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Signatures of new physics versus the ridge phenomenon in hadron-hadron collisions at the LHC

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Abstract

In this paper, we consider the possibility that a new stage of matter, stemming from hidden/dark sectors beyond the Standard Model, to be formed in *pp* collisions at the LHC, can significantly modify the correlations among final-state particles. In particular, two-particle azimuthal correlations are studied by means of a Fourier series sensitive to the near-side ridge effect while assuming that hidden/dark particles decay on top of the conventional parton shower. Then, new (fractional) harmonic terms should be included in the Fourier analysis of the azimuthal anisotropies, encoding the hypothetical new physics contribution enabling its detection in a complementary way to other signatures.

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The interest of discovering new physics (NP) beyond the Standard Model (SM) at the LHC is out of doubt. Along the last decades, many and distinct strategies have been put forward, most of them based on signatures in the transverse plane with respect to the beams axis like mono-jets, missing transverse energy, displaced vertices and so on. On the other hand, other kind of rather "diffuse" signals have been examined in the literature, e.g. [1,2], featuring the whole event (multiplicity distribution and moments, event shape variables, underlying event etc.) as a key signature of NP. For instance, the so-called "soft bomb" scenario [3] is characterized by high multiplicity events with nearly spherically distributed soft SM particles, and a large amount of missing transverse energy. In particular, a strongly coupled hidden/dark sector could lead to large angle emission of partons carrying a non-negligible amount of momentum, yielding a rather isotropic distribution of final-state particles, all sharing a similar amount of energy. Notice, however, that a likely complicated hidden sector (HS) beyond the SM may have limited observable effects at colliders, making hard the detection from SM background and especially the discimination among different models [4,5]. Thereby, alternative signatures, as proposed in this work, should be considered as complementary to other search strategies as discussed in [3].

Indeed, as is well known, (pseudo)rapidity and azimuthal particle correlations provide a crucial insight into the underlying mechanism of particle production (see [6] for a review). Moreover, from general arguments based on causality, long-range correlations should have the origin at very early times after the collision. Therefore, if the parton shower were to be altered by the presence of a non-conventional state of matter, final-state particle correlations should be sensitive to it [7].

The two-particle correlation function is often defined in pseudorapidity and azimuthal space as [8]

$$C(\Delta\eta,\Delta\phi)=\frac{S(\Delta\eta,\Delta\phi)}{B(\Delta\eta,\Delta\phi)}\ . \tag{1}$$

Here, S denotes the signal distribution built with particle pairs from the same event while B stays for the background distribution constructed by particle pairs taken from different events; $\Delta\eta=\eta_1-\eta_2$ and $\Delta\phi=\phi_1-\phi_2$ denote, respectively, the pseudorapidity and azimuthal differences of particles 1 and 2, the indexes labelling the trigger and associate particles, respectively.

Typically, a complex structure of the correlation function is observed, in particular, an enhancement of the two-particle correlations is found at $\Delta\phi\simeq 0$ in heavy-ion collisions [9]. Because of its extended longitudinal (pseudorapidity) shape, as seen in the $\Delta\eta$–$\Delta\phi$ plot, it is referred to as the (near-side) ridge. One-dimensional correlation functions $C(\Delta\phi)$ are obtained from Eq. (1) by integration over pseudorapidity along the range $2<|\Delta\eta|<5$ to focus on long-range correlations.

The observed azimuthal anisotropy in heavy-ion collisions is commonly analysed by means of a Fourier decomposition:

$$C(\Delta\phi)\ \sim 1+2\sum_{n=1}^{\infty}V_n\cos\left(n\Delta\phi\right)\ , \tag{2}$$

where the coefficients V_n are supposed to factorize as the product of the coefficients of the equivalent Fourier expansion of two single-particle densities. When applied to heavy-ion collisions, the different terms in the series of Eq. (2) find a "natural" interpretation according to a hydrodynamical model describing the very hot and dense matter resulting from the collision. In practice, up to five or six Fourier terms are taken into account in the analysis of the experimental data.

Remarkably, similar long-range ridge structures show up in proton-nucleus [8,10] and even proton-proton [10,11] collisions, under several conditions on events, like high multiplicity and a given transverse momentum range of charged particles. The interpretation of a positive V_2 in these small systems is currently highly debated, and different observables have been proposed to probe new dynamical effects related to large hadronic densities [12].

In this paper we furthermore consider that hidden particles and states, stemming from Hidden Valley (HV) models [13], can be formed at primary interactions in very high energy pp collisions. Generically, a HV model consists of three sectors: (i) a HS containing v-particles charged under a valley group $G_{\rm v}$ but

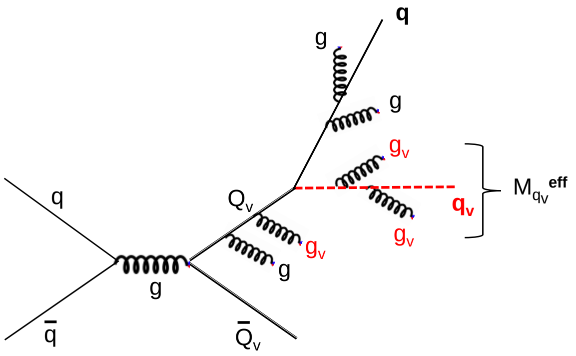

Figure 1: Pair production via $q\bar{q}$ fusion of a pair of mediators ($Q_{\rm v}\bar{Q}_{\rm v}$) bearing both SM and hidden charges. The decay $Q_{\rm v} \to q + q_{\rm v}$ can originate SM and hidden cascades from gluon and v-gluon emission.

blind to the SM interactions, (ii) a visible sector including SM particles charged under the SM group $G_{\rm SM}$ but neutral under $G_{\rm v}$, and (iii) mediators connecting both visible and hidden sectors. Usually, the masses of the hidden sector particles are assumed to lie below the electroweak scale while the mediators may have TeV-scale masses. The simplest possibility for $G_{\rm v}$ is a QCD-like scenario, with a strong (running) coupling constant $\alpha_{\rm v}$ and confinement scale $\Lambda_{\rm v}$. The SM sector could feebly couple to the HS (and the equivalent hadronic v-particles and states) via a neutral Z' or via heavy particles bearing both $G_{\rm SM}$ and $G_{\rm v}$ charges. Here, we consider the latter possibility along the lines of the Monte Carlo (MC) study using PYTHIA [14], where the hidden shower is controlled basically by two parameters: the coupling strength $\alpha_{\rm v}$ assumed to be a constant (i.e. no running is considered) and the lower cut-off scale set equal to 0.4 GeV as by default in QCD showers, consistent with a low hidden confinement scale $\Lambda_{\rm v}$. Such a simplified picture is compatible with the expected walking behaviour requiring a strong coupling over a large energy window along the showering before reaching $\Lambda_{\rm v}$, thereby yielding a large number of hidden partons and final-state particles. At the end, the energy from the primary interaction is democratically shared by soft final-state SM particles, while no classical jet structure is expected, thereby adapting quite well to a soft-bomb scenario. As commented, the ultimate goal in this paper is to show that long-range azimuthal correlations among final-state particles should emerge as a consequence of such kind of scenario.

Focusing only on the particle content relevant to the study presented here, we collectively denote by $Q_{\rm v}$ the (spin 1/2) hidden partners of the SM quarks, charged under both $G_{\rm v}$ and $G_{\rm SM}$, while $g_{\rm v}$ and $q_{\rm v}$ stand for the v-gluon and (spin 0) v-quark only charged under the $G_{\rm v}$ hidden group [3], respectively.

Special mention deserves the unparticle scenario, which can be viewed as a special case of HV models. Let us recall that, from a phenomenological point of view, an unparticle [15] does not have a fixed invariant mass squared but instead a continuous mass spectrum. As pointed out in the literature (see e.g. [16]), direct detection of unparticle stuff at colliders should rely on peculiar missing energy distributions. Unparticle production influence on particle correlations would become another useful tool to study such scenario, as shown in this work.

For some parameter values of HV models, hidden particles could promptly decay back into SM particles, altering the subsequent conventional parton shower [17] and yielding (among others [4]) observable consequences, e.g. extremely long-range correlations especially in azimuthal space [18]. In this paper we do not enter into details about specific models but limit ourselves to general features associated with the production of very massive objects on top of the parton shower and their observable consequences, mainly from kinematic constraints.

Our analysis focuses on $Q_{\rm v}$ pair-production via gg or $q\bar{q}$ fusion (see Fig. 1), subsequently decaying into a v-quark and a SM quark: $Q_{\rm v} \to q_{\rm v} + q + X$, where X stands for an ensemble of radiated gluons and v-gluons which, in turn, will originate visible and hidden parton cascades. Note that a very massive $Q_{\rm v}$

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[3] The notation used follow that of [14] for the HS in the PYTHIA 8 MC generator.

would be produced at a rather *low velocity* during the primary parton-parton interaction in pp collisions at the LHC. In fact, assuming that the centre-of-mass subenergy of the parton-parton interaction is of the order of or higher than twice magnitude of the mediator mass ($\sqrt{\hat{s}} \geq 2M_{Q_{\rm v}}$), then $Q_{\rm v}$ states can be on-shell pair-produced. Moreover, all (either SM or hidden) particles stemming from its decay should have access to a limited energy due to v-gluon radiation. In sum, final particles would "democratically" share the centre-of-mass energy released in the primary collision and rather soft and diffuse signatures are expected.

Below we use velocity of very heavy hidden sources for kinematic estimates involving angular distributions [4]. Moreover, we assume an isotropic parton emission in the hidden particle rest frame, coming out from the primary interaction, slightly boosted in the laboratory reference frame due to a non-relativistic velocity of the above-mentioned hidden source. We consider that this assumption provides the essential framework for our estimates and conclusions.

In the hidden source $Q_{\rm v}$ rest frame, the product of the velocity $v_{\rm h}$ and the Lorentz factor $\gamma_{\rm h} = (1-v_{\rm h}^2)^{-1/2}$ of the fragmenting v-quark is roughly given by

$$v_{\rm h}\gamma_{\rm h} = \frac{M_{Q_{\rm v}}^2 - M_{q_{\rm v}}^{\rm eff\,2}}{2M_{Q_{\rm v}}M_{q_{\rm v}}^{\rm eff}}\ , \tag{3}$$

where the bare q-mass was set equal to zero. The effective v-quark invariant mass, denoted as $M_{q_{\rm v}}^{\rm eff}$, is defined in a similar way as in conventional QCD jets, i.e.

$$M_{q_{\rm v}}^{\rm eff} = \sqrt{(\sum_j E_j)^2 - (\sum_j \vec{p}_j)^2}\ , \tag{4}$$

where E_j and $\vec{p}_j$ stands for the energy and three-momentum of the v-gluons emitted by the fragmenting v-quark, and the sum on j runs over all emitted v-gluons. Even though the bare $q_{\rm v}$-mass could be as light as 10 GeV, $M_{q_{\rm v}}^{\rm eff}$ can reach values close to $M_{Q_{\rm v}}$ because of radiation, as happens in QCD jets [14]. This would be especially the case for a strongly interacting hidden/dark sector, i.e. at large $\alpha_{\rm v}$. We look upon expression (3) as providing an order of magnitude estimate of the v-quark velocity. Of course, large variations of the $v_{\rm h}\gamma_{\rm h}$ factor will occur event by event because of the wide spread of $M_{q_{\rm v}}^{\rm eff}$.

In its turn, bound v-states can be formed as v-gluons create new v-quark-antiquark pairs, as happens with gluons in a conventional QCD shower. In HV models with v-hadrons promptly decaying back into SM partons, a new SM parton cascade would be originated (coexisting with invisible particles) eventually leading to final-state SM particles as well. Furthermore, as the $q_{\rm v}$ radiates more and more v-gluons, and the mean value of its effective mass $M_{Q_{\rm v}}^{\rm eff}$ distribution shifts from $M_{q_{\rm v}}$ towards $M_{Q_{\rm v}}$, more and more energy is subtracted from the visible quark and its associated system of emitted gluons. In sum, a strong coupling $\alpha_{\rm v}$ should lead to small velocities of both SM and hidden particles.

Indeed, under a Lorentz boost of velocity $v_{\rm h}$, the angular distribution of the final-state particles in the laboratory reference frame (LRF), the latter almost coinciding with the fragmenting $q_{\rm v}$ reference frame, is given by [20]

$$w(\phi - \phi_{\rm h}) = \frac{1}{\gamma_{\rm h}[1 - v_{\rm h}^2 \cos^2(\phi - \phi_{\rm h})]}\ f(\phi, g)\ . \tag{5}$$

Here, $f(\phi, g) = (g \pm \sqrt{D})/\pm\sqrt{D}$ with $D = 1 + \gamma_{\rm h}^2(1-g^2)\tan^2(\phi - \phi_{\rm h})$, and $g = v_{\rm h}/v$ with the final-state particle velocity v in the $q_{\rm v}$ rest frame. For $g \ll 1$, one can roughly set $f(\phi, g) \approx 1$. A massive hidden object of spin zero as assumed for the fragmenting v-quark in this work (leading to a nearly spherical distribution in the $q_{\rm v}$-quark reference frame), with $v_{\rm h}$ being non-relativistic, plainly justifies such an approximation.

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[4] Velocity may become a well-defined physical and meaningful quantity when dealing with heavy particles [19].

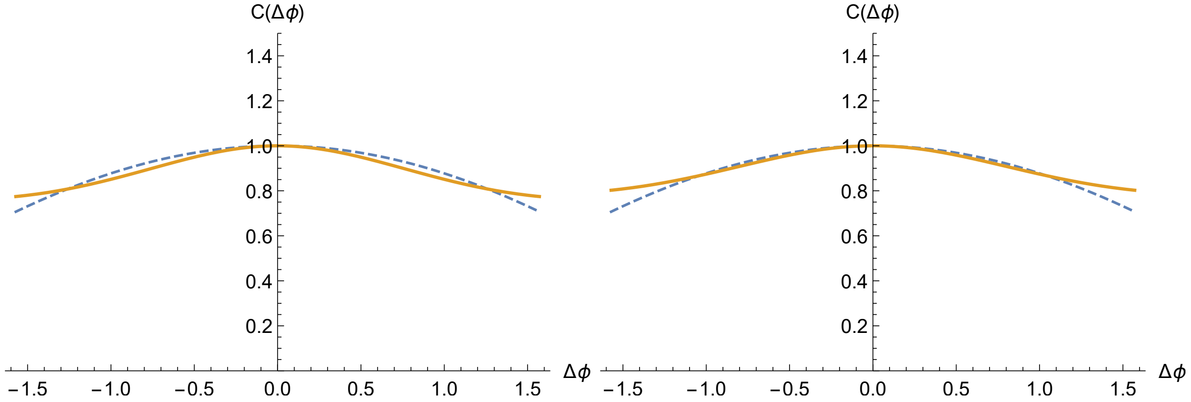

Figure 2: Expected contribution to the azimuthal dependence of the correlation function $C(\Delta\phi)$ from a massive hidden/dark sector (solid orange line). Left panel: $M_{Q_{\rm v}} = 300$ GeV and $M_{q_{\rm v}}^{\rm eff} = 200$ GeV. Right panel: $M_{Q_{\rm v}} = 1000$ GeV and $M_{q_{\rm v}}^{\rm eff} = 700$ GeV. A non-relativistic hidden source is considered in both cases. For comparison the $\cos(\Delta\phi/2)$ modulation (dotted blue line) is shown. The correlation function is normalized to unity at $\Delta\phi = 0$ to be compared with the $\cos(\Delta\phi/2)$ modulation.

The azimuthal distribution $w(\phi - \phi_{\rm h})$ can be then approximated by a Gaussian for small $\phi - \phi_{\rm h}$ angles for practical purposes, namely,

$$w(\phi - \phi_{\rm h}) \;\approx\; \exp\left[-\frac{(\phi-\phi_{\rm h})^2}{2\delta_{{\rm h}\phi}^2}\right] \quad , \quad \delta_{{\rm h}\phi} \;\simeq\; \frac{1}{\sqrt{2}\; v_{\rm h}\gamma_{\rm h}} \;, \tag{6}$$

where $\delta_{{\rm h}\phi}$ was interpreted as an azimuthal cluster “width” in [21] [5]. Large hidden source velocities lead to small $\delta_{{\rm h}\phi}$ and thereby a more pronounced peak at $\phi \simeq \phi_{\rm h}$, in accordance with Eq. (5). Conversely, small velocities of the hidden source lead to flatter azimuthal distributions.

Substituting Eq. (3) into Eq. (6) for $\delta_{{\rm h}\phi}$, one gets

$$\delta_{{\rm h}\phi} \;\simeq\; \frac{\sqrt{2} M_{Q_{\rm v}} M_{q_{\rm v}}^{\rm eff}}{M_{Q_{\rm v}}^2 - M_{q_{\rm v}}^{\rm eff 2}} \;, \tag{7}$$

where $M_{q_{\rm v}}^{\rm eff}$ stands for the effective mass resulting from v-gluon radiation as mentionned above.

Next, by Taylor expanding the exponential we can identify the above expression with a cosine function such that $1/\delta_{{\rm h}\phi}$ determines the leading Fourier component of the NP contribution from a given range of the effective v-quark mass. As reference values, we set $M_{Q_{\rm v}} = 1000$ GeV and $M_{q_{\rm v}}^{\rm eff} = 700$ GeV [14], yielding the closest fractional number

$$\frac{1}{{\rm Integer}[\delta_{{\rm h}\phi}]} \;=\; \frac{1}{2} \;. \tag{8}$$

This estimate can be extended to the mass interval of the v-quark invariant mass $M_{q_{\rm v}}^{\rm eff} \in [630, 760]$ GeV, leading to the NP contribution

$$w(\phi - \phi_{\rm h}) \;\approx\; \cos\left[(\phi - \phi_{\rm h})/2\right] \tag{9}$$

from this $M_{q_{\rm v}}^{\rm eff}$ mass “slice”.

By integration of the product of the two single particle azimuthal distributions, one gets

$$C(\Delta\phi) \;\approx\; \frac{1}{2\pi}\int_0^{2\pi} \cos\left[(\phi_1 - \phi_{\rm h})/2\right] \cos\left[(\phi_2 - \phi_{\rm h})/2\right]\, d\phi_{\rm h} \;=\; 2\; \cos\left[(\phi_1 - \phi_2)/2\right] \;. \tag{10}$$

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[5] As we are focusing on azimuthal angles, the particle trajectories are projected onto the transverse plane, hence the velocities $v_{\rm h}$, v and the Lorentz factor $\gamma_{\rm h}$ actually correspond to transverse velocities.

In Fig. 2 we show the expected angular dependence of the corresponding Fourier term in the correlation function $C(\Delta\phi)$ for two reference benchmarks: *(a)* $M_{Q_{\rm v}} = 300$ GeV and $M_{q_{\rm v}}^{\rm eff} = 200$ GeV, and *(b)* $M_{Q_{\rm v}} = 1000$ GeV and $M_{q_{\rm v}}^{\rm eff} = 700$ GeV. All hidden initial sources from the primary collision originating the subsequent visible/invisible shower are assumed to be non-relativistic (g is taken of the order of 0.1 in Eq. (5)). A comparison with the $\cos{(\Delta\phi/2)}$ modulation, shown at the same plot, points out that the HS contribution should yield a Fourier component dominated by this term (not yet considered so far in any analysis to our knowledge).

Actually, more fractional harmonic terms should be considered as the whole mass range of the effective mass $M_{q_{\rm v}}^{\rm eff}$ (up to M_{Q_v}) is taken into account in the expected continous spectrum obtained from radiation [14]. Hence, the Fourier series should be more generally written as:

$$C(\Delta\phi)\ \sim 1+2\sum_{n=1}^{\infty} V_n \cos{(n\Delta\phi)} + 2\sum_{m=1}^{\infty} V'_{1/m} \cos{(\Delta\phi/m)}\ , \tag{11}$$

where the extra $\cos{(\Delta\phi/m)}$ harmonic terms encode the angular anisotropies associated with massive hidden states modifying the parton shower and thereby correlations among final-state particles. The $V'_{1/2}$ term should be the leading component in these fractional harmonic terms. Notice that the $\cos{(\Delta\phi)}$ term has now two contributions: a negative one ($V_1 < 0$) from the conventional series of Eq. (2), and another positive one ($V'_1 > 0$) expected from a HS.

Of course, the Fourier analysis using Eq.(11) will contain contributions from both the conventional partonic cascade and from the hidden sector. In order to enhance such a hypothetical NP contribution, extra selection cuts beyond high multiplicity and usual p_T ranges of charged hadrons should be applied on events, in particular high p_T leptons and/or missing transverse energy/momentum.

Indeed, multi-lepton signatures have already been used in the search of NP at the LHC (see e.g. [22,23]) as they are predicted by many models beyond the SM. For example, a cascade of partons/particles initiated by the decay of a heavy hidden particle can proceed though intermediate states yielding electrons, muons or tau leptons in the final state. Realistic requirements would imply an electron or muon with $p_T > 20$ GeV and $|\eta| < 2.5$, a second electron or muon with slightly looser requirements and a third electron, muon or hadronically decaying tau. Moreover, lepton combinations of the same electric charge can be used to enhance the NP signature. Additional cuts can be large missing E_T, since the hidden/conventional cascade can result in invisible particles at the end of the decay chain. Lastly, as the decay of hidden particles to bottom quarks can be largely enhanced in many hidden models, b-tagging would be another technique to be applied to enrich the sample with NP events.

Notice that such proposed cuts (aside a common high-multiplicity cut) hardly could be attributed to the formation of QGP or glass condensates, but associated with the presence of NP.

Thereby the sample would be enriched with NP events enhancing the ridge effect if due to this non-standard mechanism. Then the non-vanishing values of $V'_{1/2}$, $V'_{1/3}$ and so on, resulting to a better fit than the conventional Fourier analysis, provide a hint of NP, complementary to other kinds of searches.

Let us finally remark that, from the observation of the near-side ridge effect in hadronic collisions, an integrated luminosity of order of tens of pb^{-1} would be needed to observe any possible NP emergent effect, provided that the HS production cross section turns out to be large enough, crucially depending on the $Q_{\rm v}$ mass [14]. In order to avoid pile-up effects, a dedicated low-luminosity run would be desirable at the LHC.

Summarizing, hidden/dark sectors production on top of the parton shower in pp collisions can sizeably alter final-state particle correlations which can become a signature of NP. Specifically, more fractional $\cos{(\Delta\phi/m)}$ harmonic terms should be included in the Fourier series when carrying out the analysis of the azimuthal correlation function $C(\Delta\phi)$, once appropriate selection cuts are applied to events.

Acknowledgments

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