

CERN-EP-2018-xxx
September 11, 2018

Measurement of P_T -weighted Sivers asymmetries in leptonproduction of hadrons

The COMPASS Collaboration

Abstract

The transverse spin asymmetries measured in semi-inclusive leptonproduction of hadrons, when weighted with the hadron transverse momentum P_T , allow for the extraction of important transverse-momentum-dependent distribution functions. In particular, the weighted Sivers asymmetries provide direct information on the Sivers function, which is a leading-twist distribution that arises from a correlation between the transverse momentum of an unpolarised quark in a transversely polarised nucleon and the spin of the nucleon. Using the high-statistics data collected by the COMPASS Collaboration in 2010 with a transversely polarised proton target, we have evaluated two types of P_T -weighted Sivers asymmetries, which are both proportional to the product of the first transverse moment of the Sivers function and of the fragmentation function. The results are compared to the standard unweighted Sivers asymmetries and used to extract the first transverse moments of the Sivers distributions for u and d quarks.

(to be submitted to Nuclear Physics B)

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- ^t Supported by FP7, HadronPhysics3, Grant 283286 (European Union)
- ^u Supported by MEYS, Grant LG13031 (Czech Republic)
- ^v Supported by B.Sen fund (India)
- ^w Supported by CERN-RFBR Grant 12-02-91500
- ^x Supported by FCT - Fundação para a Ciência e Tecnologia, COMPETE and QREN, Grants CERN/FP 116376/2010, 123600/2011 and CERN/FIS-NUC/0017/2015 (Portugal)
- ^y Supported by MEXT and JSPS, Grants 18002006, 20540299, 18540281 and 26247032, the Daiko and Yamada Foundations (Japan)
- ^z Supported by the Ministry of Science and Technology (Taiwan)
- ^{aa} Supported by the Israel Academy of Sciences and Humanities (Israel)
- ^{ab} Supported by the Russian Federation program “Nauka” (Contract No. 0.1764.GZB.2017) (Russia)
- ^{ac} Supported by the National Science Foundation, Grant no. PHY-1506416 (USA)
- ^{ad} Supported by NCN, Grant 2017/26/M/ST2/00498 (Poland)

1 Introduction

The traditional description of the nucleon structure in hard inclusive processes in terms of collinear parton distributions functions, which depend on the parton light-cone momentum fraction x and on a characteristic hard scale Q^2 , was recently generalised to take into account the transverse momentum k_T of the parton with respect to the nucleon direction (for reviews, see [1–3]). A complete picture of the nucleon at leading twist requires a total of eight transverse-momentum-dependent distributions (TMDs). They provide important information on the dynamics of the partons in the transverse plane in momentum space. Upon integration over the transverse momentum, three of them reduce to the number density, the helicity and the transversity collinear distributions. The other five TMDs contain prefactors that are sensitive to the direction of the quark transverse momentum vector k_T , and their contribution to the hadronic tensor vanishes when integrating over k_T .

Among the TMDs, an important rôle is played by the Sivers distribution function $f_{1T}^{\perp q}$ [4–7], which for an unpolarised quark of flavour q describes the correlation between its transverse momentum and the transverse polarisation of the nucleon. In semi-inclusive measurements of deep-inelastic scattering (SIDIS) off a transversely polarised nucleon, the Sivers TMD embodies in the cross section a sine modulation on the difference between the azimuthal angle ϕ_h of the produced hadron and that of the target nucleon spin, ϕ_S .

The Sivers effect was experimentally observed in SIDIS using transversely polarised proton targets, first by the HERMES Collaboration [8, 9] and then, at higher energy, by the COMPASS Collaboration [10, 11]. The COMPASS measurements on the deuteron [12, 13] showed asymmetries compatible with zero within the experimental accuracy. More recently, data on pion production off a transversely polarised ^3He target were made available by the Hall A Collaboration at JLab [14]. Combined analyses of these measurements [15–25] allowed for extractions of the Sivers functions and of their first transverse moments $f_{1T}^{\perp(1)q}$:

$$f_{1T}^{\perp(1)q}(x) = \int d^2k_T \frac{k_T^2}{2M^2} f_{1T}^{\perp q}(x, k_T^2), \quad (1)$$

which are found to be different from zero, a very important result in TMD physics. In particular, the u and the d distributions turn out to have similar magnitude, but opposite sign. In Eq. (1), M is the target nucleon mass.

While in most phenomenological studies the first transverse moments of the Sivers distributions are extracted by fitting the data using a given functional form for the x dependence of f_{1T}^{\perp} , in Ref. [26] a different approach was adopted: the COMPASS measurements on proton and deuteron targets in the same kinematics were used to extract point-by-point the first transverse moments of the Sivers distributions $f_{1T}^{\perp(1)}$ directly from the data by combining the various asymmetries.

The main problem in all extractions performed up to now is that the standard Sivers asymmetries involve transverse-momentum convolutions of TMDs and fragmentation functions, from which the first transverse moments of the Sivers functions can be obtained analytically only by assuming a specific form, typically a Gaussian, for the transverse-momentum dependence of all involved quantities.

Already twenty years ago an alternative method was proposed [27–29] to determine $f_{1T}^{\perp(1)}$ without making any assumption on the functional form of the transverse-momentum dependence, neither for the distribution functions nor for the fragmentation functions. The method, which consists in measuring asymmetries weighted by the measurable transverse momentum P_T of the hadron, was not pursued; the only and still preliminary results came from HERMES [30]. It is worth to mention that the first transverse moment of the Siverts function is directly entering in the Burkardt sum rule [31], which allows to constrain the gluon Siverts function using the measured Siverts functions for quarks [22]. Recently, much interest has been dedicated again to the weighted asymmetries (see e.g. [32,33]).

In this paper, we present the first measurements of two types of P_T -weighted Siverts asymmetries performed by the COMPASS collaboration using the high statistics data collected in 2010 with a 160 GeV muon beam impinging on a transversely polarised proton target. The results are compared to the standard unweighted Siverts asymmetries and used to extract the first transverse moments of the Siverts functions for u and d quarks.

2 The Siverts asymmetries

The Siverts asymmetry is associated to a $\sin \Phi_{\text{Siv}} \equiv \sin(\phi_h - \phi_S)$ modulation of the SIDIS cross section in a reference frame where the momentum vectors of virtual photon and nucleon are collinear, the z axis is taken along the virtual-photon momentum and the x axis along the lepton transverse momentum. The relevant part of the fully differential cross section is

$$d\sigma = d\sigma_U + S_T d\sigma_S \sin \Phi_{\text{Siv}}, \quad (2)$$

where S_T is the target nucleon polarisation, and $d\sigma_U$ and $d\sigma_S$ are the spin-independent and spin-dependent parts of the cross section, respectively. In the standard, i.e. unweighted case, the Siverts asymmetry is defined as

$$A_{\text{Siv}} = 2 \frac{\int d\Phi_{\text{Siv}} d\phi_h \sin \Phi_{\text{Siv}} d\sigma}{\int d\Phi_{\text{Siv}} d\phi_h d\sigma}. \quad (3)$$

At leading twist and leading order in QCD, A_{Siv} is given [29,34] in terms of the Siverts function f_{1T}^{\perp} and the transverse-momentum-dependent unpolarised distribution and fragmentation functions f_1 and D_1 by

$$A_{\text{Siv}}(x, z, P_T) = \frac{\sum_q e_q^2 x \mathcal{C} \left[\frac{P_T \cdot k_T}{MP_T} f_{1T}^{\perp q}(x, k_T^2) D_1^q(z, p_T^2) \right]}{\sum_q e_q^2 x \mathcal{C} \left[f_1^q(x, k_T^2) D_1^q(z, p_T^2) \right]}, \quad (4)$$

where the sums are over quark and antiquark flavours, e_q are the quark charges, and the transverse momentum convolutions are given by

$$\begin{aligned} & \mathcal{C} \left[\frac{P_T \cdot k_T}{MP_T} f_{1T}^{\perp q} D_1^q \right] \\ & \equiv \int d^2 k_T \int d^2 p_T \delta^2(zk_T + p_T - P_T) \frac{P_T \cdot k_T}{MP_T} f_{1T}^{\perp q}(x, k_T^2) D_1^q(z, p_T^2), \end{aligned} \quad (5)$$

and

$$\mathcal{C} [f_1^q D_1^q] \equiv \int d^2 k_T \int d^2 p_T \delta^2(zk_T + p_T - P_T) f_1^q(x, k_T^2) D_1^q(z, p_T^2). \quad (6)$$

In Eqs. (4,5,6), z is the fraction of the longitudinal momentum of the fragmenting quark carried by the produced hadron, p_T is the transverse momentum of the produced hadron with respect to the direction of the fragmenting quark momentum. For simplicity, we have omitted the Q^2 dependence of parton distributions, fragmentation functions and Siverts asymmetry.

When integrating over P_T , the denominator of Eq. (4) is easily computed yielding the familiar ‘‘collinear’’ expression

$$\sum_q e_q^2 x \int d^2 P_T \mathcal{C} [f_1^q D_1^q] = \sum_q e_q^2 x f_1^q(x) D_1^q(z), \quad (7)$$

where $f_1^q(x)$ and $D_1^q(z)$ are the above defined partonic functions integrated over the transverse momentum, while, in the general case, the numerator of Eq. (4) cannot be analytically evaluated. Hence, in order to disentangle f_{1T}^\perp and D_1 and to extract the Siverts function, some functional form must be assumed for the transverse-momentum dependence of the distribution and fragmentation functions. Assuming this form to be a Gaussian, the Siverts asymmetry becomes [15, 16, 29]

$$A_{\text{Siv,G}}(x, z) = \frac{a_G \sum_q e_q^2 x f_{1T}^{\perp(1)q}(x) z D_1^q(z)}{\sum_q e_q^2 x f_1^q(x) D_1^q(z)}. \quad (8)$$

The factor a_G in Eq. (8) is

$$a_G = \frac{\sqrt{\pi} M}{\sqrt{\langle p_T^2 \rangle + z^2 \langle k_T^2 \rangle_S}}, \quad (9)$$

where $\langle p_T^2 \rangle$ and $\langle k_T^2 \rangle_S$ are the Gaussian widths of the fragmentation function and of the Siverts function, respectively. In the Gaussian model, the average transverse momentum of the produced hadrons (integrated over its azimuthal angle) is written as

$$\langle P_T \rangle = \frac{\sqrt{\pi}}{2} \sqrt{\langle p_T^2 \rangle + z^2 \langle k_T^2 \rangle}, \quad (10)$$

where $\langle k_T^2 \rangle$ is the width of the transverse-momentum-dependent number density f_1 , which in principle differs from $\langle k_T^2 \rangle_S$. Taking approximately $\langle k_T^2 \rangle_S \simeq \langle k_T^2 \rangle$, we can write a_G as

$$a_G \simeq \frac{\pi M}{2 \langle P_T \rangle}. \quad (11)$$

The Gaussian ansatz clearly introduces a bias into the extraction of the Siverts function. In order to avoid this problem one can consider, instead of Eq. (3), an asymmetry that is weighted by the transverse momentum of the produced hadron. In particular, when choosing $w = P_T/zM$ as weight, the weighted Siverts asymmetry becomes

$$A_{\text{Siv}}^w = \frac{\int d\Phi_{\text{Siv}} \sin \Phi_{\text{Siv}} \int d^2 P_T \left(\frac{P_T}{zM} \right) d\sigma}{\int d\Phi_{\text{Siv}} \int d^2 P_T d\sigma}. \quad (12)$$

In terms of quark distribution and fragmentation functions, it reads

$$A_{\text{Siv}}^w(x, z) = \frac{\sum_q e_q^2 x \int d^2 P_T \frac{P_T}{zM} \mathcal{C} \left[\frac{P_T \cdot k_T}{MP_T} f_{1T}^{\perp q}(x, k_T^2) D_1^q(z, p_T^2) \right]}{\sum_q e_q^2 x f_1^q(x) D_1^q(z)}, \quad (13)$$

The convolution in the numerator can now be carried out in a straightforward way (see Appendix A) and the final expression is

$$A_{\text{Siv}}^w(x, z) = 2 \frac{\sum_q e_q^2 x f_{1T}^{\perp(1)q}(x) D_1^q(z)}{\sum_q e_q^2 x f_1^q(x) D_1^q(z)}, \quad (14)$$

which shows that the asymmetry contains the product of the first k_T^2 moment of the Siverts function and the unpolarised fragmentation function.

When using $w' = P_T/M$ as weight, the resulting Siverts asymmetry reads

$$A_{\text{Siv}}^{w'} = \frac{\int d\Phi_{\text{Siv}} \sin \Phi_{\text{Siv}} \int d^2 P_T \left(\frac{P_T}{M} \right) d\sigma}{\int d\Phi_{\text{Siv}} \int d^2 P_T d\sigma}. \quad (15)$$

This asymmetry is of interest because it should exhibit a z dependence close to that of the unweighted asymmetries. Its expression in the parton model,

$$A_{\text{Siv}}^{w'}(x, z) = 2 \frac{\sum_q e_q^2 x f_{1T}^{\perp(1)q}(x) z D_1^q(z)}{\sum_q e_q^2 x f_1^q(x) D_1^q(z)}, \quad (16)$$

is indeed very similar to that of the unweighted asymmetry in the Gaussian model, Eq. (8). In particular, from Eqs. (8,11,16) one sees that the ratio $A_{\text{Siv}}^{w'}/A_{\text{Siv,G}}$ is related to the average value of the hadron transverse momentum:

$$\frac{A_{\text{Siv}}^{w'}}{A_{\text{Siv,G}}} \simeq \frac{4\langle P_T \rangle}{\pi M}. \quad (17)$$

3 Experimental set-up and data analysis

The COMPASS spectrometer [35, 36] is in operation in the SPS North Area of CERN since 2002. The data used in this analysis were collected in 2010 by scattering a 160 GeV μ^+ beam on a transversely polarised target. The 1.2 m long NH_3 target was kept at 50 mK in a dilution refrigerator cryostat and segmented in three cells, 30 cm, 60 cm and 30 cm long respectively. The proton polarisation of about 80% was oriented vertically by a 0.63 T magnetic field that was provided by the saddle coils of the polarised target magnet [37]. The data were taken at a mean beam intensity of $3.5 \times 10^8 \mu/\text{spill}$, for a spill length of about 10 s every 40 s. About 37×10^9 events, corresponding to 1.9 PB of data, were collected in twelve separate periods. In order to minimize systematic errors, during each period of data taking the orientation of the proton polarisation in the three target cells was either up-down-up or down-up-down in the first subperiod, and reversed in the second one. By suitably combining the data, instrumental asymmetries could be limited to

negligible values. The principles of the measurement and the data analysis were already described in several publications [10–12] and will not be repeated here.

In order to allow for a comparison of the weighted Siverts asymmetries with the unweighted asymmetries, all constraints to select DIS events and final-state hadrons are the same as for the published data [11]. Here we only recall that in order to ensure the DIS regime only events with photon virtuality $Q^2 > 1 (\text{GeV}/c)^2$, fractional energy of the virtual photon $0.1 < y < 0.9$, and mass of the hadronic final-state system $W > 5 \text{ GeV}/c^2$ are considered. A charged hadron is required to have a transverse momentum $P_T \geq 0.1 \text{ GeV}/c$ and a fraction of the available energy $z > 0.2$. With these constraints, about 8×10^7 hadrons are left and used for the extraction of the asymmetries. This sample consists mainly of pions (about 70% for positive hadrons, 75% for negative hadrons [38]). In addition, the analysis was also done for charged hadrons in the region $0.1 < z < 0.2$.

The weighted asymmetries are measured separately for positive and negative hadrons as a function of x or z . For each bin in x or z and for each period of data taking, the asymmetries are extracted from the number of hadrons produced in each cell for the two directions of the target polarisation, and the mean of the results from the twelve periods is taken as final result.

The unweighted asymmetries were extracted using both an extended unbinned maximum likelihood method and the so-called double ratio method (DRM). The two methods led to very similar results and the small differences were added to the systematic uncertainties. In both cases, the hadrons produced in the two data-taking subperiods and in the three target cells are combined in order to ensure cancellation of the azimuthal acceptance and of the beam flux. Since only the counts in the numerator of the expression of A_{Siv}^w are weighted, a modified DRM is used in this analysis.

In each kinematic bin, we divide the Φ_{Siv} range in 12 bins, and in each of them we calculate the quantity

$$R(\Phi_{\text{Siv}}) = \frac{\Delta^w}{\sqrt{\Sigma^w \Sigma}}, \quad (18)$$

where

$$\Delta^w = N_+^w N_+^{\prime w} - N_-^w N_-^{\prime w}, \quad \Sigma^w = N_+^w N_+^{\prime w} + N_-^w N_-^{\prime w}, \quad \Sigma = N_+ N_+^{\prime} + N_- N_-^{\prime}. \quad (19)$$

Here N and N^w are the number of counts and the sum of weights, respectively, and N (N^{\prime}) refers to the first (second) subperiod. The numbers of hadrons produced in the first and in the third target cell, which are always polarised in the same direction, are added up. The subscripts $+$ and $-$ indicate the up and down orientation of the target polarisation. Both azimuthal acceptance and beam flux cancel in the ratio of Eq. (18), so that

$$R(\Phi_{\text{Siv}}) \simeq 4\bar{S}_T A_{\text{Siv}}^w \sin \Phi_{\text{Siv}}, \quad (20)$$

where \bar{S}_T is the mean transverse polarisation of the target protons. Cancellation of azimuthal acceptance is guaranteed as long as the ratios of the acceptances of the oppositely polarised cells in the two data taking subperiod are the same, which is the so-called ‘‘reasonable assumption’’ [13].

Several tests were performed to assess the correctness of the results and the size of possible systematic uncertainties. Two alternative estimators were used, which are not expected to guarantee

an as good cancellation of the azimuthal acceptance as the modified DRM but are much simpler, one of them being the mean value of $\sin\Phi_S P_T/zM$. It turned out that the results are essentially identical.

The effect of the P_T/z acceptance was also investigated. This acceptance is about 60% and rather flat in the range $0.020 < x < 0.7$ both for positive and negative hadrons. At smaller x it increases smoothly from 0.4 to about 0.8 as P_T/z increases from 0.1 GeV/ c to 10 GeV/ c . In order to evaluate the effect of the acceptance in the results, we have re-evaluated A_{Siv}^w after having corrected for the P_T/z acceptance. The difference between the results obtained with and without the corrections is at most one tenth of a standard deviation, and thus negligible.

The stability of the results was checked paying particular attention to the P_T limits. The effect of the lower P_T cut, which is expected to be negligible, was investigated by extracting the weighted Siverts asymmetries using three different lower cuts, $P_T > 0.15$ GeV/ c , $P_T > 0.20$ GeV/ c and $P_T > 0.25$ GeV/ c . Also, the effect of a cut on the upper value of P_T was investigated by extracting the asymmetries using the limits: $P_T < 1.5$ GeV/ c , $P_T < 1.25$ GeV/ c and $P_T < 1.0$ GeV/ c . In all the cases the differences to the results obtained with the standard cuts are negligibly small in all x bins.

The contributions from higher-order processes, i.e. QCD Compton and photon-gluon fusion, which are more relevant at high P_T [39], have neither been taken into account nor corrected for.

Altogether, no evidence was found for additional relevant systematic uncertainties. The systematic uncertainties are estimated to be half of the statistical uncertainties, as in the analysis of the standard Siverts asymmetries of the same data [11].

4 Siverts asymmetries weighted by P_T/zM

The distributions of the weights $w = P_T/zM$ are very similar for all nine x bins. As an example, the distribution for positive hadrons in the bin $0.080 < x < 0.130$ is shown in the left panel of Fig. 1. The mean values of w in the nine x bins are given in the right panel of the same figure and in Table 1. For negative hadrons the distributions are very much the same. The distributions of w in

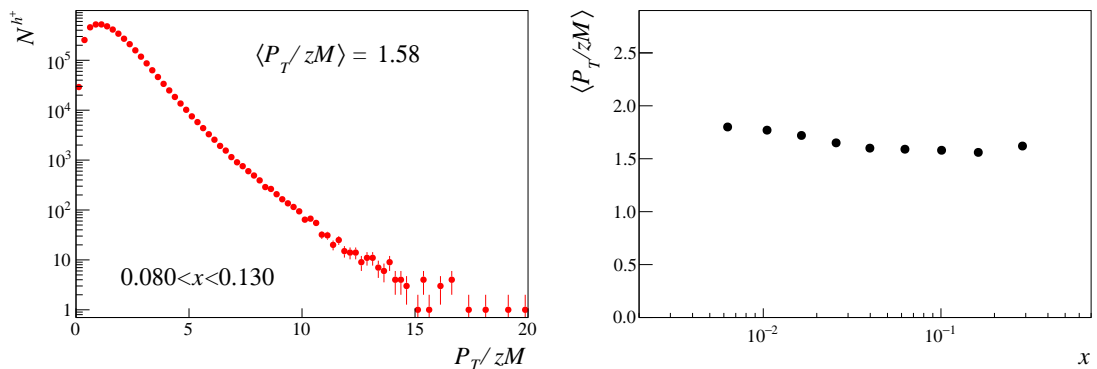


Fig. 1: Left panel: Distribution of the weight $w = P_T/zM$ for positive hadrons in the bin $0.080 < x < 0.13$. Right panel: Mean value of w as function of x . No acceptance correction applied.

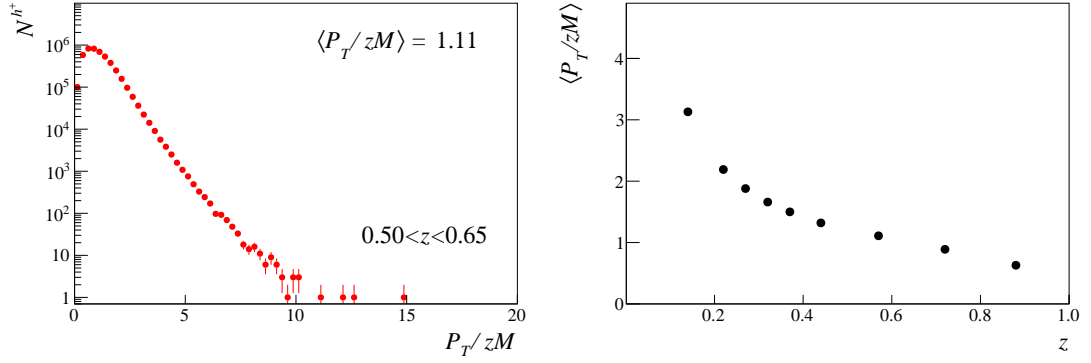


Fig. 2: Left panel: Distribution of the weight $w = P_T/zM$ for positive hadrons in the bin $0.50 < z < 0.65$. Right panel: Mean value of w as function of z . No acceptance correction applied.

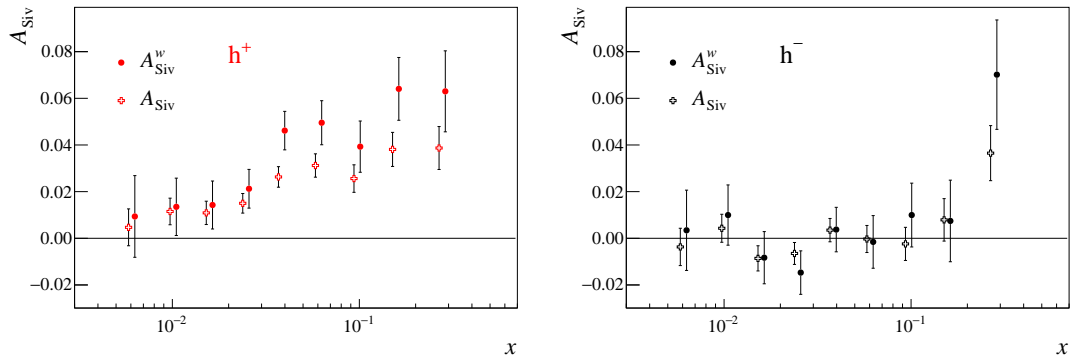


Fig. 3: Full points: A_{Siv}^w in the nine x bins for positive (left panel) and negative (right panel) hadrons. The open crosses are the unweighted Siverts asymmetries A_{Siv} [11], which are slightly shifted towards smaller x values for clarity.

the nine z bins have also similar shapes but different slopes. The distribution for $0.50 < z < 0.65$ and the mean values of w as function of z are shown in Fig. 2 for positive hadrons. Again, for negative hadrons the distributions are very much the same.

The measured weighted asymmetries are presented as a function of x in Fig. 3. The unweighted Siverts asymmetries [11] are also shown for comparison. As expected, the trends of the weighted and unweighted asymmetries are similar both for positive and negative hadrons. The asymmetry for positive hadrons is clearly different from zero, in particular at large x . In this range, the ratios $A_{\text{Siv}}^w/A_{\text{Siv}}$ are very close to the mean value of the weight. The statistical uncertainties are scaled by about the same ratio.

Assuming u -quark dominance for positive hadrons produced on a proton target, one has

$$A_{\text{Siv}}^w \simeq 2 \frac{f_{1T}^{\perp(1)u}(x, Q^2)}{f_1^u(x, Q^2)}, \quad (21)$$

and the results on A_{Siv}^w represent the first direct measurement of $f_{1T}^{\perp(1)u}/f_1^u$.

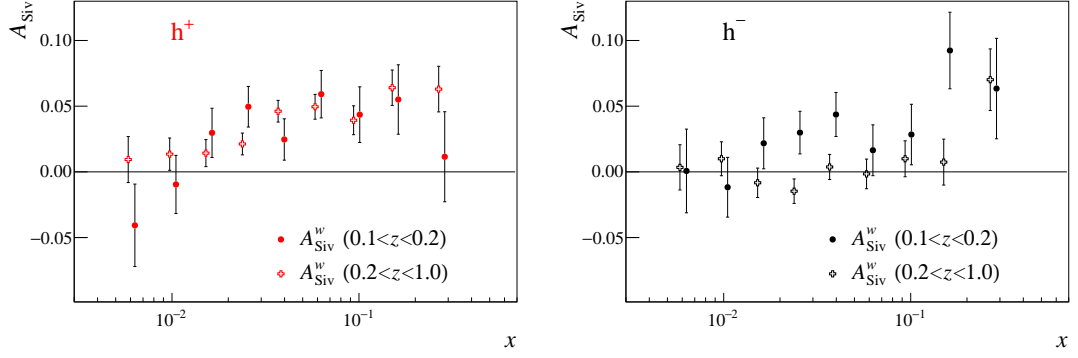


Fig. 4: Comparison of the weighted asymmetries vs. x measured in the range $(0.1 < z < 0.2)$ for positive (left) and negative (right) hadrons and the corresponding ones in the standard range $z > 0.2$, which are slightly shifted towards smaller x values for clarity.

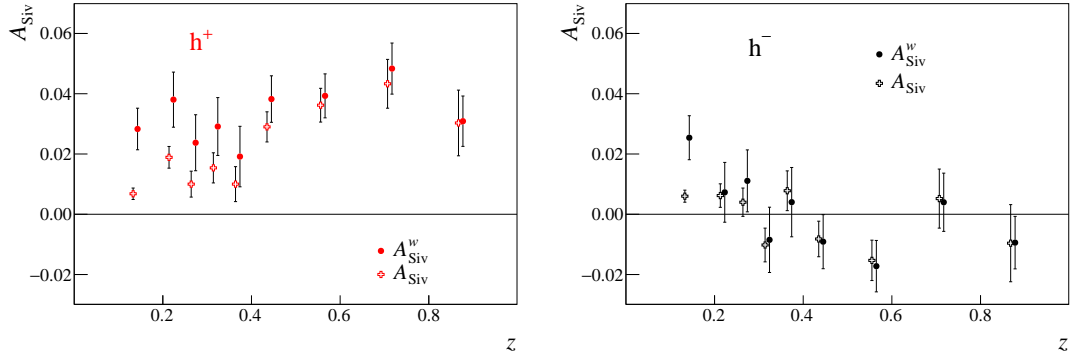


Fig. 5: Full points: A_{Siv}^w in the nine z bins for positive (left panel) and negative (right panel) hadrons. The open crosses are the corresponding unweighted Siverts asymmetries A_{Siv} [11], which are slightly shifted towards smaller x values for clarity.

In Fig. 4, the weighted Siverts asymmetries measured in our standard range $z > 0.2$ are compared with the corresponding ones in the range $0.1 < z < 0.2$. It is interesting to note that the positive-hadron asymmetries are basically unchanged, which emphasizes u -quark dominance and supports the idea that factorisation works already at small values of z in the COMPASS kinematic range. At low z , the difference between favoured and unfavoured fragmentation functions decreases, thus it is expected that the u -quark contribution to the negative-hadron asymmetry increases. The asymmetry itself is then expected to become larger and similar to the positive-hadron asymmetries, as observed in Fig. 4.

In order to further investigate the z dependence, it is of interest to look at A_{Siv}^w as a function of z , after integration over x . The results in the range $0.1 < z < 1$ are shown in Fig. 5. For positive hadrons, the values are almost constant within statistical uncertainties, as it is expected in the case of u -quark dominance if the measurement is performed in the current-fragmentation region and factorisation holds. The values of the measured P_T/zM -weighted asymmetries are given in Tables 2 and 3.

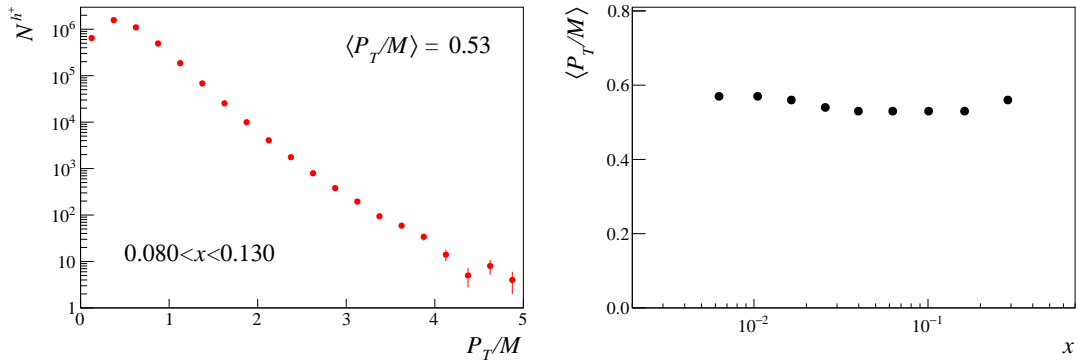


Fig. 6: Left: distribution of the weight $w' = P_T/M$ for positive hadrons in the bin $0.080 < x < 0.13$. Right: mean value of w' as a function of x . No acceptance correction applied.

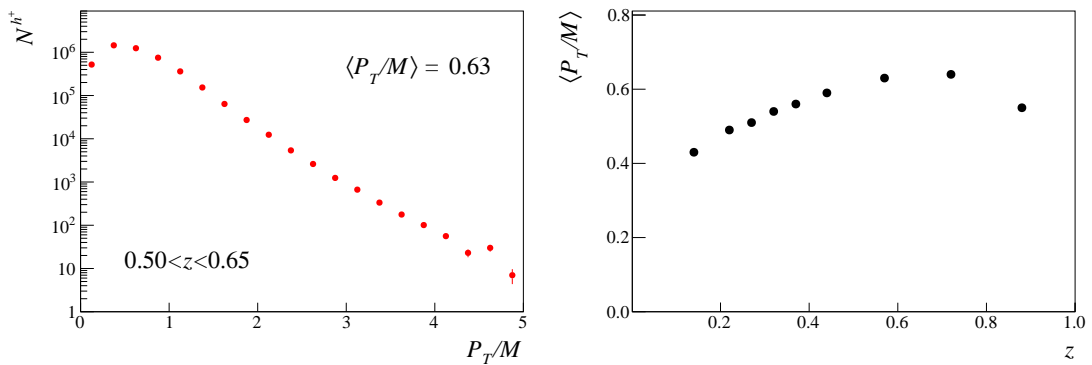


Fig. 7: Left: Distribution of the weight $w' = P_T/M$ for positive hadrons in the bin $0.50 < z < 0.65$. Right: mean value of w' as a function of z . No acceptance correction applied.

From the P_T/zM -weighted asymmetries it is straightforward to extract the first transverse moment of the Siverts function. This will be done in Section 6.

5 Siverts asymmetries weighted by P_T/M

Let us now turn to the Siverts asymmetries weighted with $w' = P_T/M$. The distributions of w' are very similar in all x and z bins. Examples of the distributions and the mean values of w' in the x and z bins for positive hadrons are given in Figs. 6 and 7, respectively, and in Table 4. Again, for negative hadrons the distributions are very much the same.

The results for $A_{\text{Siv}}^{w'}$ are shown in Fig. 8 for positive and negative hadrons. The ratio $R^{w'} = A_{\text{Siv}}^{w'}/A_{\text{Siv}}$ for positive hadrons is shown in Fig. 9. Correlations between numerator and denominator were accounted for. The ratio $R^{w'} = A_{\text{Siv}}^{w'}/A_{\text{Siv}}$ is almost constant as function of x with a mean value of 0.62, not far from that expected using the Gaussian model [see Eq. (17)], which is also shown in the figure.

In order to better investigate the z dependence, as in the case of the A_{Siv}^w asymmetries, the analysis

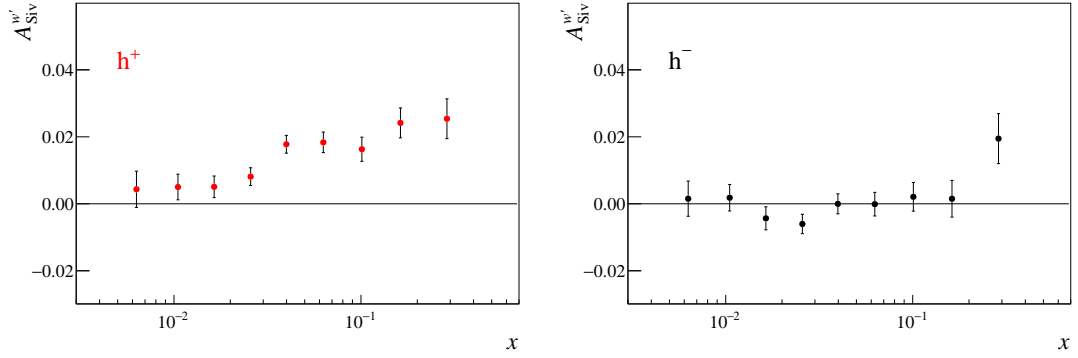


Fig. 8: The weighted asymmetry $A_{\text{Siv}}^{w'}$ with $w' = P_T^h/M$, as a function of x for positive (left) and negative (right) hadrons with $z > 0.2$.

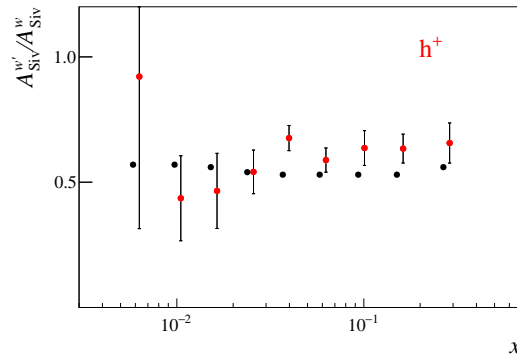


Fig. 9: Ratio $A_{\text{Siv}}^{w'}/A_{\text{Siv}}^{w'}$ as a function of x for positive hadrons and $z > 0.2$. The black points are the values of $4\langle z \rangle / \pi M \langle z / P_T \rangle$.

was repeated adding the hadrons with $0.1 < z < 0.2$. The results for the x -integrated asymmetry $A_{\text{Siv}}^{w'}$ as a function of z are shown in Fig. 10 for positive and negative hadrons. The values for positive hadrons are in qualitative agreement with the u -quark dominance approximation, i.e.:

$$A_{\text{Siv}}^{w'}(z) \sim z. \quad (22)$$

For comparison, the published Siverts asymmetries A_{Siv} [11] are also shown in the same figure. All values of the measured P_T/M -weighted asymmetries are given in Tables 5 and 6¹.

6 Point-by-point extraction of the first moments of the Siverts functions

The final goal of the measurement of the weighted Siverts asymmetries is the extraction of the first moments of the Siverts functions. Thus we consider the weighted asymmetry integrated over z (we

¹All the numerical values for the results presented in this paper, as well as the covariance matrices are available on HEPDATA [40].

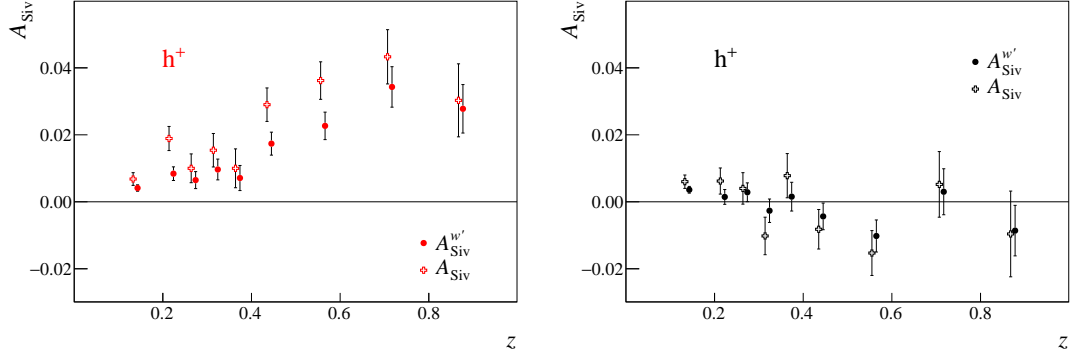


Fig. 10: Closed points: $A_{\text{Siv}}^{w'}$ with $w' = P_T^h/M$ in the nine x bins for positive (left panel) and negative (right panel) hadrons. The open crosses are the unweighted Siverson asymmetries A_{Siv} [11], which are slightly shifted towards smaller x values for clarity.

restore the Q^2 dependence):

$$A_{\text{Siv}}^w(x, Q^2) = 2 \frac{\sum_q e_q^2 x f_{1T}^{\perp(1)q}(x, Q^2) \tilde{D}_1^q(Q^2)}{\sum_q e_q^2 x f_1^q(x, Q^2) \tilde{D}_1^q(Q^2)}, \quad (23)$$

where

$$\tilde{D}_1^q(Q^2) = \int_{z_{\min}}^{z_{\max}} dz D_1^q(z, Q^2). \quad (24)$$

The denominator of Eq. (23) can be fully evaluated by resorting to global fits of distribution and fragmentation functions.

There are two sets of asymmetries, i.e. for unidentified positively (superscript +) and negatively (superscript -) charged hadrons. In our analysis, we omit the sea-quark Siverson distributions, which were shown to be negligible in a previous study [26]. The asymmetries then read (for simplicity we omit again the x and Q^2 dependence)

$$A_{\text{Siv}}^{w, \pm} = 2 \frac{4x f_{1T}^{\perp(1)u_v} \tilde{D}_1^{u, \pm} + x f_{1T}^{\perp(1)d_v} \tilde{D}_1^{d, \pm}}{9 \sum_q e_q^2 x f_1^q \tilde{D}_1^{q, \pm}}. \quad (25)$$

Denoting the denominator by δ^\pm

$$\delta^\pm \equiv 9 \sum_q e_q^2 x f_1^q \tilde{D}_1^{q, \pm}, \quad (26)$$

the valence Siverson distributions can be extracted from the asymmetries as follows

$$x f_{1T}^{\perp(1)u_v} = \frac{1}{8} \frac{\delta^+ A_{\text{Siv}}^{w, +} \tilde{D}_1^{d, -} - \delta^- A_{\text{Siv}}^{w, -} \tilde{D}_1^{d, +}}{\tilde{D}_1^{u, +} \tilde{D}_1^{d, -} - \tilde{D}_1^{d, +} \tilde{D}_1^{u, -}}, \quad (27)$$

$$x f_{1T}^{\perp(1)d_v} = \frac{1}{2} \frac{\delta^- A_{\text{Siv}}^{w, -} \tilde{D}_1^{u, +} - \delta^+ A_{\text{Siv}}^{w, +} \tilde{D}_1^{u, -}}{\tilde{D}_1^{u, +} \tilde{D}_1^{d, -} - \tilde{D}_1^{d, +} \tilde{D}_1^{u, -}}. \quad (28)$$

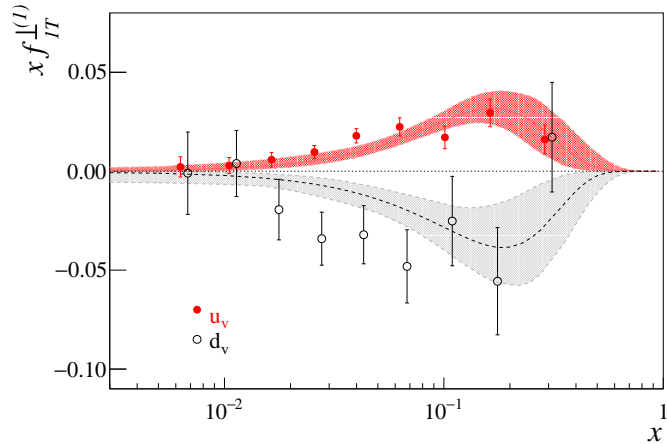


Fig. 11: Values of the first moment of the Siverts function for u (closed red dots) and d (open black dots) quarks from the P_T/zM weighted-Siverts asymmetries for charged hadrons with $z > 0.2$. The curves and the uncertainty bands are the results of the fit of Ref. [23].

Eqs. (27) and (28) allow for a point-by-point extraction of the Siverts distributions for valence quarks. For the distribution functions we use the CTEQ5D parametrisation [41] and for the fragmentation functions of unidentified hadrons the DSS parametrisation [42]. The results are displayed in Fig. 11 and tabulated in Table 7 together with the mean values of Q^2 (ranging from $1.24 \text{ (GeV}/c)^2$ to $25.6 \text{ (GeV}/c)^2$). The extracted values for $x f_{1T}^{\perp(1)u_v}$ and $x f_{1T}^{\perp(1)d_v}$ are correlated, as they are linear functions of the same two measured asymmetries, and the computed correlation coefficients are also given in Table 7.

The uncertainties are computed from the statistical uncertainties of the measured asymmetries, and no attempt was made to try to assign a systematic uncertainty to the results. The uncertainties in the extracted d_v Siverts distribution are much larger than the corresponding ones for the u_v quark. The u_v and d_v Siverts distributions are linear combinations [see Eqs. (27, 28)] of the same Siverts asymmetries for positive and negative hadrons on the proton, thus in principle sufficient for their determination, but the coefficient of proportionality is four times larger for the d quark, which makes the uncertainties of the extracted $x f_{1T}^{\perp(1)d_v}$ about four times larger than those of $x f_{1T}^{\perp(1)u_v}$.

In Fig. 11, we also show for comparison the results, i.e. central values and uncertainty bands, of the fit [23] to the HERMES proton data [9] and the COMPASS proton and deuteron data [38, 43], which uses DGLAP evolution. The results are compatible, with a slightly different trend of $x f_{1T}^{\perp(1)d_v}$ suggested by the present extraction.

It is also interesting to compare our present result with the point-by-point extraction of Ref. [26], where the pion Siverts asymmetries from the COMPASS proton [38] and deuteron [43] data are used as input. The data set used in Ref. [26] and the present one have the dominating pion data on the proton target in common, so that the results are strongly correlated. As can be seen in Fig. 12, in the present work the uncertainties on the extracted u_v and d_v Siverts function moments are on average smaller by a factor of about 1.5 with respect to the corresponding quantities in Ref. [26]. This is due to the fact that in the present analysis we had to assume the Siverts function

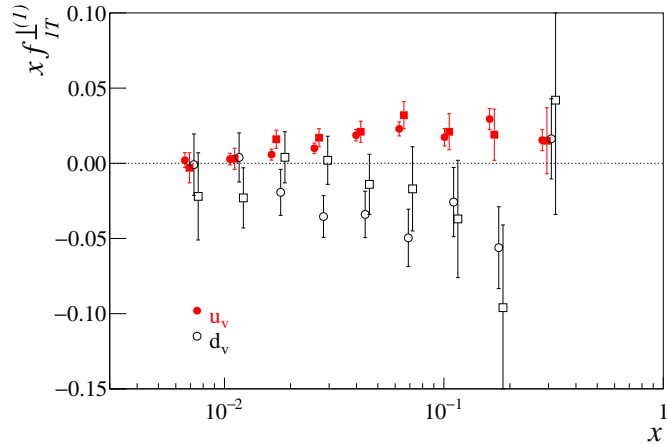


Fig. 12: Comparison of the values of the first moment of the Siverson function for u (closed red dots) and d (open black dots) quarks from the P_T/zM -weighted Siverson asymmetries for charged hadrons with $z > 0.2$, and the corresponding values obtained in Ref. [26] from the unweighted pion Siverson asymmetries measured by COMPASS on deuteron and proton (closed red and open black squares, respectively).

of the sea quarks to be zero. Following the method of Ref. [26] and imposing the sea-quark Siverson functions to be zero, we have determined the u_v and d_v functions from the π^+ and π^- proton asymmetries [38] only and verified that both the central values and the uncertainties are very similar to the ones presented in this paper. Thus the differences visible in Fig. 12 can be attributed to the impact of the deuteron data and to the extraction of the sea-quark Siverson function, rather than to the use of unweighted asymmetries. The assumption of a vanishing contribution from the sea quarks will be better verified only when more neutron data will be available.

7 Conclusions and outlook

COMPASS has measured the weighted Siverson asymmetries in SIDIS of 160 GeV muons on transversely polarised protons, extending the standard analysis of unweighted asymmetries. The weighted asymmetries were determined for positive and negative hadrons using as weight either P_T/zM or P_T/M . In both cases, the asymmetries were found to be positive for positive hadrons in the range $x > 0.013$ and compatible with zero for negative hadrons with $z > 0.2$, very much as in the case of the standard Siverson asymmetries. The z dependence for positive hadrons agrees with the expectation in the case of u -quark dominance and of a measurement performed in the current-fragmentation region.

From the P_T/zM -weighted Siverson asymmetries, and under the hypothesis of negligible Siverson functions for sea quarks, we have extracted the first moments of the Siverson functions for u_v and d_v quarks. In the leading-order pQCD formalism, the obtained values are model independent because of the use of weighted asymmetries and because of the point-by-point extraction. Previous model-dependent extractions that are based on the Gaussian ansatz compare well with our results.

The present analysis hints at the validity of the Gaussian parametrisation for the transverse-

momentum dependence of the Sivers distribution function and the fragmentation function, at least in the kinematic domain explored by our measurement. As in all other extractions of the Sivers functions from SIDIS asymmetries on transversely polarized nucleons, the d -quark Sivers function turns out to be poorly determined and strongly dependent on the assumptions on the Sivers functions of the sea quarks. This is due to the scarcity of Sivers asymmetry data taken with a transversely polarised deuteron target, as compared to the existing data taken with a transversely polarised proton target. The recently approved COMPASS run [44] with a transversely polarised deuteron target in 2021 is expected to allow for a much better extraction of the Sivers functions for both quarks and antiquarks.

Table 1: Mean values of the weight P_T/zM for positive hadrons in the nine bins of x for $z > 0.2$, and in the nine bins of z .

$\langle x \rangle$	$\langle P_T/zM \rangle$	$\langle z \rangle$	$\langle P_T/zM \rangle$
0.0063	1.80	0.14	3.13
0.0105	1.77	0.22	2.19
0.0164	1.72	0.27	1.88
0.0257	1.65	0.32	1.66
0.0399	1.60	0.37	1.50
0.0629	1.59	0.44	1.32
0.101	1.58	0.57	1.11
0.163	1.56	0.72	0.89
0.288	1.62	0.88	0.63

Table 2: Measured values of the P_T/zM -weighted Sivers asymmetries in the nine x bins.

x	$z > 0.2$		$0.1 < z < 0.2$	
	A_{Siv, h^+}^w	A_{Siv, h^-}^w	A_{Siv, h^+}^w	A_{Siv, h^-}^w
0.003–0.008	0.009 ± 0.018	0.003 ± 0.017	-0.041 ± 0.031	0.001 ± 0.032
0.008–0.013	0.013 ± 0.012	0.010 ± 0.013	-0.010 ± 0.022	-0.012 ± 0.023
0.013–0.020	0.014 ± 0.010	-0.008 ± 0.011	0.030 ± 0.019	0.022 ± 0.019
0.020–0.032	0.021 ± 0.008	-0.015 ± 0.009	0.050 ± 0.015	0.030 ± 0.016
0.032–0.050	0.046 ± 0.008	0.004 ± 0.010	0.025 ± 0.016	0.044 ± 0.017
0.050–0.080	0.050 ± 0.009	-0.002 ± 0.011	0.059 ± 0.018	0.016 ± 0.019
0.080–0.130	0.039 ± 0.011	0.010 ± 0.014	0.044 ± 0.021	0.028 ± 0.023
0.130–0.210	0.064 ± 0.013	0.007 ± 0.017	0.055 ± 0.026	0.092 ± 0.029
0.210–0.700	0.063 ± 0.017	0.070 ± 0.023	0.012 ± 0.034	0.063 ± 0.038

Table 3: Measured values of the P_T/zM -weighted Sivers asymmetries in the nine z bins.

z	A_{Siv, h^+}^w	A_{Siv, h^-}^w
0.10–0.20	0.0283 ± 0.0069	0.0254 ± 0.0073
0.20–0.25	0.0380 ± 0.0091	0.0073 ± 0.0099
0.25–0.30	0.0237 ± 0.0093	0.0111 ± 0.0103
0.30–0.35	0.0291 ± 0.0096	-0.0085 ± 0.0108
0.35–0.40	0.0191 ± 0.0100	0.0040 ± 0.0115
0.40–0.50	0.0382 ± 0.0077	-0.0091 ± 0.0090
0.50–0.65	0.0393 ± 0.0073	-0.0172 ± 0.0085
0.65–0.80	0.0483 ± 0.0085	0.0040 ± 0.0097
0.80–1.00	0.0309 ± 0.0084	-0.0094 ± 0.0087

Table 4: Mean value of the weight P_T/M for positive hadrons in the nine x bins for $z > 0.2$, and in the nine z bins.

$\langle x \rangle$	$\langle P_T/M \rangle$	$\langle z \rangle$	$\langle P_T/M \rangle$
0.0063	0.57	0.14	0.43
0.0105	0.57	0.22	0.49
0.0164	0.56	0.27	0.51
0.0257	0.54	0.32	0.54
0.0399	0.53	0.37	0.56
0.0629	0.53	0.44	0.59
0.101	0.53	0.57	0.63
0.163	0.53	0.72	0.64
0.288	0.56	0.88	0.55

Table 5: Measured values of the P_T/M -weighted Siverts asymmetries in the nine x bins.

x	$z > 0.2$		$0.1 < z < 0.2$	
	$A_{\text{Siv}}^{w'}, h^+$	$A_{\text{Siv}}^{w'}, h^-$	$A_{\text{Siv}}^{w'}, h^+$	$A_{\text{Siv}}^{w'}, h^-$
0.003–0.008	0.0094 ± 0.0175	0.0034 ± 0.0172	-0.0407 ± 0.0314	0.0007 ± 0.0318
0.008–0.013	0.0135 ± 0.0123	0.0100 ± 0.0129	-0.0096 ± 0.0221	-0.0117 ± 0.0227
0.013–0.020	0.0143 ± 0.0103	-0.0083 ± 0.0112	0.0297 ± 0.0187	0.0217 ± 0.0194
0.020–0.032	0.0212 ± 0.0083	-0.0147 ± 0.0093	0.0496 ± 0.0155	0.0299 ± 0.0162
0.032–0.050	0.0462 ± 0.0083	0.0037 ± 0.0095	0.0246 ± 0.0157	0.0437 ± 0.0168
0.050–0.080	0.0495 ± 0.0094	-0.0016 ± 0.0113	0.0591 ± 0.0180	0.0164 ± 0.0194
0.080–0.130	0.0393 ± 0.0110	0.0100 ± 0.0137	0.0436 ± 0.0212	0.0284 ± 0.0230
0.130–0.210	0.0640 ± 0.0135	0.0074 ± 0.0175	0.0551 ± 0.0264	0.0924 ± 0.0291
0.210–0.700	0.0630 ± 0.0174	0.0701 ± 0.0234	0.0115 ± 0.0343	0.0634 ± 0.0382

Table 6: Measured values of the P_T/M -weighted Siverts asymmetries in the nine z bins.

z	$A_{\text{Siv}, h^+}^{w'}$	$A_{\text{Siv}, h^-}^{w'}$
0.01–0.20	0.0041 ± 0.0010	0.0036 ± 0.0010
0.20–0.25	0.0084 ± 0.0020	0.0015 ± 0.0022
0.25–0.30	0.0065 ± 0.0025	0.0028 ± 0.0028
0.30–0.35	0.0097 ± 0.0031	-0.0027 ± 0.0035
0.35–0.40	0.0071 ± 0.0037	0.0015 ± 0.0043
0.40–0.50	0.0173 ± 0.0034	-0.0044 ± 0.0040
0.50–0.65	0.0227 ± 0.0041	-0.0102 ± 0.0048
0.65–0.80	0.0343 ± 0.0060	0.0030 ± 0.0069
0.80–1.00	0.0278 ± 0.0072	-0.0086 ± 0.0075

Table 7: Values of the first moments of the Siverts functions for u and d quarks. The last column gives their correlation coefficient ρ .

$\langle x \rangle$	$\langle Q^2 \rangle$ (GeV/c) ²	$xf_{1T}^{\perp(1)u_v}$	$xf_{1T}^{\perp(1)d_v}$	ρ
0.0063	1.27	0.0022 ± 0.0051	-0.001 ± 0.021	-0.26
0.0105	1.55	0.0029 ± 0.0040	0.004 ± 0.017	-0.31
0.0164	1.83	0.0058 ± 0.0037	-0.019 ± 0.015	-0.37
0.0257	2.17	0.0097 ± 0.0033	-0.034 ± 0.013	-0.43
0.0399	2.82	0.0179 ± 0.0036	-0.032 ± 0.015	-0.52
0.0629	4.34	0.0224 ± 0.0046	-0.048 ± 0.019	-0.63
0.101	6.76	0.0171 ± 0.0057	-0.025 ± 0.023	-0.68
0.163	10.6	0.0295 ± 0.0070	-0.056 ± 0.027	-0.65
0.288	20.7	0.0160 ± 0.0073	0.017 ± 0.028	-0.40

Appendix A - Transverse-momentum convolution in the weighted asymmetry

The Sivers asymmetry weighted with the factor $w = P_T/zM$ reads

$$A_{\text{Siv}}^w(x, z) = \frac{\sum_q e_q^2 x \int d^2 P_T \frac{P_T}{zM} \mathcal{C} \left[\frac{P_T \cdot k_T}{MP_T} f_{1T}^{\perp q}(x, k_T^2) D_1^q(z, p_T^2) \right]}{\sum_q e_q^2 x f_1^q(x) D_1^q(z)}. \quad (29)$$

The numerator contains the integral

$$\begin{aligned} & \int d^2 P_T \frac{P_T}{zM} \mathcal{C} \left[\frac{P_T \cdot k_T}{MP_T} f_{1T}^{\perp q} D_1^q \right] \\ &= \int d^2 P_T \frac{P_T}{zM} \int d^2 k_T \int d^2 p_T \delta^2(zk_T + p_T - P_T) \frac{P_T \cdot k_T}{MP_T} \\ & \quad f_{1T}^{\perp q}(x, k_T^2) D_1^q(z, p_T^2). \end{aligned} \quad (30)$$

Using the delta function to integrate over P_T gives

$$\begin{aligned} & \int d^2 P_T \frac{P_T}{zM} \mathcal{C} \left[\frac{P_T \cdot k_T}{MP_T} f_{1T}^{\perp q} D_1^q \right] \\ &= \int d^2 k_T \frac{1}{zM^2} \int d^2 p_T (zk_T^2 + k_T \cdot p_T) f_{1T}^{\perp q}(x, k_T^2) D_1^q(z, p_T^2) \\ &= \int d^2 k_T \frac{k_T^2}{M^2} f_{1T}^{\perp q}(x, k_T^2) \int d^2 p_T D_1^q(z, p_T^2) \\ &= 2 f_{1T}^{\perp(1)q}(x) D_1^q(z). \end{aligned} \quad (31)$$

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