



# LEADING AND NONLEADING $D^\pm$ PRODUCTION IN THE VALON MODEL

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## Abstract

Recent data on  $D^\pm$  production in the pion fragmentation region disagree with existing theoretical predictions. It is shown that the result obtained in the valon model without any adjustable parameters agrees well with the data. The dynamical mechanism is recombination of the projectile partons. Prediction on the ratio  $\sigma(\pi^- \rightarrow D^+)/\sigma(\pi^- \rightarrow \pi^+)$  is made as a further test of the valon model.

Recent experimental data on the production of charm mesons in the  $\pi^\pm$ - $p$  interaction show significant difference between the  $x_F$  dependences of the leading and nonleading charm particles ( $D^\pm$ ) [1, 2]. As pointed out in [2], that difference cannot be accounted for either in perturbative QCD [3] or by the Lund string model implemented by PYTHIA [4]. In this paper we show how the valon model for low- $p_T$  interaction [5] can produce the result that agrees with data without adjusting any parameters.

More specifically, the data referred to above are on the asymmetry  $A$  defined by

$$A = \frac{\sigma(\text{leading}) - \sigma(\text{nonleading})}{\sigma(\text{leading}) + \sigma(\text{nonleading})} \quad (1)$$

where  $\sigma(\text{leading})$  is the cross section for  $\pi^\pm(\text{beam})$  to  $D^\pm$ , and  $\sigma(\text{nonleading})$  is for  $\pi^\pm$  to  $D^\mp$ . All quantities in (1) are for a common value of  $x_F$ . The data exhibit a rapid rise in  $A(x_F)$ , as  $x_F$  is increased from 0.2 to 0.7, as shown in Fig. 1. They imply the approximate equality of the two cross sections at small  $x_F$ , but  $\sigma(\text{nonleading})$  decreases rapidly with increasing  $x_F$ . This feature suggests the strong connection between the quantum numbers and momenta of the quarks in the beam particle and

of those in the inclusively measured outgoing particle. That connection is the basis of the recombination model for low- $p_T$  production [6], which has been found to be very successful in a variety of inclusive processes in the beam fragmentation region [7, 8].

While the recombination model as originally described [6] is simple and physically direct, it lacks the sophistication to treat charm production quantitatively. The valon model [5] is a more complete formulation of the low- $p_T$  production process with more precise determination of the multiquark distribution in the beam particle. The role of valons in a hadronic collision problem is analogous to the role of constituent quarks in the hadronic bound-state problem. The hadronization part of the collision problem is recombination, not fragmentation [4]. The physical content of the model is first to regard the hadronization of the projectile in the beam fragmentation region as being mainly controlled by the quark (and antiquark) distributions in the projectile, minimally disturbed by the target at high energy, since the partons in the target are far in rapidity separation from those in the projectile and thus have little influence due to short-range (in rapidity) soft interaction among the partons. In this respect soft parton interaction in hadronic collisions is drastically different from hard interaction in  $e^+e^-$  annihilation or large- $p_T$  production, for which hadronization through fragmentation of the parton produced at high virtuality is more relevant.

In the valon model the valence quark and its associated sea quarks belong to one or another of the valons in a hadron. The momentum-fractions of the valons are precisely known because of the sum rules on the number and momenta of the valons. The multiquark distributions of the quarks and antiquarks in a hadron are therefore also known, even at low  $Q^2$  due to precocious scaling. The inclusive distribution of the produced meson is then determined by a convolution of the multiquark distribution with the recombination function, whose dependence on the quark and antiquark momenta is fixed by the valon distribution of the produced meson. For the production of light mesons there are no free parameters [5]. For the production of heavy mesons there is one unknown parameter governing the normalization of the probability of having a heavy quark in a valon. But that parameter is cancelled in a ratio such as  $A$  in (1). Thus in our calculation of  $A$  below there will be no adjustable parameter.

At the time when the valon model was developed, there were a number of other models also claiming success in reproducing the low- $p_T$  data on pion production [9]. Thus the pion distribution in the fragmentation region does not impose stringent tests on the models. Now that the  $D$ -meson distribution has ruled out certain models [1, 2], it is important to correct the impression made in [2] that the new data exhibit features that cannot be understood theoretically. We thus proceed to our description of what is more relevant.

The basic equation that expresses the recombination model for the invariant inclusive distribution of mesons produced in the fragmentation region is

$$\frac{x d\sigma}{\sigma dx} = \int F(x_1, x_2) R(x_1, x_2, x) \frac{dx_1}{x_1} \frac{dx_2}{x_2} \quad (2)$$

where  $x$  is Feynman's  $x_F$  of the detected meson,  $F(x_1, x_2)$  is the quark-antiquark

distribution in the projectile with momentum fractions  $x_1$  and  $x_2$ , and  $R(x_1, x_2, x)$  is the recombination function. Since our present interest is in  $\pi \rightarrow D + \text{anything}$ , we use  $x_1$  and  $x_2$  to refer to the light and charm quarks, respectively. Evidently, for the production of leading  $D^-$  from  $\pi^-$ , for example, (2) expresses the joint probability of finding  $d(x_1)$  and  $\bar{c}(x_2)$  in  $\pi^-$ , convoluted with the probability that they recombine to form a  $D^-$  at  $x$ .

By time-reversal consideration  $R_D(x_1, x_2, x)$  for  $d\bar{c} \rightarrow D^-$  is related to the invariant valon distribution of  $V_d$  and  $V_{\bar{c}}$  in  $D^-$ , where  $V_q$  denotes the valon of flavor  $q$ , i.e.,  $R_D(x_1, x_2, x) = y_1 y_2 G_D(y_1, y_2)$  with  $y_1 = x_1/x$  and  $y_2 = x_2/x$  [5]. With valons of unequal masses, we have, in general, for a meson  $M$

$$G_M(y_1, y_2) = [B(a, b)]^{-1} y_1^{a-1} y_2^{b-1} \delta(y_1 + y_2 - 1) \quad , \quad (3)$$

where  $B(a, b)$  is the beta function and the average momentum fractions of  $V_{\text{light}}$  and  $V_{\text{heavy}}$  in  $M$  are  $\bar{y}_1 = a/(a+b)$  and  $\bar{y}_2 = b/(a+b)$ , respectively. Letting the ratio of the momenta be proportional to their masses, we have, in the case of  $D^\pm$ ,  $a/b = \bar{y}_1/\bar{y}_2 = m_d/m_c \simeq 1/5$ . The requirement that  $G_D(y_2) = \int_0^1 dy_1 G_D(y_1, y_2)$  be finite at  $y_2 = 1$  is sufficient to yield  $a = 1$  and  $b = 5$ . Consequently, we have

$$R_D(x_1, x_2, x) = \frac{5x_1^1 x_2^5}{x^6} \delta\left(\frac{x_1}{x} + \frac{x_2}{x} - 1\right) \quad . \quad (4)$$

Of course, the same applies for the nonleading case:  $\bar{d}c \rightarrow D^+$ . Note that (4) implies the predominance of the heavy quark in determining the momentum of the  $D$  meson.

The primary burden in using (2) is the determination of  $F(x_1, x_2)$ . The valon model provides an unambiguous procedure for doing that [5]. The  $\pi^-$  has two valons,  $V_d$  and  $V_{\bar{u}}$ . The two quarks at  $x_1$  and  $x_2$  can either both come from the same valon, or from the two different valons. Let their contributions to  $F(x_1, x_2)$  be denoted by  $F^{(1)}(x_1, x_2)$  and  $F^{(2)}(x_1, x_2)$ , respectively, so that  $F = F^{(1)} + F^{(2)}$ . Thus  $F^{(1)}$  is a convolution of the single-valon distribution  $G_\pi(y)$  in the pion and the two-quark distribution in the valon, while  $F^{(2)}$  is a convolution of the two-valon distribution  $G_\pi(y_1, y_2)$  in the pion and the single-quark distributions in the two valons. Let the distribution of the momentum fraction  $z$  of the valence quark  $d$  in the valon  $V_d$  be  $K_{NS}(z)$ , that of the light sea quark in either valon be  $L(z)$ , and that of the charm quark in either valon be  $L_c(z)$ . Then with the subscripts  $\ell$  and  $n\ell$  denoting leading and nonleading processes (i.e.  $d\bar{c}$  and  $\bar{d}c$  in  $\pi^-$ , respectively), we have

$$F_\ell^{(1)}(x_1, x_2) = \int_{x_1+x_2}^1 dy G_\pi(y) K_{NS}\left(\frac{x_1}{y}\right) L_c\left(\frac{x_2}{y-x_1}\right) \quad (5)$$

$$F_\ell^{(2)}(x_1, x_2) = \int_{x_1}^{1-x_2} dy_1 \int_{x_2}^{1-y_1} dy_2 G_\pi(y_1, y_2) \left[ K_{NS}\left(\frac{x_1}{y_1}\right) + 2L\left(\frac{x_1}{y_1}\right) \right] L_c\left(\frac{x_2}{y_2}\right) \quad , \quad (6)$$

$$F_{n\ell}^{(2)}(x_1, x_2) = \int_{x_1}^{1-y_2} dy_1 \int_{x_2}^{1-y_1} dy_2 G_\pi(y_1, y_2) 2L\left(\frac{x_1}{y_1}\right) L_c\left(\frac{x_2}{y_2}\right). \quad (7)$$

Note that we have not written down the  $F_{n\ell}^{(1)}$  contribution because in a state where  $c\bar{c}$  is present the charm quarks being massive dominate the momentum in the sea, leaving the light sea quarks in the same valon with negligible momentum  $x_1$ . The recombination of such sea quarks with  $c$  or  $\bar{c}$  is insignificant in the  $x > 0.2$  region due to the  $x_1$  factor in  $R(x_1, x_2, x)$ . In short, in the single valon contribution only the recombination of the valence quark with charm quark is important, hence  $F_\ell^{(1)}$  only. Such arguments do not apply to  $F^{(2)}$ , since the sea quarks in the other valon can have any momentum fraction in that valon. The  $LL_c$  terms in (6) and (7) are doubled because either valon can contribute the charm quark. They cancel in the combination  $F_\ell - F_{n\ell}$ , leaving only the  $K_{NS}L_c$  contribution, which will be crucial in giving rise to the increase of  $A(x)$  in  $x$ .

The distributions  $G_\pi(y)$ ,  $G_\pi(y_1, y_2)$ ,  $K_{NS}(z)$  and  $L(z)$  are known from previous treatment of the pion production problem [5]; they are:  $G_\pi(y) = 1$ ,  $G_\pi(y_1, y_2) = \delta(y_1 + y_2 - 1)$ ,

$$K_{NS}(z) = 1.2 z^{1.1} (1-z)^{0.16}, \quad L(z) = 0.41(1-z)^{3.5}, \quad (8)$$

where  $K_{NS}(z)$  in (8) is given in [7], and  $L(z)$  describing the saturated sea (with gluon conversion) is denoted by  $\bar{L}(z)$  in [5, 10].  $L_c(z)$  is the invariant distribution of a charm quark in a valon, i.e.  $zP_{c/V}(z)$ , where  $P_{c/V}(z)$  is the corresponding probability for finding  $c$  at  $z$ . Because of the high mass of  $c\bar{c}$  relative to  $m_V$ ,  $P_{c/V}(z)$  is nearly independent of  $z$  [11], so

$$L_c(z) = cz, \quad (9)$$

where  $c \ll 1$  is the only unknown parameter in the problem, representing the suppression factor of having  $c\bar{c}$  in the pion. Fortunately, the parameter  $c$  is cancelled out in the ratio for  $A(x)$ , so we can calculate  $A(x)$  without any ambiguity.

We now have specified all the functions necessary to evaluate  $F_{\ell, n\ell}(x_1, x_2)$ , which in conjunction with (4), can then be used to carry out the integration in (2). Using the abbreviation  $H(x) = (x/\sigma)d\sigma/dx$ , we write

$$H_{\ell, n\ell}(x) = \int F_{\ell, n\ell}(x_1, x_2) R_D(x_1, x_2, x) \frac{dx_1}{x_1} \frac{dx_2}{x_2}, \quad (10)$$

and therefore have

$$A(x) = \frac{H_\ell(x) - H_{n\ell}(x)}{H_\ell(x) + H_{n\ell}(x)}. \quad (11)$$

Putting (4)-(9) into (10) and (11), we obtain

$$A(x) = \frac{I_1^{(1)}(x) + I_1^{(2)}(x)}{I_1^{(1)}(x) + I_1^{(2)}(x) + I_2^{(2)}(x)}. \quad (12)$$

where

$$\begin{aligned}
I_1^{(1)}(x) &= \int_0^x du \int_x^1 dv \frac{u^5}{u+v-x} f_1\left(\frac{x-u}{v}\right), \\
I_i^{(2)}(x) &= \int_0^x du \int_0^{1-x} dv \frac{u^5}{u+v} f_i\left(\frac{x-u}{1-u-v}\right), \\
f_1(z) &= K_{NS}(z), \quad f_2(z) = 4L(z).
\end{aligned}$$

They can directly be computed. There is no energy dependence because the model is for soft interaction at asymptotic energy, where the cross sections are scaling. We stress again that there are no free parameters in the problem. The result is shown by the solid line in Fig. 1. Evidently, the agreement with data [1, 2] is very good except at low  $x$  where the model is less reliable. For  $x$  in the central region, and for finite energies such as those used for the experiments, the assertion that the momenta of the valence quarks in the projectile are minimally affected by the partons in the target becomes less tenable. If we take the interactions between the projectile and target partons into account, we expect the creation of considerable number of partons in the central region, which at  $\sqrt{s} = 22 - 25$  GeV can extend over the range  $-0.3 < x < 0.3$ . The dominance of the sea quarks (with gluon conversion) renders  $H_\ell \approx H_{n\ell}$ , so our theoretical curve should be pushed down in that region at such a nonasymptotic beam energy, in better agreement with the data.

We have achieved our aim of showing that the new data on charm production does not present any problem to theoretical understanding. As indicated by the dashed line in Fig. 1, perturbative QCD fails to describe the data of the low- $p_T$  process. The dotted line shows that the fragmentation process of a fast quark, as described by PYTHIA [4], also does not represent the relevant physics of the observed process. These curves were presented in [1, 2] together with the data. The success of our model clearly indicates that what is responsible for the production of particles in the fragmentation region of the beam hadron is the recombination process of the beam partons. There are two parts to the problem: what the parton momenta are and how they recombine. Quantitative information about both parts is specified by the valon model. It is important to recognize that an essential property of the model is the hierarchical structure of a hadron: valons in hadrons, and partons in valons. Because of this structure it is possible for a  $d$  and a  $\bar{c}$  to have approximately equal momenta that favors recombination, if they belong to different valons. The resultant  $D^-$ -meson momentum is the sum of the  $d$  and  $\bar{c}$  momenta, thereby enhancing the probability of producing  $D^-$  at large  $x$ . For  $\pi^- \rightarrow D^+$  both  $\bar{d}$  and  $c$  are in the sea; the small  $x_1$  of  $\bar{d}$  disfavors recombination at large  $x$ . Hence  $A(x)$  is large at large  $x$  and decreases with decreasing  $x$ .

It should be remarked that (9) was obtained in [11] on the basis of a simple argument on large  $m_c$  and tight binding. Small adjustment of the  $z$  dependence can no doubt further improve the agreement with the data. However, fitting the data by adjusting some parameters is not our aim here. For that reason we have used the

previous and simple form (9) to obtain the no-free-parameter result in Fig. 1. We suggest an independent check of the credibility of the valon model by studying the ratio

$$B(x) = H_{\pi^- \rightarrow D^+}(x)/H_{\pi^- \rightarrow \pi^+}(x). \quad (13)$$

If the data confirms our prediction of the  $x$  dependence of  $B(x)$ , then even the parameter  $c$  in (9) can be fixed.

Both reactions in (13) are nonleading. Now,  $F_{n\ell\pi}^{(1)}$  is not negligible. We have

$$F_{n\ell\pi}^{(1)} = \int_x^1 dy G_\pi(y) L\left(\frac{x_1}{y}\right) L\left(\frac{x_2}{y-x_1}\right), \quad (14)$$

$$F_{n\ell\pi}^{(2)} = \int_{x_1}^{1-x_2} dy_1 \int_{x_2}^{1-y_1} dy_2 G_\pi(y_1, y_2) 2L\left(\frac{x_1}{y_1}\right) L\left(\frac{x_2}{y_2}\right), \quad (15)$$

$$R_\pi(x_1, x_2, x) = \frac{x_1 x_2}{x^2} \delta\left(\frac{x_1}{x} + \frac{x_2}{x} - 1\right). \quad (16)$$

Used in conjunction with (4) and (7), they give

$$B(x) = \frac{5c}{2(0.41)^2 x^5} \cdot \frac{I_2^{(2)}(x)}{J^{(1)}(x) + J^{(2)}(x)}, \quad (17)$$

where

$$J^{(1)}(x) = \int_x^1 dy \left(1 - \frac{x}{y}\right)^{3.5},$$

$$J^{(2)}(x) = 2 \int_0^1 ds \int_0^{1-x} dt \left[ \frac{t(1-x-t)}{(t+sx)(1-t-sx)} \right]^{3.5}.$$

The result of  $B(x)/c$  is shown in Fig. 2. Future experimental data on  $B(x)$  can further test the reliability of the valon model, and determine the normalization of the charm quark distribution.

The valon model has been successful in reproducing pion inclusive cross sections from  $\pi$ ,  $K$  and  $p$  initiated reactions without adjustable parameters. But it is in the charm production that the model now stands alone in being able to account for the existing data quantitatively. What one learns from this study is that in the projectile fragmentation region one should not think about quark fragmentation, but should consider parton recombination.

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## References

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### Figure Captions

- Fig. 1 Asymmetry  $A(x)$ : data are from Ref. [2], solid line (valon model) is from Eq. (12), dotted line (PYTHIA) and dashed line (pQCD) are from Ref. [2].
- Fig. 2 The ratio  $B(x)$  for  $\sigma(\pi^- \rightarrow D^+)/\sigma(\pi^- \rightarrow \pi^+)$  as predicted in the valon model, (17), in units of  $c$ .



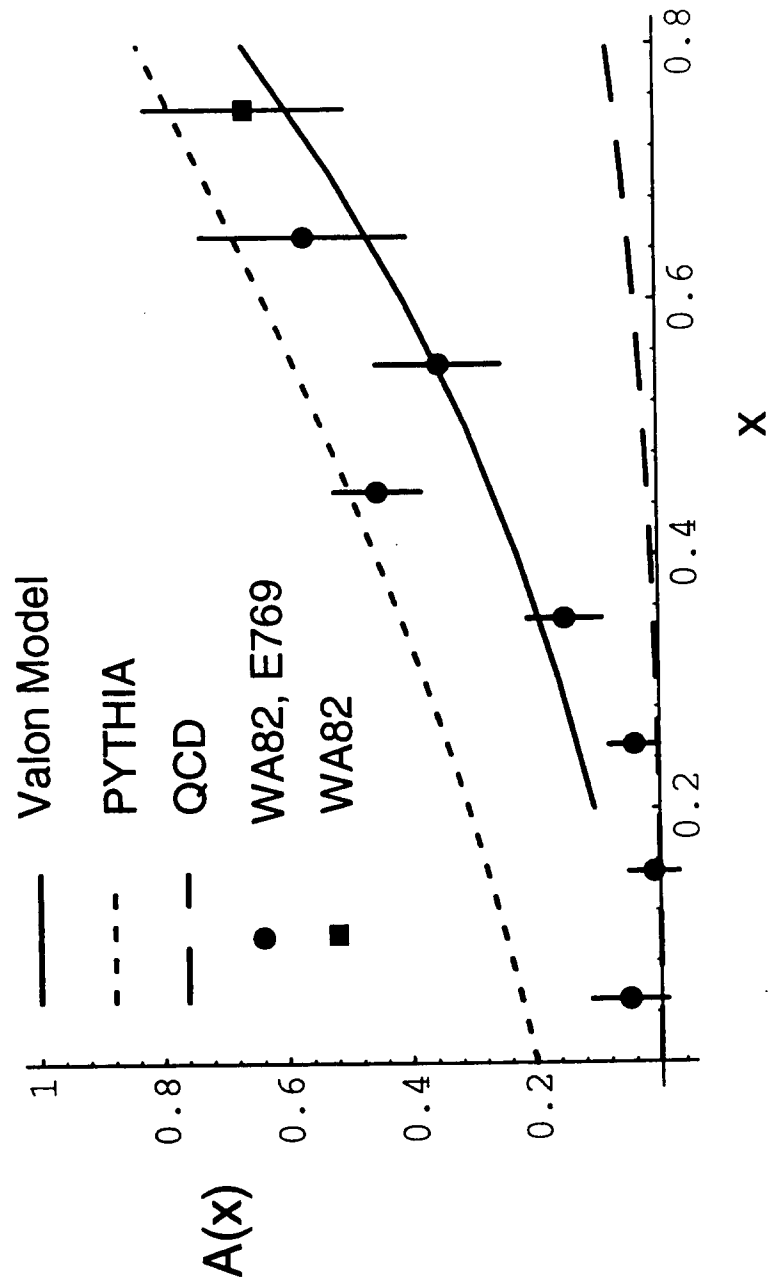


Fig. 1

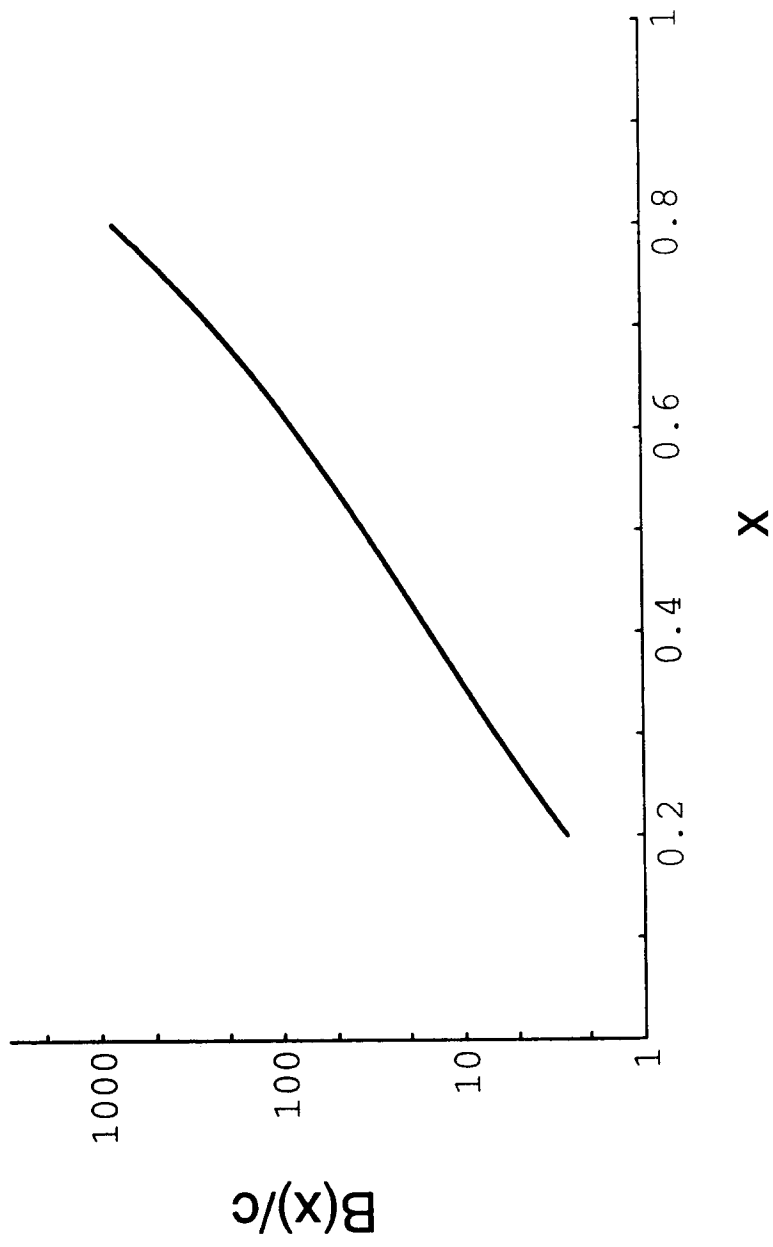


Fig. 2