CERN-TH.7043/93
DTP/93/84

Does the W Mass Reconstruction Survive QCD Effects?

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Abstract

In the hadronic decay mode of a pair of W bosons, $e^+e^- \to W^+W^- \to q_1\overline{q}_2q_3\overline{q}_4$, QCD interference effects can mix up the two colour singlets $q_1\overline{q}_2$ and $q_3\overline{q}_4$, i.e. produce hadrons that cannot be uniquely assigned to either of W^+ and W^-. We show that interference is negligible for energetic perturbative gluon emission, and develop models to help us to estimate the non-perturbative effects. The total contribution to the systematic error on the W mass reconstruction may be as large as 40 MeV.

CERN-TH.7043/93
October 1993

This letter is concerned with QCD interference effects that may occur when two unstable particles decay and hadronize close to each other in space and time. W pair production is here of particular interest because of its practical importance and relative simplicity, but many other processes are also affected. (For further details and references see Ref. [1].)

QCD interference effects between the W^+ and W^- decays undermine the traditional meaning of a W mass in the process $e^+e^- \to W^+W^- \to q_1\overline{q}_2q_3\overline{q}_4$. Specifically, it is not even in principle possible to subdivide the hadronic final state into two groups of particles, one of which is produced by the $q_1\overline{q}_2$ system of the W^+ decay and the other by the $q_3\overline{q}_4$ system of the W^- decay: some particles originate from the joint action of the two systems. Since a determination of the W mass is one of the main objectives of LEP 2, it is important to understand how large the ambiguities can be. A statistical error of 55 MeV per experiment is expected [2], so the precision of the theoretical predictions should ideally match or exceed this experimental accuracy.

A complete description of QCD interference effects is not possible since non-perturbative QCD is not well understood. The concept of colour reconnection/rearrangement is therefore useful to quantify effects (at least in a first approximation). In a reconnection two original colour singlets (such as $q_1\overline{q}_2$ and $q_3\overline{q}_4$) are transmuted into two new ones (such as $q_1\overline{q}_4$ and $q_3\overline{q}_2$). Subsequently each singlet system is assumed to hadronize independently according to the standard algorithms, which have been so successful in describing e.g. Z^0 decays. Depending on whether a reconnection has occurred or not, the hadronic final state is then going to be somewhat different.

The colour reconnection effects were first studied by Gustafson, Pettersson and Zerwas [3], but their results were mainly qualitative and were not targeted on what might actually be expected at LEP 2. Their picture represents an example of the so-called instantaneous reconnection scenario, where the alternative colour singlets are immediately formed and allowed to radiate perturbative gluons.

In order to understand which QCD interference effects can occur in hadronic W^+W^- decays, it is useful to examine the space–time picture of the process. Consider a typical c.m. energy of 170 GeV, a W mass $m_W = 80$ GeV and width $\Gamma_W = 2.08$ GeV. The averaged (over the W mass distribution) proper lifetime for a W is $\langle\tau\rangle \approx (2/3)\hbar/\Gamma_W \approx$ 0.06 fm. This gives a mean separation of the two decay vertices of 0.04 fm in space and 0.07 fm in time. A gluon with an energy $\omega \gg \Gamma_W$ therefore has a wavelength much smaller than the separation between the W^+ and W^- decay vertices, and is emitted almost incoherently either by the $q_1\overline{q}_2$ system or by the $q_3\overline{q}_4$ one [4]. Only fairly soft gluons, $\omega \lesssim \Gamma_W$, feel the joint action of all four quark colour charges. On the other hand, the typical distance scale of hadronization is about 1 fm, i.e. much larger than the decay vertex separation. Therefore the hadronization phase may contain significant interference effects.

In the following, we will first discuss perturbative effects and subsequently non-perturbative ones. (For a discussion of a possible interplay between the two stages see Ref. [1].)

Until today, perturbative QCD has mainly been applied to systems of primary partons produced almost simultaneously. The radiation accompanying such a system can be represented as a superposition of gauge-invariant terms, in which each external quark line is uniquely connected to an external antiquark line of the same colour. The system is thus decomposed into a set of colourless $q\overline{q}$ antennae/dipoles [5]. One of the simplest examples is the celebrated $q\overline{q}g$ system, which (to leading order in $1/N_C^2$, where $N_C = 3$ is the number of colours) is well approximated by the incoherent sum of two separate antennae, $\widehat{\mathrm{qg}}$ and $\widehat{\mathrm{g\overline{q}}}$. These dipoles radiate gluons, which within the perturbative scenario are the principal sources of multiple hadroproduction.

Neglecting interferences, the $\mathrm{e^+e^-} \to \mathrm{W^+W^-} \to \mathrm{q_1\overline{q}_2q_3\overline{q}_4}$ final state can be subdivided into two separate dipoles, $\widehat{\mathrm{q_1\overline{q}_2}}$ and $\widehat{\mathrm{q_3\overline{q}_4}}$. Each dipole may radiate gluons from a maximum scale m_{W} downwards. Within the perturbative approach, colour transmutations can result only from the interferences between gluons (virtual as well as real) radiated in the $\mathrm{W^+}$ and $\mathrm{W^-}$ decays. A colour reconnection then corresponds to radiation, e.g. from the dipoles $\widehat{\mathrm{q_1\overline{q}_4}}$ and $\widehat{\mathrm{q_3\overline{q}_2}}$. The emission of a single primary gluon cannot give interference effects, by colour conservation, so interference terms only enter in second order in α_s.

The general structure of the results is well illustrated by the interference between the graph where a gluon with momentum k_1 (k_2) is emitted off the $\widehat{\mathrm{q_1\overline{q}_2}}$ ($\widehat{\mathrm{q_3\overline{q}_4}}$) dipole and the same graph with k_1 and k_2 interchanged:

$$\frac{1}{\sigma_0}\,\mathrm{d}\sigma^{\mathrm{int}} \simeq \frac{\mathrm{d}^3\mathbf{k}_1}{\omega_1}\,\frac{\mathrm{d}^3\mathbf{k}_2}{\omega_2}\left(\frac{C_F\,\alpha_s}{4\pi^2}\right)^2\frac{1}{N_C^2-1}\,\chi_{12}\,H(k_1)\,H(k_2)\;, \tag{1}$$

where $C_F = (N_C^2-1)/(2N_C) = 4/3$. We proceed to comment on the non-trivial factors in this expression.

The interference is suppressed by $1/(N_C^2-1) = 1/8$ as compared to the total rate of double primary gluon emissions. This is a result of the ratio of the corresponding colour traces.

The so-called profile function χ_{12} [4, 6] controls decay–decay interferences. It quantifies the overlap of the W propagators in the interfering Feynman diagrams. Near the $\mathrm{W^+W^-}$ pair threshold, χ_{12} simplifies to

$$\chi_{12} \approx \frac{\Gamma_{\mathrm{W}}^2}{\Gamma_{\mathrm{W}}^2 + (\omega_1-\omega_2)^2}\;. \tag{2}$$

Other interferences (real or virtual) are described by somewhat different expressions, e.g. with $\omega_1-\omega_2 \to \omega_1+\omega_2$, but have the same general properties. The profile functions cut down the phase space available for gluon emissions with $\omega \gtrsim \Gamma_{\mathrm{W}}$ by the alternative quark pairs. (We can neglect the contribution from kinematical configurations with $\omega_1, \omega_2 \gg \Gamma_{\mathrm{W}}$, $|\omega_1-\omega_2| \lesssim \Gamma_{\mathrm{W}}$ since the corresponding phase-space volume is small.) The possibility for the reconnected systems to develop QCD cascades is thus reduced, i.e. the dipoles are almost sterile.

The radiation pattern $H(k)$ is given by

$$H(k) = \widehat{\mathrm{q_1\overline{q}_4}} + \widehat{\mathrm{q_3\overline{q}_2}} - \widehat{\mathrm{q_1q_3}} - \widehat{\mathrm{\overline{q}_2\overline{q}_4}}\;, \tag{3}$$

where the radiation antennae are [5]

$$\widehat{ij} = \frac{(p_i\cdot p_j)}{(p_i\cdot k)(p_j\cdot k)}\;. \tag{4}$$

In addition to the two dipoles $\widehat{\mathrm{q_1\overline{q}_4}}$ and $\widehat{\mathrm{q_3\overline{q}_2}}$, which may be interpreted in terms of reconnected colour singlets, one finds two other terms, $\widehat{\mathrm{q_1q_3}}$ and $\widehat{\mathrm{\overline{q}_2\overline{q}_4}}$, which come in with a negative sign. The signs represent the attractive and repulsive forces between quarks and antiquarks [5, 7].

It should be emphasized that, analogously to other colour-suppressed interference phenomena, rearrangement can be viewed only on a completely inclusive basis, when all the antennae are simultaneously active in the particle production. The very fact that the reconnection pieces are not positive-definite reflects their wave interference nature. Therefore the effects of reconnected almost sterile cascades should appear on top of a dominant background generated by the ordinary-looking no-reconnection dipoles $\widehat{q_1\overline{q}_2}$ and $\widehat{q_3\overline{q}_4}$.

Summing up the above discussion, it can be concluded that perturbative colour reconnection phenomena are suppressed, firstly because of the overall factor $\alpha_s^2/(N_C^2 - 1)$, and secondly because the rearranged dipoles can only radiate gluons with energies $\omega \lesssim \Gamma_W$. Only a few low-energy particles should therefore be affected, $\Delta N^{\mathrm{recon}}/N^{\mathrm{no-recon}} \lesssim \mathcal{O}(10^{-2})$.

We now turn to the possibility of reconnection occurring as a part of the non-perturbative hadronization phase. Since hadronization is not understood from first principles, this requires model building rather than exact calculations. We will use the standard Lund string fragmentation model [8] as a starting point, but have to extend it considerably. The string is here to be viewed as a Lorentz covariant representation of a linear confinement field.

The string description is entirely probabilistic, i.e. any negative-sign interference effects are absent. This means that the original colour singlets $q_1\overline{q}_2$ and $q_3\overline{q}_4$ may transmute to new singlets $q_1\overline{q}_4$ and $q_3\overline{q}_2$, but that any effects e.g. of $\widehat{q_1 q_3}$ or $\widehat{\overline{q}_2\overline{q}_4}$ dipoles are absent. In this respect, the non-perturbative discussion is more limited in outlook than the perturbative one above. However, note that dipoles such as $\widehat{q_1 q_3}$ do not correspond to colour singlets, and can therefore not survive in the long-distance limit of the theory, i.e. they have to disappear in the hadronization phase.

The imagined time sequence is the following. The W^+ and W^- fly apart from their common production vertex and decay at some distance. Around each of these decay vertices, a perturbative parton shower evolves from an original $q\overline{q}$ pair. The typical distance that a virtual parton (of mass $m \sim 10$ GeV, say, so that it can produce separate jets in the hadronic final state) travels before branching is comparable with the average W^+W^- separation, but shorter than the fragmentation time. Each W can therefore effectively be viewed as instantaneously decaying into a string spanned between the partons, from a quark end via a number of intermediate gluons to the antiquark end. The strings expand, both transversely and longitudinally, at a speed limited by that of light. They eventually fragment into hadrons and disappear. Before that time, however, the string from the W^+ and the one from the W^- may overlap. If so, there is some probability for a colour reconnection to occur in the overlap region. The fragmentation process is then modified.

The Lund string model does not constrain the nature of the string fully. At one extreme, the string may be viewed as an elongated bag, i.e. as a flux tube without any pronounced internal structure. At the other extreme, the string contains a very thin core, a vortex line, which carries all the topological information, while the energy is distributed over a larger surrounding region. The latter alternative is the chromoelectric analogue to the magnetic flux lines in a type II superconductor, whereas the former one is more akin to the structure of a type I superconductor. We use them as starting points for two contrasting approaches, with nomenclature inspired by the superconductor analogy.

In scenario I, the reconnection probability is proportional to the space–time volume over which the W^+ and W^- strings overlap, with saturation at unit probability. This probability is calculated as follows. In the rest frame of a string piece expanding along the $\pm z$ direction, the colour field strength is assumed to be given by

$$\Omega(\mathbf{x},t) = \exp\left\{-(x^2+y^2)/2r_{\mathrm{had}}^2\right\}\ \theta(t-|\mathbf{x}|)\ \exp\left\{-(t^2-z^2)/\tau_{\mathrm{frag}}^2\right\}\ . \qquad (5)$$

The first factor gives a Gaussian fall-off in the transverse directions, with a string width $r_{\mathrm{had}} \approx 0.5$ fm of typical hadronic dimensions. The time retardation factor $\theta(t-|\mathbf{x}|)$ ensures that information on the decay of the W spreads outwards with the speed of light. The last factor gives the probability that the string has not yet fragmented at a given proper time along the string axis, with $\tau_{\mathrm{frag}} \approx 1.5$ fm. For a string piece e.g. from the W^+ decay, this field strength has to be appropriately rotated, boosted and displaced to the W^+ decay vertex. In addition, since the W^+ string can be made up of many pieces, the string field strength $\Omega^+_{\mathrm{max}}(\mathbf{x},t)$ is defined as the maximum of all the contributing Ω^+'s in the given point. The probability for a reconnection to occur is now given by

$$\mathcal{P}_{\mathrm{recon}} = 1 - \exp\left(-k_{\mathrm{I}}\int \mathrm{d}^3\mathbf{x}\,\mathrm{d}t\ \Omega^+_{\mathrm{max}}(\mathbf{x},t)\,\Omega^-_{\mathrm{max}}(\mathbf{x},t)\right)\ , \qquad (6)$$

where k_{I} is a free parameter. If a reconnection occurs, the space–time point for this reconnection is selected according to the differential probability $\Omega^+_{\mathrm{max}}(\mathbf{x},t)\,\Omega^-_{\mathrm{max}}(\mathbf{x},t)$. This defines the string pieces involved and the new colour singlets.

In scenario II it is assumed that reconnections can only take place when the core regions of two string pieces cross each other. This means that the transverse extent of strings can be neglected, which leads to considerable simplifications compared with the previous scenario. The position of a string piece at time t is described by a one-parameter set $\mathbf{x}(t,\alpha)$, where $0 \leq \alpha \leq 1$ is used to denote the position along the string. To find whether two string pieces i and j from the W^+ and W^- decays cross, it is sufficient to solve the equation system $\mathbf{x}_i^+(t,\alpha^+) = \mathbf{x}_j^-(t,\alpha^-)$ and to check that this (unique) solution is in the physically allowed domain. Further, it is required that neither string piece has had time to fragment, which gives two extra suppression factors of the form $\exp\{-\tau^2/\tau_{\mathrm{frag}}^2\}$, with τ the proper lifetime of each string piece at the point of crossing, i.e. as in scenario I. If there are several string crossings, only the one that occurs first is retained.

Both scenarios are implemented in a detailed simulation of the full process of $\mathrm{W}^\pm$ production and decay, parton shower evolution and hadronization [9]. It is therefore possible to assess any experimental consequences for an ideal detector.

The reconnection probability is predicted in scenario II without any adjustable parameters, although with the possibility to vary the baseline model in a few respects. Scenario I contains a completely free strength parameter k_{I}. We have chosen k_{I} to give an average $\mathcal{P}_{\mathrm{recon}} \approx 0.35$ at 170 GeV, as is predicted in scenario II.

The resulting c.m. energy dependence of $\mathcal{P}_{\mathrm{recon}}$ is very slow: between 150 and 200 GeV the variation is less than a factor of 2. Here it is useful to remember that the $\mathrm{W}^\pm$ are never produced at rest with respect to each other: the naïve Breit–Wigner mass distributions are distorted by phase-space effects, which favour lower W masses. For 150–200 GeV the average momentum of each W is therefore in the range 22–60 GeV, rather than in the range 0–60 GeV. It is largely this momentum that indicates how fast the two W systems are flying apart, and therefore how much they overlap in the middle of the event. Also the energy variation in the perturbative description is very small. If we want to call colour reconnection a threshold effect, we have to acknowledge that the threshold region is very extended.

Comparing the scenarios I and II above with the no-reconnection scenario, it turns out that reconnection effects are very small. The change in the average charged multiplicity is at the level of a per cent or less, and similar statements hold for rapidity distributions, thrust distributions, and so on. This is below the experimental precision we may expect, and so may well go unobserved. One would like to introduce more clever measures, which are especially sensitive to the interesting features, but so far we have had little success.

Ultimately, the hope would be to distinguish scenarios I and II, and thereby to gain some insight into the nature of the confinement mechanism. In principle, there are such differences. For instance, the reconnection probability is much more sensitive to the event topology in scenario II, since the requirement of having two string cores cross is more selective than that of having two broad flux tubes overlap.

We now come to the single most critical observable for LEP 2 physics, namely the W mass. Experimentally, m_{W} depends in a non-trivial fashion on all particle momenta of an event. Errors in the W mass determination come from a number of sources [2], which we do not intend to address here. Therefore we only study the extent to which the average reconstructed W mass is shifted when reconnection effects are added, but everything else is kept the same. Even so, results do depend on the reconstruction algorithm used. We have tried a few different ones, which however all are based on the same philosophy: a jet finder is used to define at least four jets, events with two very nearby jets or with more than four jets are rejected, the remaining jets are paired to define the two W's, and the average W mass of the event is calculated. Events where this number agrees to better than 10 GeV with the input average mass are used to calculate the systematic mass shift.

In scenario I this shift is consistent with being zero, within the 10 MeV uncertainty in our results from limited Monte Carlo statistics (160,000 events per scenario). Scenario II gives a negative mass shift, of about -30 MeV; this also holds for several variations of the basic scheme. A simpler model, where reconnections are always assumed to occur at the centre of the event, instead gives a positive mass shift: about $+30$ MeV if results are rescaled to $\mathcal{P}_{\mathrm{recon}} \approx 0.35$. We are therefore forced to conclude that not even the sign of the effect can be taken for granted, but that a real uncertainty of ± 30 MeV does exist from our ignorance of non-perturbative reconnection effects.

To examine the perturbative rearrangement effect, we have used a scenario where the original $\mathrm{q}_1\overline{\mathrm{q}}_2$ and $\mathrm{q}_3\overline{\mathrm{q}}_4$ dipoles are instantaneously reconnected to $\mathrm{q}_1\overline{\mathrm{q}}_4$ and $\mathrm{q}_3\overline{\mathrm{q}}_2$ ones, and these are allowed to radiate gluons with an upper cut-off given by the respective dipole invariant mass [3]. This gives a mass shift by about $+500$ MeV. We have above argued that real effects would be suppressed by at least a factor of 10^{-2} compared to this, and thus assign a 5 MeV error from this source. Finally, the possibility of an interplay between the perturbative and non-perturbative phases must be kept in mind. We have no way of modelling it, but believe it will not be much larger than the perturbative contribution, and thus assign a further 5 MeV from this source. Since the three sources are not independent, the numbers are added linearly to get an estimated total uncertainty of 40 MeV.

In view of the aimed-for precision, 40 MeV is non-negligible. However, remember that as a fraction of the W mass itself it is a half a per mille error. Reconnection effects are therefore smaller in the W mass than in many other observables, such as the charged multiplicity.

Clearly, it is important to study how sensitive experimental mass reconstruction algorithms are, and not just rely on the numbers of this paper. We believe that the uncertainty can be reduced by a suitable tuning of the algorithms, e.g. with respect to the importance given to low-momentum particles, and with respect to the statistical treatment of the wings of the W mass distribution. The 40 MeV number should therefore be viewed as a default one, to be assigned to any algorithm that has not been properly evaluated.

In summary, we have developed the first detailed model of QCD rearrangement effects in the decay of two heavy colourless objects into quarks and gluons. Beyond the immediate use for LEP 2 physics, the same techniques may be applied to a number of other processes, e.g. top quark production and decay [4, 6, 10].

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[*] Supported by the UK Science and Engineering Research Council.

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