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DFTUZ/93/16

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Some Interesting Features of Noncompact QED$_3$

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We study the phase diagram of non compact QED_3 using the MFA method and present evidence for a continuous phase transition line at small N_f. We also analyze the chiral structure of the vacuum by means of the computation of the probability distribution function of the order parameter in the exact chiral limit.

1. Introduction

Quantum Field Theory in $2+1$ dimensions is being extensively studied in recent times. The main motivation is that it is a laboratory to explore some qualitative features of $QFT's$ in $3+1$ dimensions; moreover, the strong coupling limit of these models could be relevant for the high T_c superconductivity phenomenon. In this spirit, we have done recently an analysis of noncompact QED in $2+1$ dimensions and we report here some of the most interesting results obtained.

The numerical approach used in almost all of our computer simulations with dynamical fermions is the Microcanonical Fermionic Average Method (MFA) extensively described elsewhere [1]. For the computation of the probability distribution function of the chiral condensate, defined in Section 3, however, a different algorithm [2] was used. The MFA method has been successfully tested in the compact model in 4 dimensions and also applied to the study of the compact and noncompact abelian models in 2, 3 and 4 dimensions [1]. The crucial idea behind the MFA approach is the computation of a fermionic effective action,

$$e^{-S^F_{eff}(m,N_f,E)} = \langle \det \Delta^{\frac{N_f}{2}} \rangle_E, \qquad (1)$$

computed over gauge field configurations at fixed pure gauge energy E.

*Presented by V. Azcoiti. This work has been partly supported through a INFN(Italy)-CICYT(Spain) collaboration. V. Laliena acknowledges support from the CAICONAI mixed committee.

2. Phase Diagram from the Effective Action

As described in [1], the analytical behaviour of the Effective Action allows to draw informations about the phase structure of the theory. In noncompact QED_3 in the quenched limit we have found evidence for a continuous phase transition at finite coupling [3].

This phase transition, which is also present when dynamical flavours are switched on (at least for small N_f), reveals itself as a nonanalyticity of the fermionic effective action as a function of the pure gauge energy E. In fact S^F_{eff} shows two different regimes: at small energies it is is a linear function of the pure gauge energy, whereas for $E \geq 0.68(1)$ it exhibits a non linear behaviour.

Since in the quenched limit $E = \frac{1}{3\beta}$, we can immediately evaluate the $v.e.v.$ of the fermionic action $\langle \log\det\Delta \rangle_Q$.

The quantity

$$\Omega(\beta) = \frac{1}{V}\langle \log\det\Delta \rangle_Q - 0.145 + \frac{0.256}{3\beta}, \qquad (2)$$

where the last two terms are a fit to the linear (in E) part of the effective action, is reported in Fig. 1. It can be seen that $\Omega(\beta)$ behaves like a true order parameter $i.e.$ it is zero in the weak coupling phase and non zero in the strong coupled phase.

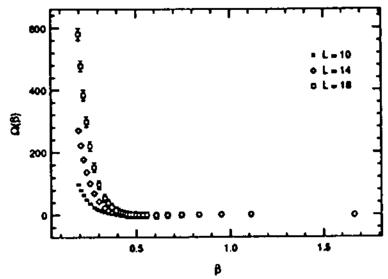

Figure 1. Order parameter in the quenched limit

This behaviour, differently with that observed in the Schwinger model [5], is qualitatively the same as that found in the noncompact abelian model in $3+1$ dimensions.

This is indeed quite surprising since there are strong qualitative differences between the noncompact model in three and four dimensions respectively. Noncompact QED in $3+1$ dimensions has a chiral transition at finite coupling whereas no such a phase transition appears in the three-dimensional model, at least for small flavour number.

On the other hand it has been shown that the phase transition, which appears in the four-dimensional model as a consequence of the anomalous behaviour of the fermionic effective action, takes place at the same critical coupling of the monopole percolation transition in the quenched limit [4], suggesting that both phase transitions are governed by the same dynamics. However in three dimensions, the phase transition we observe takes place at a critical coupling which is too small to induce percolation of topological objects such as static monopoles or vortices.

3. The Chiral Structure of the Vacuum

Another interesting feature in the dynamics of this model is what is the vacuum realization of chiral symmetry (CS). In the quenched limit and also for small flavour number, there is little doubt that CS is spontaneously broken. Indeed there are strong theoretical prejudices suggesting that a theory which confines static charges, like noncompact QED_3, breaks spontaneously CS.

Recently we have developed a new technique for the analysis of the vacuum chiral properties [6], which is based on the computation of the probability distribution function of the chiral order parameter $\bar\Psi\Psi$. The main advantage of this method, when compared with other standard approaches, is that we can work directly in the chiral limit and therefore no mass extrapolations are needed. The underlying idea, which is very simple, is based on the well known fact that in a spin system we do not need to put an external magnetic field in order to see if the system has spontaneous magnetization. The main difference between a spin system and a gauge theory with dynamical fermions is that the fermionic Grassmann variables can not be directly simulated in a computer. The path integral over the fermionic degrees of freedom must be done analytically and therefore no magnetization can be observed in the chiral limit since we are integrating out over all possible vacuum states.

In order to construct the probability distribution function ($p.d.f.$) $P(c)$ of the chiral order parameter c, let us characterize each vacuum state α by the corresponding vacuum expectation value c_α of the order parameter

$$c_\alpha = \frac{1}{V}\sum_{i=1}^{N} \langle \bar\Psi_i \Psi_i \rangle_\alpha \qquad (3)$$

The $p.d.f.$ of the chiral order parameter c will be given then by

$$P(c) = \sum_\alpha w_\alpha \delta(c - c_\alpha) \qquad (4)$$

where w_α is the probability that choosing randomly a vacuum state, we get the α state. If the vacuum state is unique, $P(c)$ will be a single δ-function. Otherwise, $P(c)$ will be a more complicated function, sum of δ-functions if we have a discrete number of ground states or a continuum function in the case we are interested in.

The function $P(c)$ can be written as

$$P(c) = \left\langle \delta\left(\frac{1}{V}\sum_i \bar\Psi_i\Psi_i - c\right) \right\rangle \qquad (5)$$

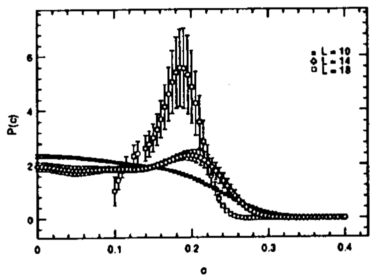

Figure 2. $P(c)$ in the two flavor model

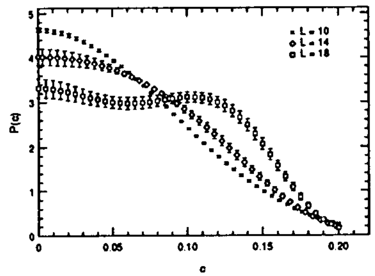

Figure 3. $P(c)$ in the four flavor model

This expression is not easily computable. However we can derive its Fourier transform $\bar{P}(q)$ which is more suitable for a numerical computation. Now, if we consider Kogut-Susskind fermions, the fermionic matrix Δ can be written as $\Delta = m + i\Lambda$, where m is the bare fermion mass and Λ an hermitian matrix whose eigenvalues λ_i are real and symmetric. Then the function $\bar{P}(q)$ can be written, in the chiral limit, as

$$\bar{P}(q) = \left\langle \prod_{j=1}^{N/2} (1 - \frac{q^2}{N^2 \lambda_j^2}) \right\rangle \tag{6}$$

where the vacuum expectation value in eq. 6 is computed with the *p.d.f.* of the effective gauge theory obtained after integration of the fermion fields.

Once we have a numerical estimate of $\bar{P}(q)$, we can get $P(c)$ by inverse Fourier transform. Fig 2 shows our results for $P(c)$ at $\beta = 0.3$ in the two flavour model. The variation of this function with the lattice size, developing a peak at a non vanishing value of c in the larger lattices, implies that the infinite volume limit of this function is not a δ-function centered at $c = 0$, as would be expected in a chirally symmetric phase. Fig. 3 is the same as Fig. 2 but for the four flavour theory and $\beta = 0.25$. This value of β is far from the critical value $\beta_c \sim 0.2$ for a chiral restoring transition reported in [7] from the results of a simulation in a 10^3 lattice. Our results in the 10^3 lattice (peaked function centered at the origin) could suggest that we are in a phase where chiral symmetry is realized. However again in this case, the lattice size dependence of the *p.d.f.* $P(c)$ shows that chiral symmetry should be spontaneously broken in the infinite volume limit.

In conclusion we find the same qualitative behaviour in the zero, two and four flavours models, without any trace of a chirally restoring phase transition in the four flavours model at $\beta = 0.25$. In all the cases finite volume effects are strong, as expected from theoretical prejudices.

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