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TWOGEN, a Simple Monte Carlo Generator for Two-Photon Reactions.

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Abstract

TWOGEN samples the transverse-transverse luminosity function for real and virtual photons, then weights events with any user-supplied cross section $\sigma_{\rm TT}(\gamma\gamma \rightarrow X)$ in a "hit or miss" sampling, using a simple method to avoid the singularity at the minimum electron angle $\vartheta_{\rm min} = 0$. Events within defined kinematic limits are accepted and the corresponding cross section is estimated, together with the effective luminosity of the run. Functions implemented for $\sigma_{\rm TT}$ include the production cross sections for lepton pairs, for the formation of narrow resonances and for hadron production in tagged, deep inelastic scattering. A comparison is made with an existing Monte Carlo Generator and a simple analytic approximation.

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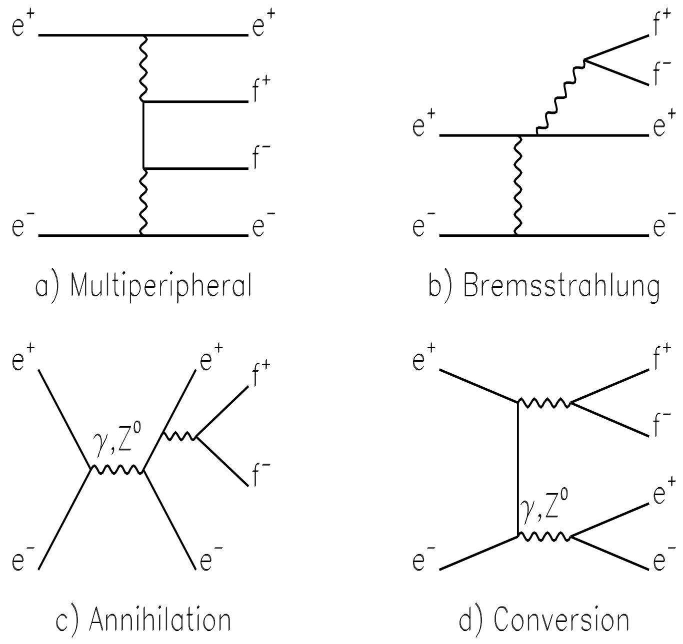

Figure 1: The four main diagrams contributing in the lowest order to the process $e^+e^- \to e^+e^- f^+ f^-$.

PROGRAM SUMMARY

```
Title of Program:   TWOGEN
Catalogue Number:
Program Obtainable from:   CPC Program Library, Queen's University
                           of Belfast, N. Ireland.

Computer for which the program is designed and others on which it
is operable:  all computers
Programming Language used:  FORTRAN 77
High-speed storage required: 410 KByte standalone,
                             1.1 Mbyte with HBOOK and JETSET
No. of lines in combined program and test case:  1800
Keywords:  Monte Carlo, Two-photon, e+e-
Nature of physical problem:  Generation of the two-photon flux in
                             e+e- colliding beam accelerators.
Typical running time: 4 sec/event (168 units)
Unusual features of the program:  calls are made to CERN library
                                  routines HBOOK and to the
                                  JETSET 7.3 library
```

1 Introduction

A number of Monte Carlo programs exist which generate fermion pairs in $e^+e^- \to e^+e^- f^+ f^-$ events[1]-[3] according to exact matrix elements (Figure 1). They can also be used to represent the point-like production of quark-antiquark pairs in the quark-parton model (QPM). These generators, however, cannot be used when events have to be generated according to a different model, for example a limited-p_t or a vector meson dom-

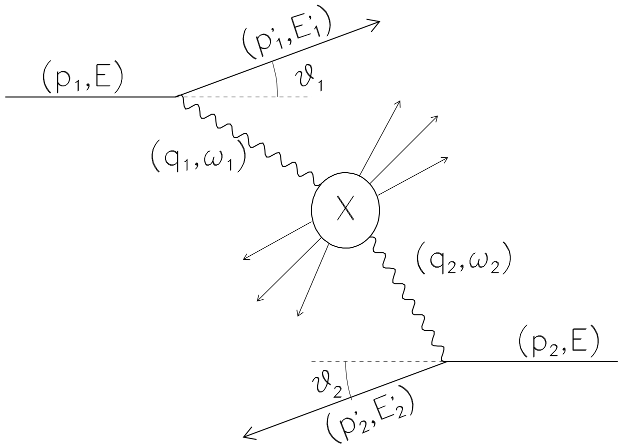

Figure 2: Definition of kinematical variables in the reaction $e^+e^- \to e^+e^-X$.

inance (VMD) model. In that case it is convenient to separate the two-photon reaction into two pieces: the generation of the luminosity function $\mathcal{L}_{\gamma\gamma}$ for the scattering between the transverse photons coming from the clouds around the colliding electron and positron beams; and the generation of the final state from the photon-photon collisions. In other words, the cross section for the production of a state X is assumed to factorise[4]-[6]:

$$\sigma(e^+e^- \to e^+e^-X) = \mathcal{L}_{\gamma\gamma}(e^+e^- \to e^+e^-\gamma_1^*\gamma_2^*)\ \sigma(\gamma_1^*\gamma_2^* \to X) \tag{1}$$

TWOGEN was written for use with datas from the TPC/2γ experiment at the PEP e^+e^- collider at SLAC[7] and has been further developed for use with the OPAL experiment at CERN[8]-[9]. It only generates events according to Figure 1.a. The core of the program is the luminosity routine, described in section 3. It samples a multi-dimensional phase space and returns equally weighted events. Models for the cross section $\sigma(\gamma_1^*\gamma_2^* \to X)$ are described in section 4. The structure of the program is given in section 5.

2 Kinematics

The incoming leptons have four-momenta p_i, E, with $i = 1, 2$ for e^+, e^- (Figure 2). The scattered leptons have four-momenta p_i', E_i'. If the beams are unpolarised, all useful quantities can be expressed in terms of six basic variables: the energies E_i' of the scattered leptons, their angles ϑ_i with respect to the beam direction, the azimuthal angle φ between the two scattering planes and ϕ_1, the azimuthal angle of one of the scattered electrons. The photon energies are $\omega_i = E - E_i'$. The invariant masses of the (space-like) photons are:

$$q_i^2 = 2m_e^2 - 2EE_i'(1 - \sqrt{1-(m_e/E)^2}\sqrt{1-(m_e/E_i')^2}\cos\vartheta_i). \tag{2}$$

Positive quantities $Q_i^2 = -q_i^2$ are defined. For $m_e/E \ll 1$ equation (2) reduces to

$$Q_i^2 \simeq 2EE_i'(1 - \cos\vartheta_i). \tag{3}$$

The invariant mass of the $\gamma\gamma$ system is given by:

$$W^2 \equiv M_{\gamma\gamma}^2. \tag{4}$$

Terms of order m_e^2 have been neglected. For small Q_1^2 and Q_2^2 this reduces to $W^2 \simeq 4\omega_1\omega_2$. Experimentally, we distinguish three types of events:

1. *Untagged* events: both electrons are scattered at such low angles with respect to the beam line that they escape detection.
2. *Single-tag* events: one of the two scattered electrons is detected. In this case the four-momentum of the corresponding photon is known and a value of Q^2 can be assigned to it.
3. *Double-tag* events: both final state electrons are detected. For such events we rename the photon invariants Q_i^2 to be $Q^2 = -q_1^2$ and $P^2 = -q_2^2$, when calculating $\sigma(\gamma_1^*\gamma_2^* \to X)$ with the structure function F_2, with particles 1 and 2 relabeled so that $Q^2 > P^2$.

3 Sampling the Transverse-Transverse Luminosity Function

The general expression for the differential cross section $d\sigma(\mathrm{e}^+\mathrm{e}^- \to \mathrm{e}^+\mathrm{e}^- X)$ involves the cross sections σ_{TT}, σ_{TL}, σ_{LT} and σ_{LL} for hadron production by scattering of two photons with the four combinations of transverse (T) and longitudinal (L) polarisation. Only σ_{TT} is nonzero for real photons. At small Q_i^2 the other cross sections are proportional to Q_i^2 or to $Q_1^2Q_2^2$ and make a small enough contribution that they can be neglected. Then the differential luminosity function of equation (1) becomes

$$\frac{d^6\mathcal{L}_{\gamma\gamma}^{\mathrm{TT}}}{d\omega_1 d\omega_2 d\vartheta_1 d\vartheta_2 d\varphi d\phi_1} = \frac{\alpha^2 E_1' E_2'}{8\pi^4 E^2}\frac{\sin\vartheta_1 \sin\vartheta_2}{q_1^2 q_2^2}\sqrt{X}\rho_1^{++}\rho_2^{++}, \tag{5}$$

with density matrix elements[10]

$$\rho_1^{++} = \frac{(k - 4E\omega_2 q_2^2)^2}{2X} + \frac{1}{2} + 2\frac{m_e^2}{q_1^2}, \tag{6}$$

$$\rho_2^{++} = \frac{(k - 4E\omega_1 q_1^2)^2}{2X} + \frac{1}{2} + 2\frac{m_e^2}{q_2^2}, \tag{7}$$

$k = (W^2 - q_1^2 - q_2^2)/2$ and $X = k^2 - q_1^2 q_2^2$. The overall dependence of the luminosity on ω_i is

$$\frac{d\mathcal{L}^{\mathrm{TT}}}{dW^2} \approx \frac{d\mathcal{L}^{\mathrm{TT}}}{4 d\omega_1 d\omega_2} \propto \frac{1}{W^2} \approx \frac{1}{4\omega_1\omega_2} \tag{8}$$

For the dependence of the luminosity on ϑ_i, we use the fact that for small ϑ_i

$$\frac{4EE_i'}{q_i^2}\sin^2\frac{1}{2}\vartheta_i \approx -1. \tag{9}$$

We can therefore isolate the dependences on ω_i and ϑ_i by rewriting equation (5) as:

$$\frac{d^6\mathcal{L}_{\gamma\gamma}^{\mathrm{TT}}}{d\omega_1 d\omega_2 d\vartheta_1 d\vartheta_2 d\varphi d\phi_1} = \frac{1}{\omega_1\omega_2}\cot\frac{1}{2}\vartheta_1 \cot\frac{1}{2}\vartheta_2 f_W, \tag{10}$$

with

$$f_W = \omega_1\omega_2 \frac{\alpha^2 E_1' E_2'}{2\pi^4 E^2}\frac{\sin^2\frac{1}{2}\vartheta_1 \sin^2\frac{1}{2}\vartheta_2}{q_1^2 q_2^2}\sqrt{X}\rho_1^{++}\rho_2^{++}. \tag{11}$$

By doing so, we have factorised equation (5) into a part which can be integrated analytically and therefore generated exactly, and a weight function f_W. The function f_W is well behaved for all values of ω_i and ϑ_i, and can easily be integrated with the hit/miss technique. We generate values of ω_i using random numbers $\mathcal{R}$:

$$\omega_i = \omega_{\rm min} \left(\frac{\omega_{\rm max}}{\omega_{\rm min}} \right)^{\mathcal{R}}, \tag{12}$$

with $\omega_{\rm max} = E$ and $\omega_{\rm min} = W^2_{\rm min}/4E$. Values of ϑ_i are generated as:

$$\vartheta_i = 2 \arcsin \left(\{\sin \frac{1}{2}\vartheta_{\rm min}\} \left[\frac{\sin \frac{1}{2}\vartheta_{\rm max}}{\sin \frac{1}{2}\vartheta_{\rm min}} \right]^{\mathcal{R}} \right). \tag{13}$$

Here $\vartheta_{\rm min}$ and $\vartheta_{\rm max}$ are the minimum and maximum scattering angles of the electrons. We may choose $\vartheta_{\rm max} = \pi$, but we have a pole at $\vartheta_{\rm min} = 0$. We know that the exact distribution has no singularity at $\vartheta_i = 0$, therefore we modify equation (10) slightly and generate the function

$$\frac{\cos \frac{1}{2}\vartheta_i}{\epsilon + \sin \frac{1}{2}\vartheta_i}, \tag{14}$$

where ϵ is a small number (studies have shown that $\epsilon = 10^{-7}$ is appropriate). Now

$$\vartheta_i = 2 \arcsin \left(\{\sin \frac{1}{2}\vartheta_{\rm min} - \epsilon\} \left[\frac{\sin \frac{1}{2}\vartheta_{\rm max} - \epsilon}{\sin \frac{1}{2}\vartheta_{\rm min} - \epsilon} \right]^{\mathcal{R}} - \epsilon \right) \tag{15}$$

gives the distribution in ϑ of Eq. (14). We correct for the addition of ϵ by replacing $\sin^2 \frac{1}{2}\vartheta_i$ by $\sin \frac{1}{2}\vartheta_i(\sin \frac{1}{2}\vartheta_i + \epsilon)$ in Eq. 11, which has very little impact on the hit/miss efficiency. The angles φ and ϕ_1 are simply generated as $2\pi\mathcal{R}$.

When ω_i and ϑ_i are generated, the weight f_W is calculated from Eq. (11). The event is accepted if $\mathcal{R}f_{\rm max}$ is less than f_W. The maximum weight $f_{\rm max}$ has to be provided by the user. Its optimal value has to be determined from a few trial runs. If with this procedure $N_{\rm kept}$ events are accepted out of $N_{\rm try}$ generated events, the integrated luminosity factor becomes:

$$\mathcal{L}^{\rm TT} = (2\pi)^2 \left[\log \left(\frac{\sin \frac{1}{2}\vartheta_{\rm max}}{\sin \frac{1}{2}\vartheta_{\rm min}} \right) \right]^2 \left[\log \left(\frac{\omega_{\rm max}}{\omega_{\rm min}} \right) \right]^2 \frac{N_{\rm kept}}{N_{\rm try}} f_{\rm max}. \tag{16}$$

This luminosity factor is an result of the generator.

4 Models for the $\gamma\gamma$ Cross Section.

4.1 The $e^+e^- \to e^+e^-\mu^+\mu^-$ Reaction in the Single-tagging Mode.

The cross section of the reaction $e^+e^- \to e^+e^-\mu^+\mu^-$ is described in terms of QED structure functions F_1 and F_2. In the case of single-tagging, and in the notation of deep inelastic scattering, the fraction x of the momentum of photon 2 carried by the muon struck by photon 1 is given by

$$x = Q^2/2(q_1.q_2) = Q^2/(Q^2 + W^2). \tag{17}$$

The kinematic variables y and z are defined as $y = 1 - (E'_1/E)\cos^2(\vartheta_1/2)$, and $z = 1 - E_2/E$. The QED structure functions are predicted to be[10]:

$$F_1(x, Q^2) = \frac{\alpha}{2\pi}\{[x^2 + (1-x)^2 + 4m_\mu^2 \frac{W^2 - 2m_\mu^2}{(W^2+Q^2)^2}] \log \left[\frac{W}{2m_\mu} + (\frac{W^2}{4m_\mu^2} - 1)^{1/2} \right]^2 - \left[(1-2x)^2 + \frac{4m_\mu^2 W^2}{(W^2+Q^2)^2} \right] (1 - \frac{4m_\mu^2}{W^2})^{1/2}\}, \tag{18}$$

and

$$F_2(x,Q^2) = 2xF_1(x,Q^2) + \tfrac{4\alpha}{\pi}x^2\{(1-x)(1-\tfrac{4m_\mu^2}{W^2})^{1/2} - \tfrac{2m_\mu^2}{W^2+Q^2}\log\left[\tfrac{W}{2m_\mu} + (\tfrac{W^2}{4m_\mu^2}-1)^{1/2}\right]^2\}. \qquad (19)$$

In the region where y is small, *i.e.* where the tag energy is large, the contribution of F_1 to the cross section is negligible, and the two-photon cross section can be written as

$$\sigma(\gamma\gamma\to\mu^+\mu^-)(x,Q^2) = \frac{8\pi^2\alpha}{Q^2}F_2(x,Q^2). \qquad (20)$$

In order to check the performance of TWOGEN, we compare its results with a well-established Monte Carlo generator by Vermaseren[1], which generates events according to matrix element calculations of the processes (a) and (b) of Figure 1. Events were generated with TWOGEN using the cross section of Eq. (20) with a beam energy of 45.6 GeV in the single tagging mode, with an angular acceptance for the tag between 20 and 200 mrad. Figure 3 shows comparisons of the resulting event distributions (dots) with the distributions of events generated with the Vermaseren generator (lines). Both event samples correspond to an integrated e^+e^- luminosity of 54 pb^{-1}. The difference in cross section between TWOGEN and Vermaseren is comparable to the uncertainty with which the cross section is calculated in the Vermaseren Monte Carlo ($\mathcal{O}(1\%)$). The statistical uncertainty in the Monte Carlo is reflected in the fluctuations of the data points.

4.2 Generation of Narrow Resonances.

TWOGEN can be used to produce events of the type $e^+e^- \to e^+e^-R$, where R is a resonance with spin J, mass m_R, full width Γ_R and two-photon decay width $\Gamma_{R\to\gamma\gamma}$. The two-photon formation cross section for such a resonance is

$$\sigma_{\gamma\gamma\to R}(W) = 8\pi^2\frac{(2J+1)\Gamma_{R\to\gamma\gamma}}{m_R}\left[\frac{1}{\pi}\frac{m_R\Gamma_R}{(W^2-m_R^2)^2+m_R^2\Gamma_R^2}\right]. \qquad (21)$$

The expression in square brackets represents the Breit-Wigner resonance shape which reduces to $\delta(W^2-m_R^2)$ for narrow resonances. In order to make the program more efficient, the Breit-Wigner factor is included in the generation of the two-photon initial state. The procedure described in section 3 is only slightly modified: instead of generating ω_2 according to Eq. (12), we first generate a value of W^2 using the integral of the Breit-Wigner curve:

$$W^2 = m_R^2 + m_R\Gamma_R\tan\pi(\mathcal{R}-0.5). \qquad (22)$$

Then, ω_2 is derived from W^2. The weight factor f_W is modified as well: the factor ω_2 is replaced by $d\omega_1/dW^2$, reflecting the fact that the integration variable for the Breit-Wigner curve is W^2 and not ω_1:

$$f_W = \frac{\alpha^2E_1'E_2'}{4\pi^4E^2}\left(\frac{\omega_1}{2\omega_1+E_1'\cos\vartheta_{12}}\right)\frac{\sin^2\frac{1}{2}\vartheta_1\sin^2\frac{1}{2}\vartheta_2}{q_1^2q_2^2}\sqrt{X}\rho_1^{++}\rho_2^{++}. \qquad (23)$$

The luminosity factor as in Eq. (16) becomes:

$$\mathcal{L}^{\rm TT} = (2\pi)^2\left[\log\left(\frac{\sin\frac{1}{2}\vartheta_{\max}}{\sin\frac{1}{2}\vartheta_{\min}}\right)\right]^2\log\left(\frac{\omega_{\max}}{\omega_{\min}}\right)m_R\Gamma_R\frac{N_{\rm kept}}{N_{\rm try}}f_{\max}. \qquad (24)$$

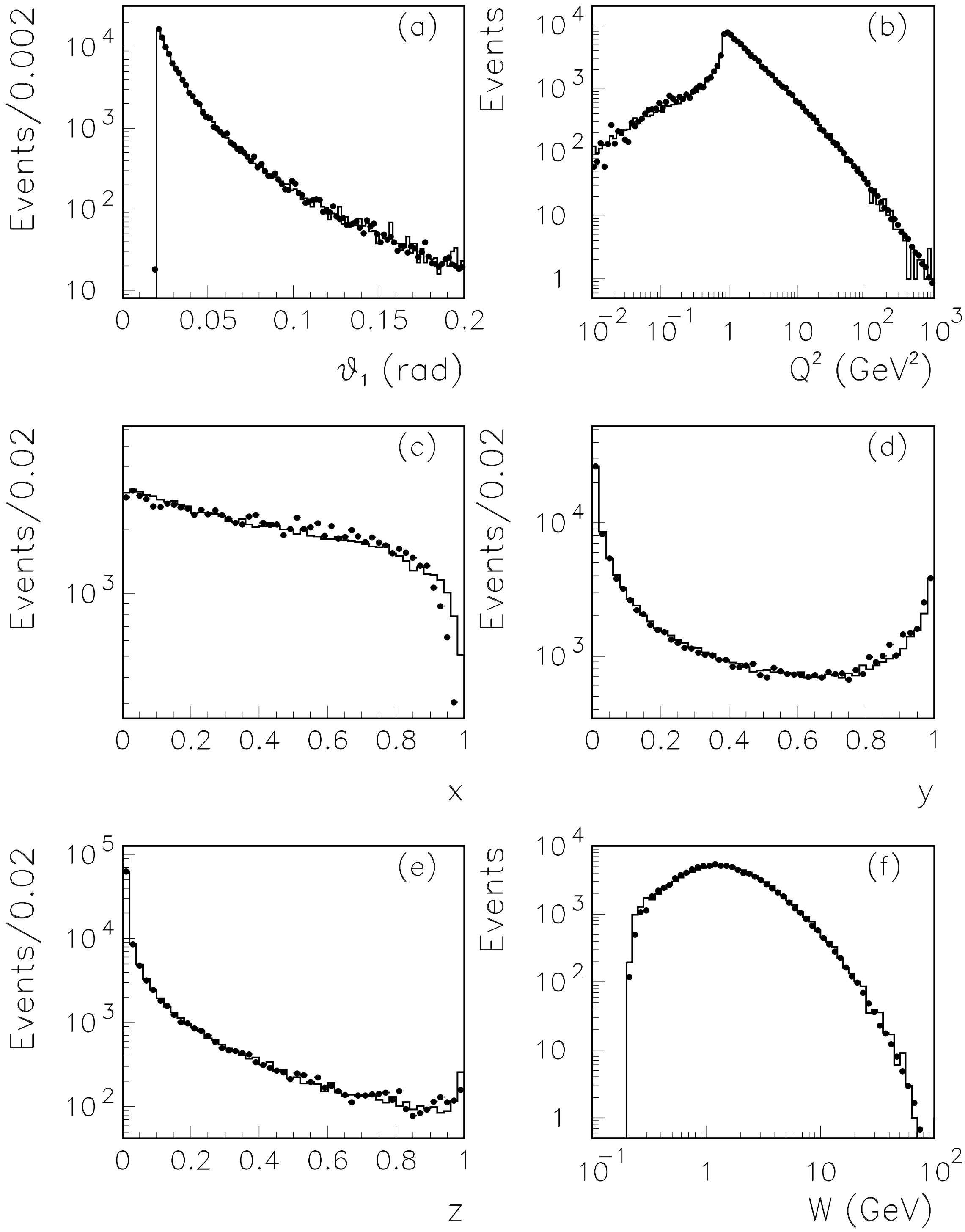

Figure 3: A comparison of event distributions as functions of ϑ_1, Q^2, x, y, z and W, generated with TWOGEN (dots) and with the Vermaseren generator (lines).

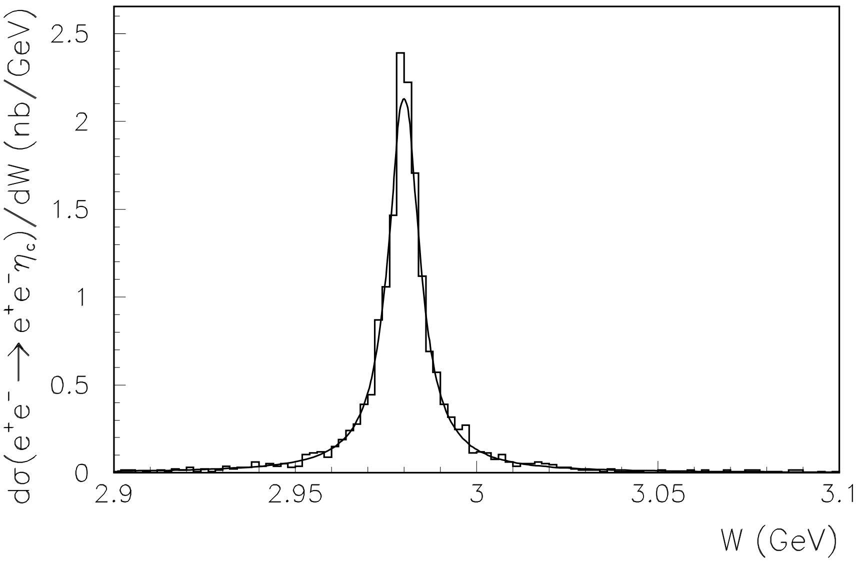

Figure 4: The distribution of the cross section $d\sigma(\mathrm{e^+e^-} \to \mathrm{e^+e^-}\eta_c)/dW$ as a function of W. The histogram corresponds to events generated with TWOGEN, the curve is an analytical calculation using the Low approximation.

As an example of the generation of resonances, we show in Figure 4 the distribution of the cross section $d\sigma(\mathrm{e^+e^-} \to \mathrm{e^+e^-}\eta_c)/dW$ as a function of W. The histogram represents the events generated by TWOGEN, the solid line is an analytical calculation using the Low approximation[11]:

$$\frac{d\sigma(\mathrm{e^+e^-} \to \mathrm{e^+e^-}\eta_c)}{dW} = 16\pi^2 \frac{(2J+1)\Gamma_{\eta_c\to\gamma\gamma}}{m_{\eta_c}} \chi(W) \ln^2\left(\frac{E}{m_e}\right) \frac{f\left(\frac{m_{\eta_c}}{2E}\right)}{W}, \tag{25}$$

with

$$\chi(W) = \frac{1}{\pi} \frac{m_{\eta_c}\Gamma_{\eta_c}}{(W^2 - m_{\eta_c}^2)^2 + m_{\eta_c}^2 \Gamma_{\eta_c}^2} \tag{26}$$

and the Low function

$$f(z) = -(2+z^2)^2 \ln z - (1-z^2)(3+z^2) \tag{27}$$

In this example the beam energy corresponds to the LEP energy, $E = 45.6$ GeV, the two-photon width of the η_c was chosen to be $\Gamma_{\eta_c\to\gamma\gamma} = 1$ keV, its total width $\Gamma_{\eta_c} = 10.3$ MeV and $J = 0$.

4.3 Single-Tagged Hadronic Deep Inelastic Scattering.

TWOGEN has been used in the OPAL measurement of the hadronic F_2 structure function of the photon[9]. The photon-photon cross section is calculated from one of a range of possible parametrisations of F_2, in a way analagous to that used for dimuon production in subsection 4.1 above. The quark-antiquark pair is allowed to fragment into hadrons via the LUND string model[12]. Using a simpler form of the QED/QPM structure function than equation (18), the overall normalisation of TWOGEN is found to disagree with Vermaseren[1] by only 1.4%, much less than the statistical errors on the experimental datas.

5 The Structure of the Program

The TWOGEN program consists of a package of FORTRAN-77 subroutines, of which the user has to call three: `TWINIT` to initialise the generator, `TWGGEN` to be called inside an event loop, and `TWEXIT` to print out the generated cross section and related statistics. The calling sequences are as follows:

```
SUBROUTINE TWINIT(LOUTX,IPAR,XPAR)
```

with input parameters:

LOUTX		Logical unit for print-out	
XPAR(1)		Beam energy	GeV
XPAR(2)		Minimum two-gamma mass	GeV
XPAR(3)		Maximum two-gamma mass	GeV
XPAR(4)		Minimum scattering angle tag 1	rad
XPAR(5)		Maximum scattering angle tag 1	rad
XPAR(6)		Minimum scattering angle tag 2	rad
XPAR(7)		Maximum scattering angle tag 2	rad
XPAR(8)		Maximum weight	
XPAR(9)		Mass of resonance	GeV
XPAR(10)		Total width of resonance	GeV
IPAR(1)	0	Unweighted events	
	1	Weighted events	
IPAR(2)	0	Continuum production	
	1	Resonance formation	

Inside the event loop, a call to TWGGEN will return a weighted or unweighted event with the following parameters:

```
SUBROUTINE TWGGEN(WSQ,Q1SQ,Q2SQ,PGAM1,PGAM2,PTAG1,PTAG2,WEIGHT)
```

The outputs of this routine are:

WSQ	The gamma-gamma invariant mass squared	GeV**2
Q1SQ	Virtual mass squared of photon 1	GeV**2
Q2SQ	Virtual mass squared of photon 2	GeV**2
PGAM1(1:4)	Four-vector of photon 1	GeV
PGAM2(1:4)	Four-vector of photon 2	GeV
PTAG1(1:4)	Four-vector of tag 1	GeV
PTAG2(1:4)	Four-vector of tag 2	GeV
WEIGHT	Weight of the event	

The user can now generate the $\gamma\gamma$ final state he wishes, and accept or reject this event applying a hit-miss technique. At the end of the job a subroutine call must be made to `TWEXIT(XNORM,DXNORM)`, which prints out some statistical information and returns in XNORM the normalisation factor for the run (Eq. 16), and an estimate of its error `DXNORM`.

The standard random number generator RANMAR[13] is included in the package. The code is a self-contained FORTRAN-77 program. Its MAIN part calls two example routines (TWEXA1 and TWEXA2, respectively), which produce the plots of figures 3 and 4. A third example, TWEXA3 shows the generation of hadronic events as described in subsection 4.3. Starting from these examples, the user can implement his own final state model. In the code, references are made to standard HBOOK routines[14] for creating the histograms. The routine TWNENT can be called for writing the generated four-vectors to a file. This routine references routines from the JETSET library [12], version 7.3.

6 Summary

We have presented here a simple generator for the two-photon luminosity function in e^+e^- colliders. This generator can be used for generating with reasonable accuracies either continuum production of two-photon final states or resonances produced in two-photon collisions. It can therefore be a useful tool for studying simple two-photon reactions at the LEP or SLC accelerators and also for estimating the backgrounds to various processes from two-photon reactions. We have shown that the results of the generator are in good agreement with the Vermaseren Monte Carlo generator for the $\gamma\gamma \to \mu^+\mu^-$ channel and with an approximative analytical formula for the $\gamma\gamma \to \eta_c$ channel.

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