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PRECISION PHYSICS AT THE ELECTROWEAK SCALE IN SUSY UNIFICATION[1]

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ABSTRACT

Unification of the electroweak and the strong coupling constants implied by the LEP data in SUSY GUT framework has renewed interest in precision calculations in models with supergravity unification. A brief review is given of these developments. The implication of the top Yukawa coupling fixed point in this context is discussed. A brief discussion is also given of precision analysis of neutralino relic density and of the event rates of neutralino nucleus scattering.

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I. INTRODUCTION

The observation that the LEP data[1)] implies unification of the electro-weak and the strong coupling constants when extrapolated from the weak scale to the GUT scale[2)] within SUSY SU(5) model, has led to renewed interest in precision calculations of the mass spectra and other calculable quantities in supergravity unified models[3-9)]. In this paper we give a brief discussion of the framework of these calculations with special emphasis on accurate computation of neutralino relic density and event rate for neutralino-nucleus scattering. The outline of the paper is as follows: In sec. II we give a brief discussion of the parameter space of supergravity models and the implications of the Yukawa coupling fixed point for precision calculations. In sec III we discuss the precision computation of the relic density of the LSP neutralino. In sec. IV we discuss the computation of the event rate in elastic neutralino-nucleus scattering which is relevant for dark matter detectors. Conclusions are given in sec. V.

II. PARAMETER SPACE OF SUPERGRAVITY UNIFIED MODELS

Supergravity unified models are based on the framework of applied N=1 supergravity[10,11)]. The basic tenets of this formalism are that the effective theory below the Planck scale is an N=1 supergravity coupled to n number of chiral multiplets and a vector multiplet belonging to the adjoint representation of a unified gauge group G. It is assumed that supersymmetry breaks via a so called hidden sector below the Planck scale, and the GUT group G breaks to the Standard Model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ at the scale M_G. The effective theory below the scale M_G is then described by the parameters: $m_o, m_{1/2}, A_o, B_o; \mu_o; \alpha_G, M_G$, where the first four parameters are associated with soft SUSY breaking, μ_o is the Higgs mixing parameter for a two Higgs doublet case, and α_G, M_G are the GUT parameters. The current analyses in supergravity unification are all based on radiative electro-weak symmetry breaking. Here one evolves the gauge and Yukawa couplings and the soft SUSY breaking terms from the GUT scale down to the weak scale using renormalization group equations. After radiative breaking of the electro-weak symmetry one can reduce to four the free parameters of the theory using LEP data, and choose these parameters to be $m_o, m_{1/2}, A_t$ and $\tan\beta$ where $\tan\beta = \langle H_2 \rangle / \langle H_1 \rangle$ and where H_2 gives mass to the up quark and H_1 gives mass to the down quark.

Since radiative breaking of the electro-weak symmetry involves large extrapolations, from scales $0(10^{16} GeV)$ to scales $0(10^2 GeV)$, the stability of solutions to small corrections should be addressed. A number of possible corrections form various sources have been discussed in the literature. These include two and higher loop corrections to beta functions, threshold corrections at the GUT scale due to heavy thresholds, and at low energy due to the light SUSY spectrum, loop corrections to the effective potential and corrections due to possible higher dimensional operators from quantum gravity effects. All of these corrections have influence on precision calculations at the electro-weak scale. In addition to the above there is also the possibility that errors may be magnified sometimes due to proximity to the fixed point of the top quark Yukawa coupling. As is well known the one-loop renormalization group analysis using $SU(3)_C \times SU(2)_L \times U(1)_Y$ evolution below M_G gives a fixed point corresponding to $D_o = o$ where[12]

$$Y_o = \frac{Y_t}{ED_o},\ D_o = 1 - 6Y_t\frac{E}{F},\quad E = (1+\beta_3 t)^{\frac{16}{3b_3}}(1+\beta_2 t)^{\frac{3}{b_2}}(1+\beta_1 t)^{\frac{13}{9b_1}} \tag{2.1}$$

where $F = \int_o^t E dt$, $Y_t = h_t^2/(4\pi)^2$, $t = 2\log(M_G/Q)$, and h_t is the top Yukawa coupling, and Y_o is the value of Y_t at the GUT scale. In Eq. (2.1) $\beta_i = \alpha_i(o)b_i/4\pi$ with $(b_3,b_2,b_1) = (3,1,11)$. From Eq. (2.1) we see that there is a fixed point in the top quark Yukawa coupling corresponding to $Y_t = F/(6E)$ which implies a fixed point mass m_t^f of

$$m_t^f = (4\pi\sin^2\beta/3\alpha_2)^{1/2}(E/F)^{1/2}M_W \tag{2.3}$$

Because of the coupled nature of the renormalization group equations, the same fixed point appears in the soft SUSY breaking parameter of the trilinear coupling[13,14]. Near the fixed point small variations in some of the input quantities can be magnified[13] in the output. Specifically the output in certain regions of the parameter space can be very sensitive to variations in α_G, m_t and $\tan\beta$ near a fixed point. An example of this is shown in Fig. (1), where rapid variations in the stop1 (the lighter stop) mass is shown as one approaches the fixed point. We note that $m_t^f \approx 140GeV$ for $\tan\beta \approx 1$ and $m_t^f \approx 183GeV$ for $\tan\beta = 2.5$. From Fig. 1, one finds that the variations in stop1 mass with $\tan\beta$ and m_t become rapid for lower values of $\tan\beta$, which lowers the value of m_t^f and brings it closer to the running top mass. Typically in all cases shown in Fig. (1) the stop1 mass turns tachyonic 5-6 GeV below the Landau-Pole. In the region of the parameter-space where stop1 mass variation is rapid, even a few GeV ambiguity in the top mass can lead to enormous changes in the predictions of the stop1 mass. This phenomenon is thus of significance for precision predictions at the electro-weak scale in supergravity unification.

III. PRECISION ANALYSIS OF RELIC DENSITY

An aspect of supergravity unification within the assumed naturalness constraint is that the lightest neutralino $(\tilde{Z}_1)$ is also the lightest supersymmetric particle (LSP) in models with R-parity conservation[15]. The LSP in this case will be stable and hence an acceptable dark matter candidate which would contribute to matter density of the universe. A minimal requirement one would like to impose is that the neutralino matter density not overclose the universe, i.e., $\Omega_{\tilde{Z}_1} h^2 < 1$ where $\Omega_{\tilde{Z}_1} = \rho_{\tilde{Z}_1}/\rho_c$ ($\rho_{\tilde{Z}_1}$ is the neutralino matter density and ρ_c is the critical matter density), and h is the hubble parameter in units of 100 km/sMpc. One could also impose more stringent constraints on $\Omega_{\tilde{Z}_1} h^2$. For example, in the inflationary scenario one requires that $\Omega = 1$. Assuming that the visible (baryonic) matter satisfies $\Omega_B \le 0.1$, and using the COBE data, which is consistent with a mix of cold and hot dark matter in the ratio of 2:1, one finds $0.1 < \Omega_{\tilde{Z}_1} h^2 < 0.35$. Theoretically the standard techniques for the computation of relic density give[16]

$$\Omega_{\tilde{Z}_1} h^2 \approx 2.53 \times 10^{-11}\left(\frac{T_{\tilde{Z}_1}}{T\gamma}\right)^3\left(\frac{T\gamma}{2.75}\right)^3 \frac{N_f^{\frac{1}{2}}}{J(x_f)} \tag{3.1}$$

where $J(x_f) = \int_o^{x_f} dx\langle\sigma v\rangle$, $\langle\sigma v\rangle$ is the thermally averaged annihilation cross-section times the relative neutralino velocity, N_f is the effective number of degrees of freedom and x_f is the "freeze-out" temperature. Precision computations of the relic density, however, hinge upon accurate evaluation of the thermally averaged quantity $\langle\sigma v\rangle$. In usual analyses one makes the following approximation in evaluating this quantity: $\langle\sigma v\rangle = a + bv^2/6 + \ldots$ However, it has been known for sometime that this approximation breaks down near thresholds and poles[17]. It is actually found that for the physical situation in supergravity the annihilation of the relic neutralinos can occur often close to the Higgs and the Z-poles. Here one must use the pole amplitude[15,18]. For example, for the Higgs pole one has $\sigma v \approx A_h v^2\left((v^2 - \varepsilon_R)^2 + \gamma_h\right)^{-1}$. Here $\varepsilon_h = \left(m_h^2 - 4m_{\tilde{Z}_1}^2\right)/m_{\tilde{Z}_1}^2$ and $\gamma_h = m_h\Gamma_h/m_{\tilde{Z}_1}^2$ where Γ_h is Higgs width and A_h is a constant. It is found very useful to carry out the x integration in $J(x_f)$ and write

$$J_h = \frac{A_h}{m_{\tilde{Z}_1}^4 2\pi^{\frac{1}{2}}} \int_0^\infty d\xi e^{-\xi} \xi^{-\frac{1}{2}} \left\{ \frac{1}{2} \ell n \left[\frac{(4\xi x_f - \varepsilon_h)^2 + \gamma_h^2}{\varepsilon_h^2 + \gamma_h^2} \right] + \frac{\varepsilon_h}{\gamma_h} \left[\tan^{-1}\left(\frac{4\xi x_f - \varepsilon_h}{\gamma_h} \right) + \tan^{-1}\left(\frac{\varepsilon_h}{\gamma_h} \right) \right] \right\}$$

The integral above can be computed with great precision, leading to precise evaluation of $\Omega_{\tilde{Z}_1} h^2$ in Eq. (3.1). It is found that precise evaluations give significantly different results compared to approximate results.

IV. DETECTION OF NEUTRALINO DARK MATTER

Estimates of dark matter density in the solar neighborhood give $\rho \approx 0.3$ GeV cm^{-3} and the velocity with which they impact a target are estimated to be $\langle v \rangle \approx 320 kms^{-1}$. The recoil kinetic energy in the scattering of a neutralino of mass $m_{\tilde{Z}_1}$ from a nucleus of mass m_N is $\varepsilon \approx 2(m_{\tilde{Z}_1}\langle v \rangle)^2 / m_N$. For $m_{\tilde{Z}_1} \langle 150 GeV$ and $m_N \langle 300 GeV$, one expects ε in the range (1-100) keV. It appears feasible that detectors sensitive to recoil energies in the above range can be built[19] and some experiments are currently underway (For a recent overview see Ref. (20)). Recently[21] there have been a number of works devoted to the computation of the event rate in neutralino-nucleus scattering to determine the region of the parameter space most conducive to the detection of the neutralino in the current and future dark matter experiments. In the analysis being reported here the event rate for the full parameter space of supergravity models was computed using radiative breaking of the electro-weak symmetry and the accurate method for the computation of relic density described in Sec. III[22]. The effective Lagrangian that governs the elastic scattering of $\tilde{Z}_1$ on nucleus can be described by

$$L_{eff} = \bar{\tilde{Z}}_1 \gamma_\mu \gamma_5 \tilde{Z}_1 \bar{q} \gamma^\mu \Gamma_{SD} q + m_q \bar{\tilde{Z}}_1 \tilde{Z}_1 \bar{q} \Gamma_{SI} q \qquad (4.1)$$

where the term involving $\Gamma_{S.D}$ is the sum of spin dependent terms arising from exchange of t-channel Z-pole and s-channel squark pole, and the term involving $\Gamma_{S.I}$ of the sum of spin independent terms involving the sum of t-channel Higgs exchange and s-channel chirality flip (i.e. involving mass insertion) squark exchange. The computation of elastic neutralino-nucleus scattering involves two parts: a nuclear physics part involving matrix elements of quark-operators between nucleus states and a SUSY part which enters via the composition of the $\tilde{Z}_1$ neutralino which is given by $\tilde{Z}_1 = \alpha \tilde{W}_3 + \beta \tilde{B} + \gamma \tilde{H}_1 + \delta \tilde{H}_2$ where $\tilde{W}_3, \tilde{B}$ are the gaugino counterparts of W_3 and B gauge bosons and $\tilde{H}_{1,2}$ are the Higgsino fields. Thus the unknowns on which $\Gamma_{S.D.}$ and $\Gamma_{S.I.}$ depend are: $\alpha, \beta, \gamma, \delta, m_{h^0}, m_{\tilde{q}_L}, m_{\tilde{q}_R}$. These seven parameters are further determined in

terms of just four: $m_o, m_{\tilde{g}}, A_t$ and $\tan\beta$. We have taken into account both the light and the heavy Higgs in the calculationThe event rate is given by[21]

$$R = (R_{inc} + R_{coh}) \left(\frac{\rho}{0.3 GeV cm^{-3}} \right) \left(\frac{\langle v \rangle}{320 kms^{-1}} \right) \left(\frac{event}{kgd} \right) \qquad (4.2)$$

Analysis was carried out for a number of target nuclei[22]: $Ge, CaF_2, He, NaI, Pb, GaAs$. In general as expected it is found that R_{coh} dominates for large atomic numbers, while for small atomic numbers both R_{coh} and R_{inc} are important depending on the region of the parameter space being explored. Fig. (2) exhibits the event rate for Ge as a function of the lightest neutralino mass. Typically the analysis implies that there is a region of the parameter space where $R \rangle 0.1$ and hence in this region of space neutralinos will be detectable in dark matter detectors being planned with sensitivity of $R \rangle 0.1$. However, this region is not large and it shrinks as the atomic number of the target nucleus gets smaller. A full coverage of the parameter space would require detectors which are $0(10^2)$ more sensitive than those currently feasible.

V. CONCLUSION

Precision computations in supergravity unified theories were discussed. The importance of the Landau pole effects was emphasized. Accurate methods for the computation of the neutralino relic density and of the event rate in neutralino-nucleus scattering was discussed.

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REFERENCES

[1] M. Davier, Proc. Lepton-Photon High Energy Phys. Conference, Geneva, 1991 eds. S. Hegarty et. al. (World Scientific, Singapore, 1991); H. Bethke, Proc. XXVI Conf. on HIgh Energy Physics, Dallas, 1992, ed. J. Sanford, AIP Proc. No. 272 (1993).

[2] J.Ellis, S. Kelley and D.V. Nanopoulos, Phys. Lett. 2Y9B (1990) 441; B260 (1990) 131; P. Langacker and M. Lou, Phys. Rev. D44 (1991) 817; U Amaldi, W. deBoer and H. Furstenau, Phys, Lett. B260 (1991) 447; F. Anselmo, L. Cifarelle A. Peterman and A. Zichichi, N.C. 104 (1991) 1817.

[3] R. Arnowitt and P. Nath, Phys. Rev. lett. 69 (1992) 725.

[4] P. Nath and R. Arnowitt Phys. Lett. B289 (1992) 368

[5] S. Kelley, J. Lopez, D.V. Nanopoulos, H. Pois and K. Yuan, Phys. Lett. B272 (1991) 423.

[6] M. Drees amd M.M. Nojiri, Nucl. Phys. B369 (1992) 54; G.G. Ross and R.G. Roberts, Nucl Phys. B377 (1992) 971; K. Inoue et. al. Phys. rev. D45 (1992) 387; D.J. Castano, E.J. Piard and P. Ramond, Univ. Florida preprint UFIFI- HGP-93-18 (1993).

[7] G. L. Kane, C. Kolda, L. Roskowski and J. D. Wells, Univ. Michigan preprint UM-TH-93-24 (1993).

[8] V. Barger, M.S. Berger and P. Ohman, Univ. Wisconsin preprint MAD/PH/901 (1993).

[9] W. de Boer, R. Ehret and D. Kazakov, Karlsruhe preprint IEKP-KA/93/13 (1993) and reference, quoted therein.

[10] A.H. Chamseddine, R. Arnowitt and P. Nath, Phys. Rev. Lett. 29 (1982) 970.

[11] For a review see P. Nath, R. Arnowitt an A. H. Chamseddine, "Applied N=1 Supergravity", (World Scientific, Singapore, 1984).

[12] See e.g. L. E. Ibañez, C. Lopez and C. Munoz, Nucl. Phys. B256 (1985) 218.

[13] P. Nath, R. Arnowitt and J. Wu, in preparation.

[14] Implications of the Yukawa coupling fixed point have also been examined in this context by M. Carena, M. Olechowski, S. Pokorski and C. E. M. Wagner, CERN-TH-7060193.

[15] R. Arnowitt and P. Nath, Phys. Lett. B303 (1993).

[16] J. Ellis, J. S. Hagelin, D. V. Nanopoulos, K. Olive and M. Srednicki, Nucl. Phys. B238 (1984) 453.

[17] K. Griest and D. Seckel, Phys. Rev. D43 (1991) 3191; P. Gondolo and G. Gelmini, Nucl. Phys. B360 (1991) 145.

[18] P. Nath and R. Arnowitt, Phys Rev. Lett. 70 (1993) 3696.

[19] M. W. Goodman and E. Witten, Phys. Rev. D30 (1985) 3059.

[20] D. O. Caldwell, in "Supersymmetry and Unification of Fundamental Interactions" edited by P. Nath, World Scientific (Singapore 1993). p. 186.

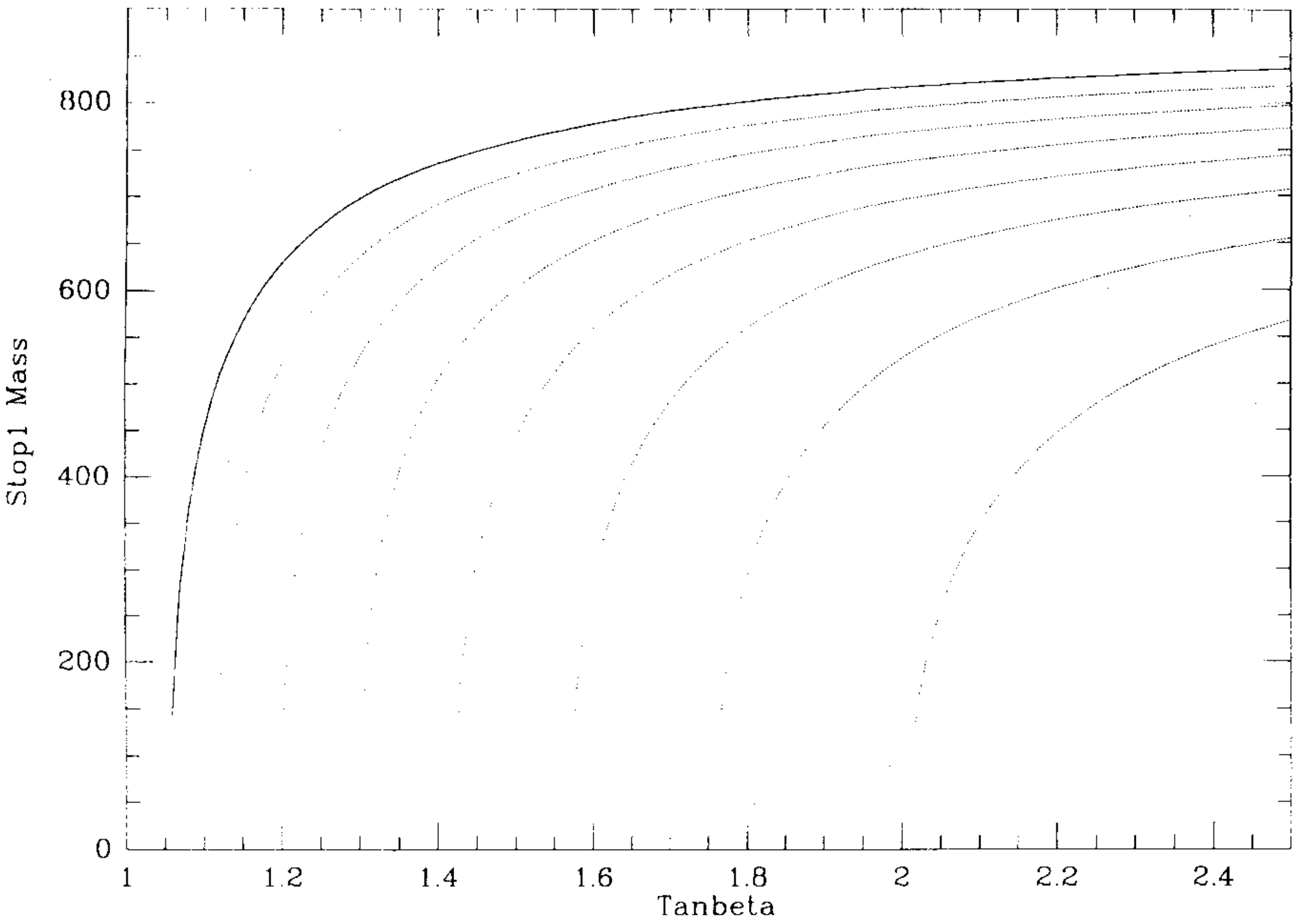
[21] J. Ellis and R. Flores, Phys. Rev. Lett. 62 (1988) 666, M. Drees and M. M. Nojiri, Phys. Rev. 48 (1993) 3483; A. Bottino et. al., Astroparticle Phys. 2 (1994) 67.

[22] R. Arnowitt and P. Nath, in preparation

FIGURE CAPTIONS

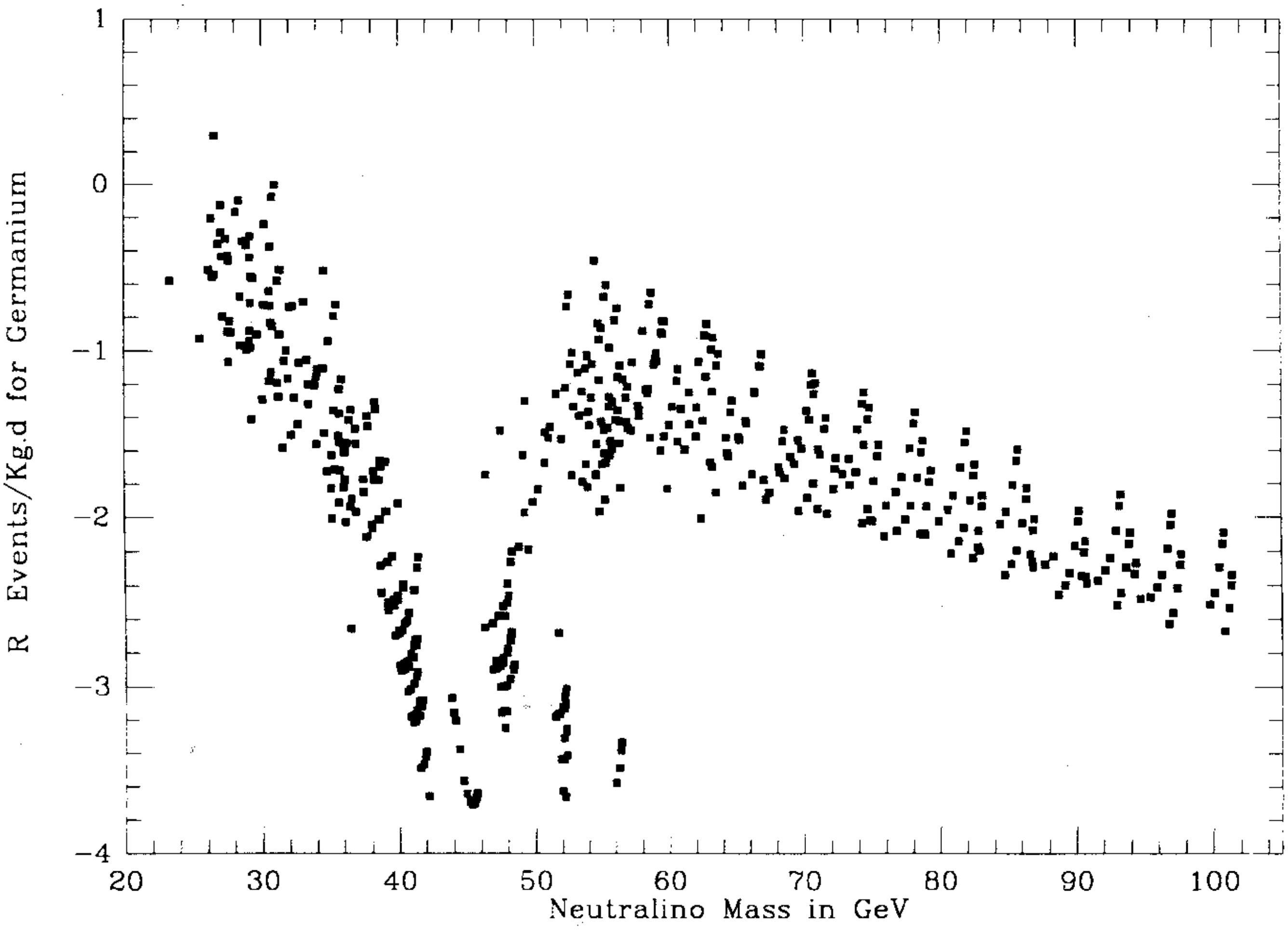
Fig. 1: Plot of stop1 mass Vs $\tan\beta$ for $m_t = 135-170$ GeV in 5 GeV units is ascending order as we go vertically down. $A_t = 0.5$, $m_0 = 500GeV$, $m\tilde{g} = 1000GeV$.

Fig. 2.: Event rate (events/kg.d) as a function of the lightest neutralino mass for Ge target for the case $m_t = 160(GeV,\mu)0$ when one integrates over the full parameter space.



800
600
400
200
0
Stop1 Mass
1
1.2
1.4
1.6
1.8
2
2.2
2.4
Tanbeta

Fig. 1



1
0
-1
-2
-3
-4
R Events/Kg.d for Germanium
20
30
40
50
60
70
80
90
100
Neutralino Mass in GeV

Fig. 2