_1\right)^2 + \frac{\lambda_2}{2}\left(H_2^\dagger H_2\right)^2 + \lambda_3\left(H_1^\dagger H_1\right)\left(H_2^\dagger H_2\right) + \lambda_4\left|H_2^\dagger i\tau_2 H_1^*\right|^2,
\end{aligned} \tag{20}
$$

with $m_i^2 = \mu^2 + m_{H_i}^2$, $i = 1, 2$, and $m_3^2 = B|\mu|$, and where at scales at which the theory is supersymmetric the running quartic couplings λ_j, with $j = 1$–4, must satisfy the following conditions:

$$\lambda_1 = \lambda_2 = \frac{g_1^2 + g_2^2}{4} = \frac{M_Z^2}{2\,v^2}, \qquad \lambda_3 = \frac{g_2^2 - g_1^2}{4}, \qquad \lambda_4 = -\frac{g_2^2}{2} = \frac{M_W^2}{v^2}. \tag{21}$$

Hence, in order to obtain the low energy values of the quartic couplings, they must be evolved using the appropriate renormalization group equations, as was explained in Refs. [28]–[31]. The mass parameters m_i^2, with $i = 1$–3 must also be evolved in a consistent way and their RG equations may be found in the literature [4]–[6], [32],[33]. The minimization conditions $\partial V/\partial H_i|_{<H_i>=v_i} = 0$, which are necessary to impose the proper breakdown of the electroweak symmetry, read

$$\sin(2\beta) = \frac{2m_3^2}{m_A^2} \tag{22}$$

$$\tan^2 \beta = \frac{m_1^2 + \lambda_2 v^2 + (\lambda_1 - \lambda_2)\,v_1^2}{m_2^2 + \lambda_2 v^2}, \tag{23}$$

where m_A is the CP-odd Higgs mass,

$$m_A^2 = m_1^2 + m_2^2 + \lambda_1 v_1^2 + \lambda_2 v_2^2 + (\lambda_3 + \lambda_4)\,v^2. \tag{24}$$

Considering the one–loop leading order contribution to the running of the quartic couplings, which, in the limit of stop mass degeneracy, transforms λ_2 into $\lambda_2 + \Delta\lambda_2$, with $\Delta\lambda_2 = (3/8\pi^2)h_t^4 \ln(m_{\tilde t}^2/m_t^2)$, the minimization condition Eq. (23) can be written as:

$$\tan^2 \beta = \frac{m_1^2 + M_Z^2/2}{m_2^2 + M_Z^2/2 + \Delta\lambda_2 v_2^2}. \tag{25}$$

Therefore, using Eq. (25) and considering the approximate analytical expressions for the mass parameters m_i, Eq. (16), the supersymmetric mass parameter μ is determined as a function of five parameters:

$$\mu^2 = \mathcal{F}(m_0, M_{1/2}, A_0, \tan\beta, Y_t/Y_f). \tag{26}$$

Furthermore, the ratio of the top quark Yukawa coupling to its infrared fixed point value may be expressed as a function of the top quark mass and the angle β,

$$\frac{Y_t}{Y_f} = \left(\frac{m_t}{m_t^{IRmax}}\right)^2 \frac{1}{\sin^2\beta}, \tag{27}$$

where the exact value of m_t^{IRmax}, Eq. (6), depends on the value of the strong gauge coupling considered and, for the experimentally allowed range, varies approximately between 190 and 200 GeV. Depending on the precise value of the running top quark mass m_t and $\tan\beta$, the above equation gives a measure of the proximity to the infrared fixed point solution.

The other minimization condition, Eq. (22), depends on the soft supersymmetry breaking parameter B and, hence, on its boundary condition B_0. Both minimization conditions put restrictions on the soft supersymmetry breaking parameters. However, B (and thus B_0) is not involved in the renormalization group evolution of the (s)particle masses, implying that Eq. (22) is not relevant in defining the range of possible mass values of the Higgs and supersymmetric particle spectra.

Considering the relation between the physical and the running top quark mass, Eq. (7), for a given value of the physical top quark mass, the running top quark mass is fixed and then Eq. (27) fixes the ratio Y_t/Y_f as a function of $\sin\beta$. Then, the correlation implied by the minimization condition, Eqs. (25) and (26), determines the Higgs and supersymmetric spectrum as a function of four parameters. However, if one is at the infrared fixed point, $Y_t \to Y_f$, the model becomes much more predictive. This is partially due to the strong correlation between the top quark mass and the value of $\tan\beta$, Eq. (5), which allows a reduction by one of the number of free parameters. Moreover, there is an additional reduction by one in the number of effective free parameters, which follows from the infrared fixed point structure of the theory. Indeed, the expressions for the low energy parameters, Eqs. (14)–(19), show important properties of the solution when $Y_t \to Y_f$ [8]:

a) The term Δm^2, and hence the mass parameters $m_{H_2}^2$, m_Q^2 and m_U^2, become very weakly dependent on the supersymmetry breaking parameter A_0. In fact, the dependence on A_0 vanishes in the formal limit $Y_t \to Y_f$. The only relevant dependence on A_0 enters through the mass parameter m_3^2, that is to say, through B. This leads to property (b).

b) There is an effective reduction in the number of free, independent, soft supersymmetry breaking parameters. In fact, the dependence on B_0 and A_0 of the low energy solutions is effectively replaced by a dependence on the parameter

$$\delta = B_0 - \frac{A_0}{2}. \tag{28}$$

c) There is also a very interesting dependence of the low energy mass parameters on m_0. For example, the m_0 dependence of the combination $m_Q^2 + m_{H_2}^2$ vanishes in the formal limit $Y_t \to Y_f$. Moreover, the right stop mass m_U^2 becomes itself independent of m_0^2 in this limit, a property that is very important for the analysis of the bounds on the stop sector.

From properties (a) and (b) it follows that, at the infrared fixed point, the dependence of the Higgs and supersymmetric spectrum on the parameter A_0 is negligible. Indeed, considering only the one–loop leading order radiative corrections to the quartic couplings, the minimization condition at the infrared fixed point reads,

$$\mu^2 + \frac{M_Z^2}{2} = \left[m_0^2 \left(1 + 0.5\tan^2\beta \right) + M_{1/2}^2 \left(0.5 + 3.5\tan^2\beta \right) - \omega_t\, 0.5\tan^2\beta \right] \frac{1}{\tan^2\beta - 1}, \tag{29}$$

where we define $\omega_t = 2\Delta\lambda_2 v_2^2$, which depends only logarithmically on m_0 and $M_{1/2}$. In the limit $\tan\beta \to 1$, one has $\mu^2 \gg m_0^2, M_{1/2}^2$.

Hence, due to the independence of the spectrum on the parameter A_0 and the strong correlation of the top quark mass with $\tan\beta$, for a given top quark mass the Higgs and supersymmetric particle spectrum is completely determined as a function of only two parameters, m_0 and $M_{1/2}$. It is now possible to perform a scanning of all the possible values for m_0 and $M_{1/2}$, bounding the squark masses to be, for example, below 1 TeV, and the whole allowed parameter space for the Higgs and superparticle masses may be studied.

3.1 Colour–breaking Minima

There are several conditions that need to be fulfilled to ensure the stability of the electroweak symmetry breaking vacuum. In particular, one should check that no charge– or colour– breaking minima are induced at low energies. A well–known condition for the absence of colour–breaking minima is given by the relation [34]

$$A_t^2 \leq 3(m_Q^2 + m_U^2 + m_{H_2}^2) + 3\mu^2. \tag{30}$$

At the fixed point, however, since $A_t \simeq -2.1 M_{1/2}$ and $m_Q^2 + m_U^2 + m_{H_2}^2 \simeq 6M_{1/2}^2$, this relation is trivially fulfilled (see also Ref. [35]).

For values of $\tan\beta$ close to 1, large values of μ are induced, and a more appropriate relation is obtained by looking for possible colour–breaking minima in the direction $\langle H_2 \rangle \simeq \langle H_1 \rangle$ and $\langle Q \rangle \simeq \langle U \rangle$. The requirement of stability of the physically acceptable vacuum implies the following sufficient condition

$$(A_t - \mu)^2 \leq 2\left(m_Q^2 + m_U^2\right) + \tilde{m}_{12}^2 \,, \tag{31}$$

where $\tilde{m}_{12}^2 = (m_1^2 + m_2^2)(\tan\beta - 1)^2/(\tan^2\beta + 1)$.

If Eq. (31) is not fulfilled, a second sufficient condition is given by

$$\left[(A_t - \mu)^2 - 2\left(m_Q^2 + m_U^2\right) - \tilde{m}_{12}^2\right]^2 \leq 8\left(m_Q^2 + m_U^2\right)\tilde{m}_{12}^2. \tag{32}$$

The above relations, Eqs. (31) and (32), are sufficient conditions since they assure that a colour–breaking minimum lower than the trivial minimum does not develop in the theory. If the above conditions are violated, a necessary condition to avoid the existence of a colour– breaking minimum lower than the physically acceptable one is given by

$$V_{col} \geq V_{ph} \,, \tag{33}$$

with

$$V_{col} = \frac{(A_t - \mu)^2 \alpha_{min}^2}{h_t^2 (2\alpha_{min}^2 + 1)^3}\left[(m_Q^2 + m_U^2) - 2\tilde{m}_{12}^2 \alpha_{min}^4\right], \tag{34}$$

$$V_{ph} = -\frac{M_Z^4}{2(g_1^2 + g_2^2)} \cos^2(2\beta), \tag{35}$$

and

$$\alpha_{min}^2 = \left[(A_t - \mu)^2 - 2(m_Q^2 + m_U^2) - \tilde{m}_{12}^2 \right] / (4\tilde{m}_{12}^2). \tag{36}$$

The above conditions impose strong constraints on the possible fixed point solutions, particularly for low values of $\tan\beta$, for which the value of μ rapidly increases. Therefore, they have implications in determining the Higgs and supersymmetric particle mass predictions of the model.

4 Higgs and Supersymmetric Particle Spectrum

The infrared fixed point solution yields, as we said before, a quite predictive framework for the Higgs and supersymmetric spectrum of the MSSM. Indeed most of the properties of the masses may be understood analytically through their dependence on the mass parameters m_0 and $M_{1/2}$, which govern their behaviour. Experimental bounds on the sparticle masses, as well as theoretical restrictions to avoid inconsistencies in the predicted spectrum, are useful in constraining the allowed values of the defining parameters m_0 and $M_{1/2}$. As a matter of fact, the only two sectors of the theory that one should be particularly careful about, at the infrared fixed point, are those related to the Higgs and the stop.

Let us first summarize the results for the relevant low energy mass parameters at the fixed point solution:

$$
\begin{aligned}
m_{H_2}^2 &\simeq -0.5m_0^2 - 3.5M_{1/2}^2 , & m_{H_1}^2 &\simeq m_0^2 + 0.5M_{1/2}^2 , \\
m_Q^2 &\simeq 0.5m_0^2 + 6M_{1/2}^2 , & m_U^2 &\simeq 4M_{1/2}^2 , \\
A_t &\simeq -2.1M_{1/2} , \\
\mu^2 &\simeq \left[m_0^2 \left(1 + 0.5\tan^2\beta \right) + M_{1/2}^2 \left(0.5 + 3.5\tan^2\beta \right) \right] \frac{1}{\tan^2\beta - 1} .
\end{aligned}
\tag{37}
$$

As we mentioned before, since the whole spectrum may be given as a function of $\tan\beta$, μ and the soft supersymmetry breaking parameters, for low values of $\tan\beta$ and for a given value of the top quark mass, it is completely determined as a function of two free independent parameters, which we take to be m_0 and $M_{1/2}$.

4.1 Stop Sector

We shall first analyse the stop sector, considering the stop mass matrix given by

$$M_{\tilde{t}}^2 = \left[\begin{array}{cc} m_Q^2 + m_t^2 + D_{t_L} & m_t(A_t - \mu/\tan\beta) \\ m_t(A_t - \mu/\tan\beta) & m_U^2 + m_t^2 + D_{t_R} \end{array} \right] , \tag{38}$$

where $D_{t_L} \simeq -0.35 M_Z^2 |\cos 2\beta|$ and $D_{t_R} \simeq -0.15 M_Z^2 |\cos 2\beta|$ are the D-term contributions to the left– and right– handed stops, respectively. The above mass matrix, after diagonalization, leads to the two stop mass eigenvalues, $m_{\tilde{t}_1}$ and $m_{\tilde{t}_2}$. To avoid a tachyon in the theory, it is necessary to require $\det M_{\tilde{t}}^2 \geq 0$. At the infrared fixed point, the values of the parameters involved in the mass matrix are given in Eq. (37). Already from an analytical study it is possible to conclude that, for values of $\tan\beta$ close to 1, the off-diagonal term contribution will be enhanced, due to the large values of μ associated with such low values of $\tan\beta$ and, consequently, the mixing may be sufficiently large to yield a tachyonic solution. Thus, depending on the hierarchy between m_0 and $M_{1/2}$ and on the sign of μ, important constraints on the parameter space may be obtained. For example, for $\tan\beta = 1.2$, which implies $M_t \simeq 160$ GeV and for which the value of the supersymmetric mass parameter $\mu^2 \simeq 4m_0^2 + 12M_{1/2}^2$, it is straightforward to show that, if one considers the regime $M_{1/2}^2 \ll m_0^2$, then for both signs of μ a tachyon state will develop unless $M_{1/2} \geq 0.9\ m_t$. If, instead, one considers the regime $M_{1/2}^2 \gg m_0^2$, then for $\mu > 0$ it follows that in order to avoid a tachyon it is necessary to require $M_{1/2} \geq 1.2\ m_t$. For negative values of μ, since there is a partial cancellation of the off-diagonal term, which suppresses the mixing, no tachyonic solution may develop and, hence, no constraint is derived. However, as we shall show below, restrictions coming from the Higgs sector will constrain this region of parameter space as well. Observe that for these low values of $\tan\beta$, the necessary and sufficient conditions to avoid colour–breaking minima, Eqs. (31), (32) and (33), put strong restrictions on the solutions with large left–right stop mixing.

For slightly larger values of $\tan\beta \simeq 1.8$, which correspond to much larger values of the top quark mass, $M_t \simeq 180$ GeV, the value of $\mu \simeq 1.2m_0^2 + 5.3M_{1/2}^2$ is sufficiently small so that, helped by the factor $1/\tan\beta$, there is no possibility for a tachyon to develop in this case and, hence, no constraints on $M_{1/2}$ are obtained. Of course, this result holds for larger values of $\tan\beta$ as well. Is is interesting to notice that, although there is no necessity to be concerned about tachyons for values of $\tan\beta \simeq 1.8$, it is still possible to have light stops, close to the experimental bound, if the value of $M_{1/2} \leq 100$ GeV. Figure 2 gives the value of the lightest stop quark mass as a function of the gluino mass, $M_{\tilde{g}} \simeq 3M_{1/2}$, while considering various values of $\tan\beta$ and M_t close to the infrared fixed point solution, and it shows the results obtained from the full numerical study [8]. The dots in Fig. 2 denote solutions forbidden by experimental bounds. In particular, for $\tan\beta = 1.2$ the restrictions on $M_{\tilde{g}}$ come through bounds on $M_{1/2}$ derived from the fulfilment of the lower bounds on the lightest CP-even Higgs mass m_h (see below). For larger values of $\tan\beta$ a light stop is no longer possible. Indeed, for $\tan\beta \geq 5$ the stop mass becomes heavier than the top mass, $M_t \simeq 200$ GeV, for most of the parameter space and the experimental bound on the gluino mass, taken as $M_{\tilde{g}} > 120$ GeV, becomes an important constraint for the solutions.

Fig. 2. Running gluino mass $M_{\tilde{g}}$ as a function of the lightest stop mass $m_{\tilde{t}}$, for different values of the top quark mass at the infrared fixed point solution: A) $M_t = 160$ GeV, $\tan\beta = 1.2$; B) $M_t = 180$ GeV, $\tan\beta = 1.8$; C) $M_t = 202$ GeV, $\tan\beta = 5$; D) $M_t = 205$ GeV, $\tan\beta = 10$. Crosses (dots) denote solutions allowed (excluded) experimentally.

4.2   Higgs Spectrum

Other important features of the spectrum at the infrared fixed point are associated with the Higgs sector. The Higgs spectrum is composed by three neutral scalar states, two CP-even, h and H, and one CP-odd, A, and two charged scalar states $H^\pm$. Considering the one–loop leading order corrections to the running of the quartic couplings –those proportional to m_t^4– and neglecting in a first approximation the squark mixing, the masses of the scalar states are given by

$$
m_{h,H}^2 = \frac{1}{2}\Big[m_A^2 + M_Z^2 + \omega_t
$$
$$
\pm\sqrt{(m_A^2 + M_Z^2)^2 + \omega_t^2 - 4m_A^2 M_Z^2 \cos^2(2\beta) + 2\omega_t \cos(2\beta)\left(m_A^2 - M_Z^2\right)}\Big] \tag{39}
$$

$$
m_A^2 + M_Z^2 = m_1^2 + m_2^2 + M_Z^2 + \frac{\omega_t}{2} = \left[\frac{3}{2}m_0^2 + 4M_{1/2}^2 - \frac{\omega_t}{2}\right]\frac{(1 + \tan^2\beta)}{(\tan^2\beta - 1)} \tag{40}
$$

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$$m_{H^\pm}^2 \quad = \quad m_A^2 + M_W^2 \; . \tag{41}$$

In the above, we have omitted the one–loop contributions proportional to ω_t/m_t^2, since for $\tan\beta > 1$ they are negligible with respect to the other contributions. Indeed, the radiative corrections to the Higgs mass become relevant only for large values of the heaviest stop mass, $m_{\tilde{t}}^2 \gg M_Z^2$. To obtain these stop mass values we need moderate values of the soft supersymmetry breaking parameters, which for low values of $\tan\beta \leq 2$, induce large values of the CP-odd mass, $m_A^2 \gg M_Z^2$. In this case, the ω_t contribution is of order M_Z^2 and, hence, it gives a decisive contribution to m_h in Eq. (39), but does not give a relevant contribution to m_A, Eq. (40). If $m_A^2 \gg M_Z^2$, then m_H and $m_{H^\pm}$ are also large, of the order of the CP-odd mass. If, instead, $m_A^2 = \mathcal{O}(M_Z^2)$, then m_0^2 and $M_{1/2}^2$ are also small and, due to the logarithmic dependence of ω_t on these two parameters, its contribution is small, both in Eq. (39) and in Eq. (40).

For the lightest CP-even mass a finite upper bound on its value, m_h^{max}, is reached in the limit of very large values of the CP-odd mass, $m_A^2 \gg M_Z^2$,

$$(m_h^{max})^2 = M_Z^2 \cos^2(2\beta) + \frac{3}{4\pi^2}\frac{m_t^4}{v^2}\left[\ln\left(\frac{m_{\tilde{t}_1}m_{\tilde{t}_2}}{m_t^2}\right) + \Delta_{\theta_{\tilde{t}}}\right] \; . \tag{42}$$

In the above, we have now considered the expression in the case of non-negligible squark mixing [35]–[37]; $\Delta_{\theta_{\tilde{t}}}$ is a function of the left–right mixing angle in the stop sector, and it vanishes in the limit in which the two mass eigenstates are equal, $m_{\tilde{t}_1} = m_{\tilde{t}_2}$. From Eq. (40) it follows that, for lower values of $\tan\beta$, the value of the CP-odd eigenstate mass is enhanced. This means that in such a case the expression for the lightest Higgs mass is given by Eq. (42), and it is independent of the exact value of the CP-odd mass. The fact that values of $\tan\beta$ close to 1 yield larger values of m_A, implies as well that the charged Higgs and the heaviest CP-even Higgs will become heavier in such regime.

Furthermore, the infrared fixed point solution for the top quark mass has explicit important implications for the lightest Higgs mass. For a given value of the physical top quark mass, the infrared fixed point solution is associated with the minimun value of $\tan\beta$ compatible with the perturbative consistency of the theory. For values of $\tan\beta \geq 1$, lower values of $\tan\beta$ correspond to lower values of the tree level lightest CP-even mass, $m_h^{tree} = M_Z|\cos 2\beta|$. Therefore, the infrared fixed point solution minimizes the tree level contribution and after the inclusion of the radiative corrections it still gives the lowest possible value of m_h for a fixed value of M_t [8], [23], [38]. This property is very appealing, in particular, in relation to future Higgs searches at LEP2, as we shall show explicitly below.

Due to the specific dependence of the lightest Higgs mass with $\tan\beta$, it occurs that, for values of $\tan\beta$ close to 1, restrictions on the allowed high energy parameter space and, hence, on the spectrum, may be derived by the requirement that m_h is above its experimental bound. Indeed, if $\tan\beta \simeq 1.2$ ($|\cos 2\beta| \simeq 0.2$), the tree level value is very small and, in order

to satisfy the experimental constraint on m_h, it is necessary to impose a bound on the radiative correction contribution. One may choose to push m_0 to large values, but this will induce a tachyon in the stop spectrum unless $M_{1/2} \geq m_t$. If, instead, one keeps moderate values of m_0, values of $M_{1/2} > 100$ GeV are needed to generate the appropriate radiative corrections. Summarizing, for values of $\tan\beta$ close to one, $M_t \leq 160$ GeV ($\tan\beta \leq 1.2$), to avoid conflicts in the Higgs and stop sectors one needs

$$
\begin{aligned}
M_{1/2} &> m_t & \text{if} \quad \mu &> 0 \\
M_{1/2} &\geq 100\,\text{GeV} & \text{if} \quad \mu &< 0.
\end{aligned}
\tag{43}
$$

For larger values of $\tan\beta \simeq 1.8$, the Higgs sector constraints are still important, although they do not lead to a lower bound on $M_{1/2}$ independent of the experimental bounds on the gaugino sectors. In this case, one has $m_A^2 \simeq 8M_{1/2}^2 + 3m_0^2$ and for values of the defining parameters consistent with the experimental constraints in the gaugino and slepton sectors, the CP-odd mass is still sufficiently large, so that the lightest CP-even mass is given by its upper bound, Eq. (42). Then, $m_h^{tree} \simeq 50$ GeV and if $m_0 \geq m_t \simeq 170$ GeV, no bounds on $M_{1/2}$ are obtained from the experimental constraint on m_h. In Fig. 3 we show the m_A–m_h plane for various values of $\tan\beta$ and M_t extremely close to the infrared fixed point, as derived from the full numerical study [8]. The results from Fig. 3 are in perfect agreement with the behaviour described above. Moreover, it follows that for values of $M_t \leq 180$ GeV the lightest Higgs mass is expected to be in the 50–100 GeV range, while for larger values of $M_t \simeq 200$ GeV it is mostly larger than 100 GeV with a range $m_h \simeq 125 \pm 25$ GeV. In general, for larger values of $\tan\beta$ the tree level value becomes larger and the experimental bounds on gauginos and gluinos also contribute to push the lightest Higgs mass to larger values.

All the above analysis is done under the assumption of being at the infrared fixed point of the top quark mass. However, it is also interesting to observe how the predictions for the lightest Higgs mass are altered if one considers a departure from the infrared fixed point solution. As we said before, in this case a fixed value of the top quark mass may be considered and still the value of $\tan\beta$ may vary, implying in each case a different degree of departure from the infrared fixed point solution. In Fig. 4 we show the value of the lightest Higgs mass as a function of $\tan\beta$, performing a scanning over all possible values of m_0 and $M_{1/2}$ for a top quark mass $M_t = 175$ GeV, so that the squark masses have an upper bound of 1 TeV. (In this plot we have considered the Higgs mass value obtained from the one–loop effective potential computation, Eq. (42). The upper bound obtained within this approach differs by approximately 5 GeV from the one obtained through the RG procedure in which the squark mixing is directly considered through the matching conditions for the quartic couplings, as done in Fig. 3. These results show the degree of uncertainty in the Higgs mass computation [35].)

Fig. 3. The same as Fig. 2, but for the CP-odd Higgs mass m_A vs. the lightest CP-even Higgs mass m_h.

For each value of the top quark mass, the lowest possible value of $\tan\beta$ is associated with the infrared fixed point value. The larger values of $\tan\beta$, for which the solutions are increasingly away from the infrared fixed point, show larger values for the lightest Higgs mass, which, however, become stagnant for values of $\tan\beta$ close to 10. Away from the infrared fixed point solution a scanning over A_0 is also done. The most remarkable feature, for solutions that depart from the infrared fixed point, is that not only the upper bound on m_h becomes larger, but the whole set of solutions lies in a region of the parameter space that renders a lightest Higgs, which is predominantly out of the reach of LEP2. On the contrary, the predictions from the infrared fixed point solution are very appealing in this respect, since there are good chances that, for values of the top quark mass experimentally favoured at present, $M_t = 174 \pm 16$ GeV [40], the lightest Higgs may be within the reach of LEP2.

Fig. 4. Lightest CP-even Higgs mass for different values of $\tan\beta$. The lowest value of $\tan\beta$ (crosses) corresponds to the infrared fixed point solutions for the considered value of the top quark mass, $M_t = 175$ GeV.

4.3 Chargino and neutralino spectrum

From the restrictions on $M_{1/2}$ that follow from the analysis of the Higgs and stop sectors at the infrared fixed point of the top quark mass, a very interesting result can be obtained. Indeed, due to the large values of the mass parameter μ in this framework, there is small mixing in the chargino and neutralino sectors. Hence, to a good approximation the lightest chargino mass and the lightest and next–to–lightest neutralino masses are given by $m_{\tilde{\chi}_l^\pm} \simeq m_{\tilde{\chi}_2^0} \simeq 2m_{\tilde{\chi}_1^0} \simeq 0.8 M_{1/2}$. For values of the top quark mass $M_t \leq 160$ GeV, light charginos, with masses very close to its present experimental bounds, are forbidden due to the lower bounds on the gaugino masses, Eq. (43). Quite generally, we obtain $m_{\tilde{\chi}_l^\pm} > 70$ GeV in this case. On the contrary, due to the large mixing in the stop sector, small values of the lightest stop mass, $m_{\tilde{t}_1} \leq 150$ GeV, may be easily achieved (see Fig. 2). For values of $M_t \geq 185$ GeV, the situation is basically reversed. As can be observed in Fig. 2, light stops are harder to obtain, due to the reduced mixing, while light charginos are possible, since there is no constraint on $M_{1/2}$ either from the stop or from the Higgs sector. Most interesting, just for the phenomenologically preferred region, 165 GeV $\leq M_t \leq 185$ GeV,

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both the charginos and the stops may become light.

Fig. 5. Running lightest stop mass as a function of the lightest chargino mass for a top quark mass $M_t = 175$ GeV at the infrared fixed point solution.

Figure 5 shows the correlation between the lightest chargino mass and the lightest stop mass, for the infrared fixed point solution, for a value of the top quark mass $M_t = 175$ GeV. Light stops and charginos are very interesting, both for direct experimental searches and for indirect searches through deviations from the Standard Model predictions for the leptonic and hadronic variables measured at LEP.

5   Unification of Couplings and the Infrared Fixed Point

In the above, we have assumed the infrared fixed point solution for the top quark mass, and we have analysed its implications in the MSSM under the general requirement of unification of the gauge couplings at some high energy scale $M_{GUT} = \mathcal{O}(10^{16})$ GeV and considering universal conditions for the soft supersymmetry breaking parameters at the grand unification scale. It is now interesting to investigate which physical scenarios may predict the infrared fixed point solution for the top quark mass. One possibility would be the onset of non–perturbative physics at scales of the order of M_{GUT}, as it occurs for example in the supersymmetric extension of the so–called top condensate models [3]. In this case an

analysis of the non–perturbative effects would be necessary before considering the precise unification conditions. Other possibility would be perturbative grand unification with the large values of the top quark Yukawa coupling at the unification scale necessary to induce its infrared fixed point behaviour, followed by the onset of non-perturbative physics for scales just above M_{GUT}. Moreover, the infrared fixed point solution also appears as a prediction in some interesting class of string theories, where the stability of the cosmological constant against corrections of order $M_{SUSY}^2 M_P^2$ is ensured [39]. Another option, which may be the most appealing case to treat, is to have a perturbative theory up to the Planck scale, but with large values of the top quark Yukawa coupling –close to its perturbative limit– and with the onset of new physics above the unification scale. In this context it is possible to consider grand unified models with an SU(5) or SO(10) symmetry, which include also unification of Yukawa couplings. In particular, as we are going to show, the unification of bottom and tau Yukawa couplings at the high energy scale yields a very interesting framework, which naturally renders large values of the top quark Yukawa coupling at M_{GUT}.

The condition of gauge coupling unification in itself gives predictions for the strong gauge coupling as a function of the electroweak gauge couplings. Considering a two–loop RG analysis, it is necessary to include the supersymmetric threshold corrections at one–loop, to take into account the decoupling of the different supersymmetric particles above M_Z. These supersymmetric threshold corrections may be parametrized in terms of a single effective scale T_{SUSY} [17], which, in the limit of common characteristic values for the masses of electroweak gauginos, $m_{\tilde{w}}$, gluinos, $m_{\tilde{g}}$, sleptons, $m_{\tilde{l}}$, squarks, $m_{\tilde{q}}$, Higgsinos, $m_{\tilde{H}}$, and the heavy Higgs doublet, m_H, is given by [18]

$$T_{SUSY} = m_{\tilde{H}} \left(\frac{m_{\tilde{w}}}{m_{\tilde{g}}} \right)^{28/19} \left[\left(\frac{m_{\tilde{l}}}{m_{\tilde{q}}} \right)^{3/19} \left(\frac{m_H}{m_{\tilde{H}}} \right)^{3/19} \left(\frac{m_{\tilde{w}}}{m_{\tilde{H}}} \right)^{4/19} \right] . \qquad (44)$$

The above equation shows that the main contribution to the supersymmetric threshold corrections comes from the gaugino and Higgsino sectors. For the models under study, in which a common gaugino mass $M_{1/2}$ at M_{GUT} is assumed and in the case of large values of μ for which the mixing in the gaugino–Higgsino sector is negligible, Eq. (44) reads

$$T_{SUSY} \simeq |\mu| \left(\frac{\alpha_2(M_Z)}{\alpha_3(M_Z)} \right)^{3/2} \simeq \frac{|\mu|}{6} . \qquad (45)$$

The strong gauge coupling at M_Z can then be computed as follows [17], [18]

$$\frac{1}{\alpha_3(M_Z)} = \frac{1}{\alpha_3^{SUSY}(M_Z)} + \frac{19}{28\pi} \ln \left(\frac{T_{SUSY}}{M_Z} \right) , \qquad (46)$$

where $1/\alpha_3^{SUSY}(M_Z)$ would be the value of the strong gauge coupling coming from the two–loop RG running if the theory were supersymmetric all the way down to M_Z. The effective scale T_{SUSY} is quite useful, since it permits to parametrize the uncertainty about the exact

SUSY spectrum in a very general way. Indeed, to vary T_{SUSY} from 15 GeV to 1 TeV is equivalent to considering the supersymmetric threshold corrections due to variations in the sparticle masses within a very conservative wide range.

Performing a complete two–loop numerical analysis, we have as inputs the value of $1/\alpha_{em} = 127.9$, which has only a logarithmic dependence on the top quark mass, and the value of $\sin^2 \theta_W(M_Z)$, which is given by the electroweak parameters G_F, M_Z and α_{em} as a function of M_t (at the one–loop level) by the formula,

$$\sin^2 \theta_W(M_Z) = 0.2324 - 10^{-7} \times \text{ GeV}^{-2} \times \left(M_t^2 - (138 \text{ GeV})^2\right) \pm 0.0003 . \qquad (47)$$

Then, the unification condition implies the following numerical correlation [17], [24],

$$\sin^2 \theta_W(M_Z) = 0.2324 - 0.25 \times (\alpha_3(M_Z) - 0.123) \pm 0.0025 . \qquad (48)$$

The above central value corresponds to $T_{SUSY} = M_Z$ and the error ± 0.0025 is the estimated uncertainty in the prediction arising from possible supersymmetric threshold corrections and including also possible effects from threshold corrections at the unification scale and from higher dimensional operators, but assuming that they are not larger than the supersymmetric threshold corrections. Thus, considering the α_3–$\sin^2 \theta_W$ correlation predicted by the unification of the gauge couplings together with the $\sin^2 \theta_w$–M_t correlation obtained from the fit of the experimental data (both within their uncertainties), a band of correlated values between $\alpha_3(M_Z)$ and M_t is obtained [24],

$$\alpha_3(M_Z) = 0.123 + 4 \times 10^{-7} \times \text{GeV}^{-2} \times \left(M_t^2 - (138 \text{ GeV})^2\right) \pm 0.01 . \qquad (49)$$

As we shall show below, this correlation is crucial in the analysis of the top quark mass predictions coming from bottom–tau Yukawa coupling unification.

For given values of the gauge coupling the requirement of bottom and tau Yukawa coupling unification determines the value of the top quark mass as a function of $\tan \beta$, depending on the input value of the bottom quark mass. Indeed, the additional inputs with respect to the gauge coupling unification analysis are the value of the tau mass, $M_\tau = 1.78$ GeV and the value of the bottom mass, which involves a large uncertainty. In fact, the range of experimentally allowed values for the physical bottom quark mass is $M_b = 4.6$–5.2 GeV [41]. Moreover, a significant difference, of the order of 12%, between the running bottom quark mass, which is the one directly related to the bottom Yukawa coupling, and the physical bottom quark mass arises from QCD corrections. At the two–loop level the relation is $M_b = m_b(M_b)[1 + (4/3\pi)\alpha_3(M_b) + 12.4(\alpha_3(M_b)/\pi)^2]$. Assuming bottom–tau Yukawa coupling unification, the exact range of values to be considered for the physical bottom mass as well as the appropriate treatment of the difference between the physical and running bottom quark masses have important consequences on the determination of the top quark Yukawa coupling. This is due to the fact that the bottom mass fixes the overall scale of the bottom

quark Yukawa coupling. We shall return to the dependence of our predictions for the exact value of the bottom mass after presenting the numerical study. The other decisive variable in the bottom–tau Yukawa unification scheme is the exact value of the strong gauge coupling. Indeed, for relatively large values of the strong gauge coupling, $\alpha_3(M_Z) \geq 0.115$, large values of the top quark Yukawa coupling at the high energy scale are needed in order to partially contravene the strong renormalization effect of the strong gauge coupling in the running of the bottom quark Yukawa coupling. This is the reason why, for such values of the strong gauge coupling, the condition of bottom–tau unification yields predictions for the top quark mass close to its infrared fixed point values –the exact value of M_b defining the precise degree of closeness.

Fig. 6. Top quark mass predictions as a function of the strong gauge coupling for the condition of unification of Yukawa couplings $h_b(M_G) = h_\tau(M_G)$, for different values of $\tan\beta$. The solid line shows the infrared fixed point solutions, while the dashed, long-dashed and dot-dashed lines show the results for $M_b = 4.7, 4.9$ and 5.2, respectively. Here the unification scale M_G is defined as the scale at which the weak gauge couplings unify and the region to the right of the dashed–long-dashed line shows the regime of $\alpha_3(M_Z)$ preferred by the gauge coupling unification condition.

For smaller values of the strong gauge coupling, $\alpha_3(M_Z) \leq 0.110$, which may still be

compatible with its experimental bound, the necessity of a large top quark Yukawa coupling becomes weaker and as a result the infrared fixed point prediction for the top quark mass would not be a necessary outcome of the Yukawa coupling unification condition. However, for those smaller values of $\alpha_3(M_Z)$ the condition of gauge coupling unification is not consistent with the experimentally allowed values for $\sin^2\theta_W$. Therefore, large values of Y_t at M_{GUT}, which imply the proximity to the infrared fixed point solution for M_t, are always a necessary outcome in the low and moderate $\tan\beta$ region, if gauge and bottom-tau Yukawa coupling unification are required [17], [24].

In Fig. 6 we show a detailed numerical study of the degree of proximity to the infrared fixed point solution implied by the unification conditions. The value of the top quark mass is plotted as a function of the strong gauge coupling for the exact infrared fixed point solution as well as for the case of bottom–tau Yukawa coupling unification for three different values of the bottom quark mass, which define the allowed domain of solutions compatible with the experimental predictions for M_b. Moreover, the condition of gauge coupling unification, Eq. (49), implies that the region in $\alpha_3(M_Z)$ to the right of the dashed–long-dashed curve is the allowed one. Indeed, Eq. (49) defines a band whose upper bound is $\alpha_3(M_Z)^u \geq 0.13$ and, thus, it does not appear on the figure. The intersection of this region with the M_t–α_3 curves that follow from $h_b = h_\tau$ at M_{GUT}, for the range $M_b = 4.9 \pm 0.3$ GeV, determines the predicted values for M_t to be within 10% of its infrared fixed point values. The above is fulfilled for small and moderate values of $\tan\beta$. For larger values of $\tan\beta \geq 30$, the behaviour is drastically changed and, as we said, we shall not concentrate in such case here (see Refs. [26], [27] these proceedings). It is interesting to notice that low values of $\alpha_3(M_Z) \simeq 0.113$ are only possible for $M_t \simeq 140$ GeV ($\tan\beta \simeq 1$). For larger values of $\tan\beta$, the lower bound on the strong gauge coupling increases together with the top quark mass, which then has a stronger convergence to its infrared fixed point. For $M_t \simeq 180$ GeV ($\tan\beta \simeq 2$) a value $\alpha_3(M_Z) \geq 0.118$ is already necessary.

Concerning the relevance of the experimental bounds on the physical bottom quark mass, it is worth mentioning that, if values of $M_b < 4.6$ GeV were allowed, it would induce a top quark Yukawa coupling which may become too large. For a consistent perturbative treatment of the theory, one requires $Y_t(M_{GUT}) \leq 1$, which implies that the two–loop contribution to the renormalization group evolution of h_t is less than 30% of the one–loop one. As a matter of fact, observe that in Fig. 6 the curves for $M_b = 4.7$ GeV and $M_b = 4.9$ GeV do not continue up to $\alpha_3(M_Z) = 0.13$, since the top quark Yukawa coupling would then develop a Landau pole before reaching the unification scale. Larger values of the bottom mass, $M_b > 5.2$–5.3 GeV, would destroy the proximity to the infrared fixed point solution. Let us mention, however, that a recent analysis based on QCD sum rules, gives values for the perturbative bottom quark pole mass M_b close to the lower experimental bound considered above ($M_b \simeq 4.6$ GeV) [42].

Concerning possible threshold corrections which may affect the unification of both Yukawa couplings, it follows that a relaxation in the exact unification condition of the order of 10% for $M_b = 4.9$ GeV gives approximately the same behaviour as if one considers exact bottom–tau unification, but with $M_b = 5.2$ GeV. Hence, values of $M_b \leq 4.9$ GeV secure the infrared fixed point behaviour even against possible supersymmetric threshold corrections to the Yukawa couplings. It is necessary to say that, if there are large threshold corrections at the grand unification scale, then all the above study can be significantly changed. These high energy threshold corrections depend, however, on the particular physics above the scale M_{GUT} and may not be computed in a general framework. The study of the Higgs and supersymmetric spectrum performed in section 4 is only based on the infrared fixed point solution or in the proximity to it whithin the MSSM, and has general validity. Then, depending on the exact, complete grand unified model under study, one has to compute the degree of proximity to the infrared fixed point solution. In this section we showed that provided the threshold corrections at M_{GUT} are not very large, a grand unified gauged theory with the extra ingredient of bottom–tau Yukawa coupling unification provides a framework in which the infrared fixed point solution of the top quark mass is realized. Most interesting is the fact that this result depends crucially on the values of the bottom mass and the electroweak parameters being exactly within their experimentally allowed range.

6   Conclusions

We have studied the properties of the MSSM with unification of gauge couplings and universal soft supersymmetry breaking parameters, for the case in which the top quark mass is close to its infrared quasi fixed point solution and $\tan \beta < 10$. To study the regime of the infrared fixed point solution for m_t is of interest for various reasons.

i) It appears as a prediction in many interesting theoretical scenarios. In particular, it has been shown that for the values of the bottom quark mass and the electroweak parameters allowed at present, for small and moderate values of $\tan \beta$, the conditions of gauge and bottom–tau Yukawa coupling unification imply a strong convergence of the top quark mass to its infrared fixed point value.

ii) It gives a very predictive framework in which, given the value of the top quark mass, the properties of the Higgs and supersymmetric spectrum in the minimal supergravity model are determined as a function of two high energy parameters, the common scalar mass m_0 and the common gaugino mass $M_{1/2}$. The implementation of the radiative electroweak symmetry breaking condition is a crucial ingredient for this result.

iii) For the range of top quark mass values suggested by the recent experimental measurements at CDF [40], $M_t = 174 \pm 16$ GeV, the value of $\tan \beta$, within its low and moderate regime, is bounded to be $1 < \tan \beta < 2.5$. For $M_t \leq 175$ GeV, the lightest Higgs mass is

bounded to be $m_h \leq 105$ GeV, implying that there are good chances to observe it at the LEP2 experiment. Moreover, light charginos and light stops may appear in the spectrum. If they are present, they will have many interesting phenomenological implications.

iv) The correlations among the free parameters of the theory derived from the conditions of a proper breakdown of the electroweak symmetry may be very useful in probing models of dynamical supersymmetry breakdown, in which the soft supersymmetry breaking parameters are predicted.

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