SCALING LAWS FOR THE TIME DEPENDENCE OF LUMINOSITY IN HADRON CIRCULAR ACCELERATORS BASED ON SIMPLE MODELS OF DYNAMIC APERTURE EVOLUTION*

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Abstract

In recent years, models for the time-evolution of the dynamic aperture have been proposed and applied to the analysis of non-linear betatronic motion in circular accelerators. In this paper, these models are used to derive scaling laws for the luminosity evolution and are applied to the analysis of the data collected during the LHC physics runs. An extended set of fills from the LHC proton physics has been analysed and the results presented and discussed in detail. The long-term goal of these studies is to improve the estimate of the performance reach of the HL-LHC.

INTRODUCTION

Since the advent of the generation of superconducting colliders, the unavoidable non-linear magnetic field errors have plagued the dynamics of charged particles inducing new and potential harmful effects. This required the development of new approaches to perform more powerful analyses and to gain insight in the beam dynamics. It is worth mentioning the work done on the scaling law of the DA as a function of time [1, 2] for the case of single-particle beam dynamics. Indeed, such a scaling law was later successfully extended to the case in which weak-strong beam-beam effects are added to the beam dynamics [3]. More importantly, such a scaling law was proposed to describe the time evolution of beam losses in a circular particle accelerator under the influence of non-linear effects [4], and the proposed model was verified experimentally, using data from CERN accelerators and the Tevatron. Note that such a scaling law for beam intensity as a function of time is at the heart of a novel method to measure experimentally the DA in a circular ring [5].

The model developed represents a bridge between the concept of DA, which is rather abstract, and the beam losses observed in a particle accelerator. Clearly, the next step was to extend the model to describe the luminosity evolution in a circular collider. The first attempts are reported in [6, 7]. However, in those papers the DA scaling law was used without disentangling the contribution of burn off. Although the results were rather encouraging, to recover the correct physical meaning of the model parameters it was necessary to include as many known effects as possible.

This limitation is removed in the model discussed in this paper. In fact, the proposed scaling law is combined with the well-known intensity decay from particle burn off so that a coherent description of the physical process is provided. Moreover, it is worth stressing that the proposed model can be generalised so to consider a time-dependence for some

of the beam parameters describing the luminosity evolution, such as emittance. All detail can be found in Refs. [8, 9]. It is worth mentioning that the scaling law [4] has also been used in the analysis of beam-beam experiments performed at the LHC [10, 11].

LUMINOSITY EVOLUTION WITH PROTON BURN OFF LOSSES

The starting point is the expression of luminosity, which is a key figure-of-merit for colliders and, neglecting the hourglass effect, reads

$$L = \frac{\gamma_{\rm r}\, f_{\rm rev}\, k_{\rm b}\, n_1\, n_2}{4\,\pi\epsilon^*\beta^*}\, F(\theta_{\rm c}, \sigma_z, \sigma^*), \qquad (1)$$

where $\gamma_{\rm r}$ is the relativistic γ-factor, $f_{\rm rev}$ the revolution frequency, $k_{\rm b}$ the number of colliding bunches, $n_{\rm i}$ the number of particles per bunch in each colliding beam, ϵ^* is the RMS normalised transverse emittance, and β^* is the value of the beta-function at the collision point. The total beam population is defined as $N_j = k_{\rm b}\, n_j$ and the fact that not all bunches are colliding in the high-luminosity experimental points is taken into account by introducing a scale factor.

The factor F accounts for the reduction in volume overlap between the colliding bunches due to the presence of a crossing angle and is a function of the half crossing angle $\theta_{\rm c}$ and the transverse and longitudinal RMS dimensions σ^*, σ_z, respectively according to:

$$F(\theta_{\rm c}, \sigma_z, \sigma^*) = \frac{1}{\sqrt{1 + \left(\frac{\theta_{\rm c}}{2}\, \frac{\sigma_z}{\sigma^*}\right)^2}}\,. \qquad (2)$$

Note that $\sigma^* = \sqrt{\beta^*\,\epsilon^*/(\beta_{\rm r}\,\gamma_{\rm r})}$, where $\beta_{\rm r}$ is the relativistic β-factor. Equation (1) is valid in the case of round beams and round optics. For our scope, Eq. (1) will be recast in the following form:

$$L = \Xi\, N_1\, N_2, \qquad \Xi = \frac{\gamma_{\rm r}\, f_{\rm rev}}{4\,\pi\epsilon^*\beta^*\, k_{\rm b}}\, F(\theta_{\rm c}, \sigma_z, \sigma^*) \qquad (3)$$

in which the dependence on the total intensity of the colliding beams is highlighted and the other quantities are included in the term Ξ.

Under normal conditions, i.e. excluding any levelling gymnastics or dynamic-beta effects, only the emittances and the bunch intensities can change over time. Therefore, Eq. (1) is more correctly interpreted as peak luminosity at the beginning of the fill, as in general L is a function of time. When the burn off is the only relevant mechanism for

a time-variation of the beam parameters, it is possible to estimate the time evolution of the luminosity, which turns out to be derived from the following equation

$$N'(t) = -\sigma_{\text{int}} \, n_c \, L(t) = -\sigma_{\text{int}} \, n_c \, \Xi \, N^2(t) \qquad (4)$$

where σ_{int} represents the cross section for the interaction of charged particles and the two colliding beams have been assumed to be of equal intensity. The value used is 73.5 mb for 3.5 TeV and 76 mb for 4 TeV [12, 13] for protons, representing the total inelastic cross-section. Here, n_c stands for the number of collision points.

In the most general case, where both beams can have different intensities, the intensity evolution is described by the following equations

$$\begin{cases} N_1'(t) & = -\sigma_{\text{int}} \, n_c \, \Xi \, N_1(t) \, N_2(t) \\ N_2'(t) & = -\sigma_{\text{int}} \, n_c \, \Xi \, N_1(t) \, N_2(t) \, . \end{cases} \qquad (5)$$

It is useful to change to a different time variable, namely

$$\tau - 1 = f_{\text{rev}} \, t \qquad \text{giving} \qquad \frac{\mathrm{d}}{\mathrm{d}t} = f_{\text{rev}} \frac{\mathrm{d}}{\mathrm{d}\tau}, \qquad (6)$$

τ being an adimensional variable representing the number of turns, where a shift of the origin of τ with respect to t has been introduced.

The solution of Eq. (5), indicated as $N_{1,2}^{\text{bo}}(\tau)$ to highlight that it only includes the burn off contribution, can be obtained by re-writing:

$$\begin{cases} \dot{N}_1^{\text{bo}}(\tau) + \dot{N}_2^{\text{bo}}(\tau) & = -2 \, \varepsilon \, N_1^{\text{bo}}(\tau) \, N_2^{\text{bo}}(\tau) \\ \dot{N}_1^{\text{bo}}(\tau) - \dot{N}_2^{\text{bo}}(\tau) & = 0 \end{cases} \qquad (7)$$

with

$$\varepsilon = \frac{\sigma_{\text{int}} \, n_c \, \Xi}{f_{\text{rev}}} \qquad (8)$$

and from which one finds

$$\begin{cases} N_1^{\text{bo}}(\tau) & = N_2^{\text{bo}}(\tau) + \xi \\ \dot{N}_2^{\text{bo}}(\tau) & = -\varepsilon \, N_2^{\text{bo}}(\tau) \left[N_2^{\text{bo}}(\tau) + \xi \right]. \end{cases} \qquad (9)$$

Equation (9) has two solutions depending on the value of ξ. If $\xi = 0$ then

$$\begin{cases} N_1^{\text{bo}}(\tau) & = \dfrac{N_i}{1 + \varepsilon \, N_i \, (\tau - 1)} \\ N_2^{\text{bo}}(\tau) & = N_1^{\text{bo}}(\tau), \end{cases} \qquad (10)$$

where $N_i = N_{i,1} = N_{i,2}$ stands for the initial beam intensity.

Otherwise, if $\xi \neq 0$ then

$$\begin{cases} N_1^{\text{bo}}(\tau) & = \xi \, \dfrac{1}{1 - N_r \, e^{-\varepsilon \, \xi \, (\tau-1)}} \\ \\ N_2^{\text{bo}}(\tau) & = \xi \, \dfrac{N_r \, e^{-\varepsilon \, \xi \, (\tau-1)}}{1 - N_r \, e^{-\varepsilon \, \xi \, (\tau-1)}}, \end{cases} \qquad (11)$$

where $\xi = N_{i,1} - N_{i,2}$ and $N_r = \frac{N_{i,2}}{N_{i,1}}$.

Whenever additional time dependence in the luminosity evolution needs to be taken into account, the solutions (10) and (11) can be extended to take into account these effects (see Refs. [8]).

LUMINOSITY EVOLUTION INCLUDING PSEUDO-DIFFUSIVE EFFECTS

An efficient modelling of the luminosity evolution in a real collider can be obtained either by means of numerical tracking, see e.g. Ref. [14], or by means of analytical or semi-analytical models, see e.g. Refs. [15–17]. However, none of the models studied included the effect of non-linear motion and this is at the heart of the approach proposed in Ref. [7]. The basis for such a model is the evolution of the dynamic aperture (DA) with time in a hadron collider. The analysis of single-particle tracking results showed that the time evolution of the DA follows a simple law [1, 2], whose justification is not only phenomenological. Recently, this approach has been successfully applied to the analysis of intensity evolution in hadron machines [4]. So far, however, the results were obtained in the case of single-particle simulations or whenever the conditions in a particle accelerator were not under the influence of any collective effect. To extend the proposed scaling law to luminosity evolution, it is necessary to show that it is valid also in the presence of beam-beam effects. This is the case at least for the results of numerical simulations in the weak-strong regime, as discussed in Ref. [3], thus opening the possibility to justify the proposed interpretation.

The proposed approach is a refinement of what is presented in Ref. [7] and assumes that all possible pseudo-diffusive effects can be modelled by a scaling of the intensity with time as

$$N(\tau) = N_i \left[1 - \int_{D(\tau)}^{+\infty} \mathrm{d}r \, \hat{\rho}(r) \right] = N_i \left[1 - e^{-\frac{D^2(\tau)}{2}} \right], \quad (12)$$

where

$$D(\tau) = D_\infty + \frac{b}{[\log \tau]^\kappa}. \qquad (13)$$

The parameters D_∞, b, κ are normally fitted to the experimental data and the variable τ represents the turn number and satisfies $\tau \in [1, +\infty[$. It is worthwhile stressing some properties of the parameters as highlighted in Refs. [2, 4], where two regimes were identified depending on the signs of the fit parameters.

The further step in view of using this scaling law for the analysis of the evolution of the luminosity requires a number of additional considerations, namely

- The proton burn off occurs mainly in the core of the beam distribution, corresponding to the region of largest particle density. On the other hand, the diffusive processes are mainly affecting the tails of the beam distribution. This, in turn, implies that proton burn off and diffusive phenomena are acting on different parts of the beam distributions and are, hence, essentially decoupled and independent.

- The characteristic times of the two processes are rather different. The burn off takes place at a sub-turn time

scale (for instance, in the case of the LHC, considering only the high-luminosity experiments, the burn off occurs twice per turn), while the pseudo-diffusive phenomena take place on a much longer time scale, as a continuous process.

- The fit parameters in Eq. (13) might depend on the beam intensity. However, if one assumes that the overall intensity variation over one physics fill is not too large, it is then reasonable to consider that the pseudo-diffusive effects are, to a good extent, almost constant.

Then, under these assumptions, it is justified to describe the intensity evolution as

$$\begin{cases} \dot{N}_1(\tau) &= -\varepsilon \, N_1(\tau) \, N_2(\tau) - \mathcal{D}_1(\tau) \\ \dot{N}_2(\tau) &= -\varepsilon \, N_1(\tau) \, N_2(\tau) - \mathcal{D}_2(\tau), \end{cases} \quad (14)$$

where the terms $\mathcal{D}_i$ represent the intensity-independent pseudo-diffusive effects. Typical values of ε are 1.1×10^{-24} assuming the beam parameters during the 2011 physics run for protons. Therefore, about 3.1×10^4 particles are removed from the bunches each turn, corresponding to 0.24 ppb.

The explicit expression for $\mathcal{D}_i(\tau)$ can be found by noting that these functions are the solutions of

$$\begin{cases} \dot{N}_1(\tau) &= -\mathcal{D}_1(\tau) \\ \dot{N}_2(\tau) &= -\mathcal{D}_2(\tau) \end{cases} \quad (15)$$

and that the explicit solution has been assumed to be of the form (12) [3,4,7]. Therefore, one obtains

$$\mathcal{D}_j(\tau) = -N_{i,j} \, D_j(\tau) \, \dot{D}_j(\tau) \, e^{-\frac{D_j^2(\tau)}{2}} \qquad j = 1, 2. \quad (16)$$

Under the assumptions that the initial beam intensities are the same as well as the terms $\mathcal{D}_j$, then an explicit expression at the lowest order in ε (see Eq. (8)) can be given for both intensity and luminosity, namely

$$\frac{N(\tau)}{N_i} = \frac{1}{1 + \varepsilon \, N_i \, (\tau - 1)} - \left[e^{-\frac{D^2(\tau)}{2}} - e^{-\frac{D^2(1)}{2}} \right] \quad (17)$$

and

$$\frac{L(\tau)}{L_i} = \frac{1}{[1 + \varepsilon \, N_i \, (\tau - 1)]^2} - \left[e^{-\frac{D^2(\tau)}{2}} - e^{-\frac{D^2(1)}{2}} \right] \times$$
$$\times \left\{ 2 - \left[e^{-\frac{D^2(\tau)}{2}} - e^{-\frac{D^2(1)}{2}} \right] \right\} \quad (18)$$

where $L_i = \Xi \, N_{rm}^2$ is the initial value of the luminosity.

INTEGRATED LUMINOSITY OVER A PHYSICS FILL

The models analysed in the previous sections can be used to derive some useful scaling laws for the integrated luminosity as a function of the length of the physics fill. Indeed,

assuming the simple case of equal intensities for both beams, it is possible to obtain for the burn off part

$$L_{int}^{bo}(\tau) = \int_1^\tau d\tilde{\tau} \, L^{bo}(\tilde{\tau}) = \frac{N_i \, \Xi}{\varepsilon} \, \frac{\varepsilon \, N_i \, (\tau - 1)}{1 + \varepsilon \, N_i \, (\tau - 1)}. \quad (19)$$

Note that because

$$L_{int}^{bo}(\tau \to \infty) = \frac{N_i \, \Xi}{\varepsilon}, \quad (20)$$

one can normalise the integrated luminosity as

$$L_{norm}^{bo}(\tau) = \frac{L_{int}^{bo}(\tau)}{L_{int}^{bo}(\tau \to \infty)} = \frac{\varepsilon \, N_i \, (\tau - 1)}{1 + \varepsilon \, N_i \, (\tau - 1)}. \quad (21)$$

Furthermore, by using the normalised turn variable $\bar{\tau} = \varepsilon \, N_i \, (\tau - 1)$, L_{norm}^{bo} can be recast in the following form

$$L_{norm}^{bo}(\bar{\tau}) = \frac{\bar{\tau}}{1 + \bar{\tau}}. \quad (22)$$

Hence, $L_{norm}^{bo}(\bar{\tau})$ has a very simple scaling law in terms of $\bar{\tau}$. This allows comparing experimental data from physics runs with different beam parameters, such as β^*, crossing angle, bunch intensity, and number of bunches (see [9]).

To include pseudo-diffusive effects it is enough to apply the computations made before to the general solution of the intensity-evolution equation, based on the sum of components $N_{1,2}^{bo}(\tau)$ and $N_{1,2}^{pd}(\tau)$, hence giving

$$L_{norm}(\tau) = L_{norm}^{bo}(\tau) + L^{pd}(\tau) \quad (23)$$

where L_{norm}^{bo} stands for the burn off component of the luminosity evolution derived above, and L^{pd} is the integral of the pseudo-diffusive contribution in Eq. (18):

$$L^{pd}(\tau) = -N_i \, \varepsilon \int_1^\tau d\tilde{\tau} \left[e^{-\frac{D^2(\tilde{\tau})}{2}} - e^{-\frac{D^2(1)}{2}} \right] \times$$
$$\times \left\{ 2 - \left[e^{-\frac{D^2(\tilde{\tau})}{2}} - e^{-\frac{D^2(1)}{2}} \right] \right\}. \quad (24)$$

ANALYSIS OF LHC RUN 1 DATA

The models derived will be applied to the analysis of the LHC performance data collected during Run 1. Detailed information on this topic can be found in Refs. [18–21], while in Ref. [22] a preliminary analysis was made, without focusing on models to describe the luminosity and its time evolution. Here, the focus will be on the proton physics run and the data analysed can be found at [23]. Among the full data set available from [23] a selection has been considered including only the fills that resulted in successful physics runs, the so-called stable beams, of a total duration exceeding 10^3 s and featuring $N_{i,1,2} > 10^{13}$ p. Such a filtering allows removing data corresponding to beam commissioning stages or low-luminosity fills, which would not be representative of the typical LHC performance. Additionally we only select those fills that have a number of bunches $k_b > 1300$.

Equations (1) and (2) show that while σ_z has an impact on F, only, the transverse normalised emittances $\epsilon_{x,y}^*$ affect both F and the peak luminosity. The measured data revealed that the variation of σ_z over a typical physics fill does not exceed ≈ 7 %. Therefore, the time-dependence of σ_z can be safely neglected in the analyses presented in the following sections.

The time-dependence of ε needs to be assessed to decide the approach to be applied to the data analyses. The data have been fitted using an exponential function and the result is given by

$$\Delta \varepsilon(t) = 34.69 \, e^{-0.1358 \, t} - 35.39 \tag{25}$$

where t is expressed in hours and $\Delta \varepsilon$ in percent. For the majority of fills $\Delta \varepsilon$ does not exceed $\approx$ 30 % and it has been decided to perform the numerical analyses assuming a time-independent ε.

A close inspection of the Run 1 data [22] reveals that for a typical physics fill the quantity $2 \, |N_{i,1} - N_{i,2}|/(N_{i,1} + N_{i,2})$ does not exceed ≈ 10 %. Hence, in the analysis reported in the following sections, the two initial beam intensities have always been assumed equal. Given that a similar estimate holds also for the intensities at the end of the physics fills, the pseudo-diffusive effects have been assumed to be the same for both beams.

LUMINOSITY EVOLUTION OVER A FILL

The first step in the analysis of the LHC Run 1 data is the fit of the pseudo-diffusive component of the luminosity evolution based on the expression given in Eq. (18).

For this, 24 fills, 10 from 2011 and 14 from 2012, have been selected and fitted individually, also separating the results for the two high-luminosity experiments, ATLAS and CMS. The results are listed in Table 1. Also shown is R_{adj}^2, the so-called adjusted coefficient of determination, given by

$$R_{adj}^2 = 1 - \frac{N-1}{N-\nu-1} \frac{\Sigma^2}{\sigma^2}, \tag{26}$$

where N is the sample size, ν the number of fit parameters, Σ^2 the sum of residues squared. Note that R_{adj}^2 compares the fit under consideration to the most naive fit possible, i.e. a constant line through the mean. When $R_{adj}^2 \ll 1$ (or possibly even negative), the fit is of poor quality as the mean of the data provides a better fit than the proposed model. A good fit has $R_{adj}^2 \to 1$, indicating that the residues are small compared to the data variance.

If we look at the results in Fig. 1, we notice that all fits are of particular good quality, as all except one have $R_{adj}^2 > 90$%, while for all fits from 2011 this is even $R_{adj}^2 > 99$%. There is a clear distinction between the results for 2011 and those for 2012, both in spread, but also in behaviour, as the yearly average value of D_∞ is negative for 2011 whereas it is positive for 2012. Furthermore, it is worth noting that from the lower plots of Fig. 1 no systematic difference between the fitted models based on the ATLAS or CMS data is found.
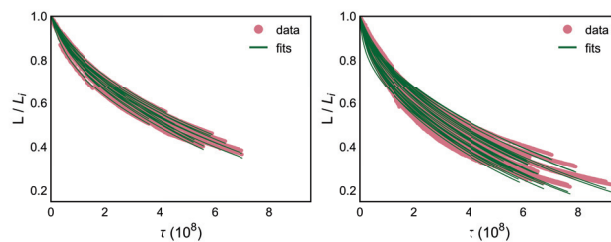

Figure 1: The plots show the measured and fitted curves for L (normalised to the initial fill luminosity L_i) for 2011 (left) and 2012 (right) fills and a very good agreement is clearly visible.

It is useful to fit the data to a slightly adapted model, which has a reduced set of parameters. To this end, we selected three different configurations: one where we fix $\kappa = 2$ (according to the Nekhoroshev estimate [24]) and fit b and D_∞; one where we fix $D_\infty = 0$ (as it is approximately the average of Run 1) and fit b and κ; and one where we fix both $\kappa = 2$ and $D_\infty = 0$ and fit only b, thus leaving only one model parameter.

The resulting weighted average parameter values are listed in Table 1. The difference between the fills from 2011 and from 2012 persists in all fit versions, for this reason we did not calculate the total average parameter values over the two years of Run 1. When one parameter is fixed ($\kappa = 2$ or $D_\infty = 0$) the fit quality is almost unaffected, but when two parameters are fixed (both $\kappa = 2$ and $D_\infty = 0$ at the same time), there is a clear worsening of the fit (even though the overall quality remains rather good). This indicates that fixing one parameter delivers a fit that is as good as using the full parameter set, given the existence of an approximate degeneracy of the parameter space. The case $\kappa = 2$ is preferred over $D_\infty = 0$, because of its justification on the basis of the Nekhoroshev theorem.

ANALYSIS OF INTEGRATED LUMINOSITY

The second step consists of establishing the model for the integrated luminosity delivered in a single fill for physics.

As a first investigation, the pseudo-diffusion model has been fitted to the complete Run 1 dataset. This is shown on the left side of Fig. 2, and the values of the fit parameters including the associated errors are reported in Table 1.

The pseudo-diffusive effect on a yearly basis is shown in the right plot of Fig. 2, and a difference between the two years is seen, which does not not exceed 20 %. Careful inspection reveals that the same difference exists in the data, thus confirming that the model reproduces closely the features of the dataset. The parameter values for the yearly fits are also given in Table 1. Note that now $D_\infty < 0$ for 2011, exactly like in the non-integrated case.

A comparison of the fit parameters for the unintegrated and integrated cases reported in Table 1 shows that the values are compatible, within the errors, for the case of three-

Table 1: Summary of the fit parameters and associated errors corresponding to the expression of $L^{pd}(\tau)$, for different model parameters and for different data subsets, and both for the unintegrated and integrated luminosity. The error on the fit parameters is estimated using the BCa interval [25], and in the unintegrated case the presented values are the weighted averages over the fills.

	D_∞	b	κ	$\bar{R}^2_{\mathrm{adj}}[\%]$
unintegrated: 2011	-0.61 ± 0.71	180 ± 210	1.64 ± 0.40	99.759
$\kappa = 2$	-0.44 ± 0.19	920 ± 73	$-$	99.736
$D_\infty = 0$	$-$	1900 ± 940	2.41 ± 0.19	99.726
$\kappa = 2, D_\infty = 0$	$-$	752 ± 18	$-$	97.469
2012	0.36 ± 0.41	1200 ± 680	2.19 ± 0.24	96.531
$\kappa = 2$	0.20 ± 0.26	670 ± 110	$-$	96.232
$D_\infty = 0$	$-$	200 ± 200	1.84 ± 0.26	96.037
$\kappa = 2, D_\infty = 0$	$-$	748 ± 23	$-$	93.492
integrated: Run 1 (2011+2012)	0.44 ± 0.54	460 ± 110	1.92 ± 0.31	96.433
$\kappa = 2$	$0.497^{+0.095}_{-0.054}$	556^{+20}_{-37}	$-$	96.440
$D_\infty = 0$	$-$	177^{+30}_{-43}	$1.517^{+0.052}_{-0.094}$	96.434
$\kappa = 2, D_\infty = 0$	$-$	740.0 ± 1.1	$-$	96.208
2011	$-0.43^{+0.38}_{-0.14}$	350^{+150}_{-80}	$1.68^{+0.16}_{-0.13}$	97.835
$\kappa = 2$	$-0.03^{+0.10}_{-0.13}$	757^{+49}_{-35}	$-$	97.847
$D_\infty = 0$	$-$	830^{+370}_{-200}	$2.04^{+0.13}_{-0.09}$	97.848
$\kappa = 2, D_\infty = 0$	$-$	$744.0^{+1.6}_{-1.8}$	$-$	97.857
2012	0.82 ± 0.52	560 ± 114	2.08 ± 0.35	95.746
$\kappa = 2$	$0.77^{+0.13}_{-0.06}$	455^{+21}_{-49}	$-$	95.754
$D_\infty = 0$	$-$	81^{+15}_{-26}	$1.25^{+0.06}_{-0.13}$	95.737
$\kappa = 2, D_\infty = 0$	$-$	738.2 ± 1.4	$-$	95.166

parameter fit, while the compatibility degrades as the number of fit parameters is reduced, the case with fixed κ being more compatible between the non-integrated and integrated luminosity models, than that with D_∞. This confirms once more that fixing κ is the best option among those with reduced fit parameters.

CONCLUSIONS

The luminosity models proposed have been benchmarked against the data from the LHC Run 1, with special emphasis on the years 2011 and 2012, showing a remarkable power in reproducing and describing the observed behaviours of luminosity as a function of time and of integrated luminosity.

Given the encouraging results of the analyses reported in this paper, the data from Run 2 will be considered next, as the higher beam energy that characterises the proton physics in Run 2 opens a new domain in terms of beam behaviour, such as strong longitudinal emittance damping due to synchrotron radiation as well as a burn-off dominated regime.

Ultimately, we aim at applying these model to the HL-LHC in order to provide more accurate estimates of its performance reach.
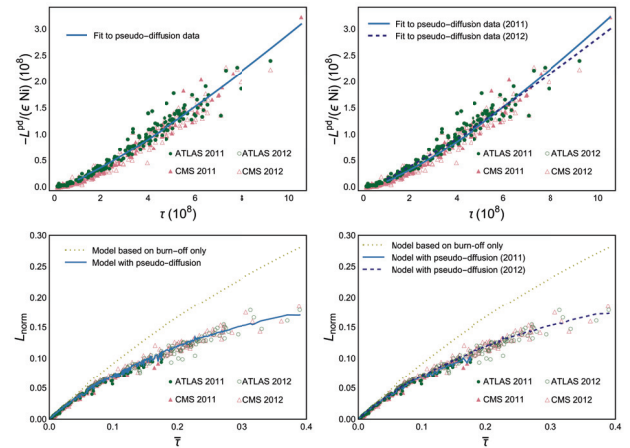
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Figure 2: Pseudo-diffusive contribution to the integrated luminosity (upper row) and normalised luminosity (lower row) as a function of turn number resp. normalised time. The model has been fitted using the complete set of 2011 and 2012 data (left), and separately year by year (right).

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