

QCD Calculation of the $B \rightarrow \pi, K$ Form Factors ¹

V.M. Belyaev^a, A. Khodjamirian^{b,c*}, R. Rückl^{b,d,e}

^a Institute of Theoretical and Experimental Physics, 117259 Moscow, Russia

^b Sektion Physik der Universität München, D-8000 München 2, FRG

^c Yerevan Physics Institute, 375036 Yerevan, Armenia

^d Max-Planck-Institut für Physik, Werner-Heisenberg-Institut, D-8000 München 40, FRG

^e CERN, CH-1211 Geneva 23, Switzerland

Abstract

We calculate the form factors for the heavy-to-light transitions $B \rightarrow \pi, K$ by means of QCD sum rules using π and K light-cone wave functions. Higher twist contributions as well as gluonic corrections are taken into account. The sensitivity to the shape of the leading-twist wave functions and effects of SU(3)-breaking are discussed. The results are compared with quark model predictions and with the results from QCD sum rules for three-point correlators.

¹ Work supported by the German Federal Ministry for Research and Technology under contract No. 05 6MU93P

* *Alexander von Humboldt Fellow*

1 Introduction

The development of reliable quantitative methods to calculate hadronic matrix elements of quark and gluon operators in QCD is a task of great theoretical and phenomenological importance. In particular, the understanding of weak decays of charmed and beautiful hadrons and their use in determining fundamental parameters of the standard model and testing the theory depend crucially on the progress made in solving this difficult problem.

In this paper we consider the heavy-to-light transition form factors appearing in the matrix elements $\langle \pi | \bar{q}\gamma_\mu b | \bar{B} \rangle$, $q = u, d$ and $\langle K | \bar{s}\gamma_\mu b | \bar{B} \rangle$ which play an important role in B -meson decays. The former of these matrix elements determines CKM suppressed semileptonic B -decays, while the latter one enters in factorizable amplitudes of nonleptonic B -decays. One of the most interesting examples is the mode $B \rightarrow J/\psi K$. Theoretically, these form factors represent relatively simple hadronic matrix elements and are therefore very useful to test and compare different approaches.

The early calculations of heavy-to-light B -meson form factors were performed at zero momentum transfer in the framework of a quark model [1] as well as using standard QCD sum rules [2, 3]. The dependence on the four-momentum transfer p was assumed to be given by a simple pole factor $(1 - p^2/m_*^2)^{-1}$, with m_* being the mass of the lowest lying resonance in a given channel. The first complete calculation of the $B \rightarrow \pi$ form factor for nonzero momentum transfer was carried out quite recently [4] using QCD sum rules for three-point correlators. The results actually support the pole dominance model.

The aim of the present paper is to call attention to an alternative way of calculating heavy-to-light form factors which is also based on short-distance operator product expansion and QCD sum rule methods. However, in the present approach the basic correlator is taken between the vacuum and a light meson state, and all information about large distances is incorporated into a set of so-called light-cone wave functions for that particular light meson [5]. These wave functions represent distributions of the light-cone momentum of the constituents, and can be classified by their twist defined as the difference between the canonical dimension and the Lorentz spin of the corresponding operator. The light-cone wave functions are universal quantities similarly as the vacuum condensates, and are by now quite well studied. Their asymptotic form is fixed by perturbative QCD, while the nonasymptotic effects at lower momentum scales can be estimated from QCD sum rules for two-point correlators of appropriate currents [5, 6].

In refs. [7, 8] the light-cone method was demonstrated to be very suitable for exclusive processes. Moreover, it is technically much simpler than the standard QCD sum rule approach where all participating hadrons are replaced by corresponding currents with euclidean momenta. Previous applications of light-cone sum rules to weak form factors of heavy mesons focussed on the $B \rightarrow \pi$ form factor at zero momentum transfer [9] and on the $D \rightarrow K$ form factor [10].

Here, we present and discuss a comprehensive calculation of the $B \rightarrow \pi, K$ form factors which takes into account all twist 2, 3 and major twist 4 contributions. In addition to the pure quark wave functions, we also evaluate the influence of quark-gluon wave functions. The sensitivity to the precise shape of the leading-twist wave function and the impact of $SU(3)$ -breaking effects are investigated in some details. Finally, we compare our results with quark model [1] and conventional QCD sum rule calculations [4].

2 Formal derivation of the sum rules

For definiteness, we show here and in the next section the derivation of the sum rule for the $B^- \rightarrow K^-$ form factor. The results can be easily generalized to the corresponding expressions for $B \rightarrow \pi$ form factors. We start with the following matrix element of the time-ordered product of two currents between the vacuum state and the K-meson at momentum q :

$$F_\mu(p, q) = i \int d^4x e^{ipx} \langle K(q) | T\{\bar{s}(x)\gamma_\mu b(x), \bar{b}(0)i\gamma_5 u(0)\} | 0 \rangle . \quad (1)$$

The hadronic representation of (1) is obtained by inserting a complete set of states including the B -meson ground state, higher resonances and nonresonant states with B -meson quantum numbers:

$$\begin{aligned} F_\mu(p, q) &= \frac{\langle K | \bar{s}\gamma_\mu b | B \rangle \langle B | \bar{b}i\gamma_5 u | 0 \rangle}{m_B^2 - (p+q)^2} + \\ &+ \sum_h \frac{\langle K | \bar{s}\gamma_\mu b | h \rangle \langle h | \bar{b}i\gamma_5 u | 0 \rangle}{m_h^2 - (p+q)^2} \\ &= F(p^2, (p+q)^2)q_\mu + \tilde{F}(p^2, (p+q)^2)p_\mu . \end{aligned} \quad (2)$$

Here, p denotes the four-momentum transfer. Otherwise, the notation should be self-explaining. From now on, we shall concentrate on the invariant amplitude F which is physically more interesting than the amplitude \tilde{F} . For F one can write a general dispersion relation in the momentum squared $(p+q)^2$ of the B -meson:

$$F(p^2, (p+q)^2) = \int_{m_B^2}^{\infty} \frac{\rho(p^2, s)ds}{s - (p+q)^2} \quad (3)$$

where possible subtractions are neglected and the spectral density is given by

$$\rho(p^2, s) = \delta(s - m_B^2)2f_K^+(p^2)\frac{m_B^2 f_B}{m_b} + \rho^h(p^2, s) . \quad (4)$$

The first term on the r.h.s. of (4) represents the B -meson contribution and follows from (2) by inserting the matrix elements

$$\langle K | \bar{s}\gamma_\mu b | B \rangle = 2f_K^+(p^2)q_\mu + (f_K^+(p^2) + f_K^-(p^2))p_\mu \quad (5)$$

and

$$\langle B | \bar{b}i\gamma_5 u | 0 \rangle = \frac{m_B^2 f_B}{m_b} . \quad (6)$$

In the above, $f_K^+(p^2)$ is the transition form factor for $B^- \rightarrow K^-$ which we want to calculate, f_B is the B -meson decay constant, m_b is the b -quark mass, and $\rho^h(p^2, s)$ denotes the spectral density of higher resonances and of the continuum of states. In accordance with the standard procedure in the QCD sum rule applications, we invoke quark-hadron duality and replace ρ^h by

$$\rho^h(p^2, s) = \frac{1}{\pi} \text{Im}F_{QCD}(p^2, s)\Theta(s - s_0) \quad (7)$$

where $\text{Im}F_{QCD}(p^2, s)$ is obtained from the imaginary part of the correlation function (1) calculated in QCD and s_0 is the threshold parameter. For consistency, we shall take both

parameters f_B and s_0 from a QCD sum rule analysis of the correlator of two $\bar{b}\gamma_5 u$ currents, as explained in Section 3.

In order to suppress the contribution from the excited states and from the continuum exponentially and to get rid of possible subtraction terms, we apply the Borel operator \hat{B} with respect to the variable $(p+q)^2$ to the dispersion integral in (3) :

$$F(p^2, M^2) = \hat{B}F(p^2, (p+q)^2) = \int_{m_B^2}^{\infty} \rho(p^2, s) e^{-s/M^2} ds. \quad (8)$$

Using then (4) and (7) , one obtains

$$F(p^2, M^2) = 2 \frac{f_B m_B^2}{m_b} f_K^+(p^2) e^{-m_B^2/M^2} + \frac{1}{\pi} \int_{s_0}^{\infty} \text{Im} F_{QCD}(p^2, s) e^{-s/M^2} ds. \quad (9)$$

The next step involves the calculation of the correlation function (1) or the invariant amplitude $F(p^2, (p+q)^2)$ in QCD. This calculation is of course essentially the same as the one proposed above for (7). After Borel transformation the result can be written in the form

$$F(p^2, M^2) = \frac{1}{\pi} \int_{m_b^2}^{\infty} \text{Im} F_{QCD}(p^2, s) e^{-s/M^2} ds. \quad (10)$$

Finally , equating (10) with (9) yields the desired sum rule for the form factor f_K^+ :

$$f_K^+(p^2) = \frac{m_b}{2\pi m_B^2 f_B} \int_{m_b^2}^{s_0} \text{Im} F_{QCD}(p^2, s) e^{-\frac{s-m_B^2}{M^2}} ds. \quad (11)$$

3 QCD calculation with light-cone wave functions

The possibility to calculate the correlator (1) in the region of large space-like momenta $(p+q)^2 < 0$, keeping the momentum q at the physical point $q^2 = m_K^2 \simeq 0$, is based on the expansion of the T -product of the currents in (1) near the light-cone $x^2 = 0$. The leading contribution to the operator product expansion arises from the contraction of the b -quark operators in (1) to the free b -quark propagator $\langle 0 | \bar{b}\bar{b} | 0 \rangle$. The light quark operators are left uncontracted . Diagrammatically, this contribution is depicted in Fig. 1a. The formal expression is easily obtained from (1):

$$F_\mu(p, q) = \int_0^\infty \frac{d\alpha}{16\pi^2 \alpha^2} \int d^4 x e^{ipx - m_b^2 \alpha + \frac{x^2}{4\alpha}} \quad (12)$$

$$\times \left(m_b \langle K(q) | \bar{s}(x) \gamma_\mu \gamma_5 u(0) | 0 \rangle + \frac{ix^\rho}{2\alpha} \langle K(q) | \bar{s}(x) \gamma_\mu \gamma_\rho \gamma_5 u(0) | 0 \rangle \right)$$

where we made use of the following representation of the free propagator \hat{S}_b^0 :

$$\langle 0 | T \{ b(x) \bar{b}(0) \} | 0 \rangle = i \hat{S}_b^0(x) = \int \frac{d^4 p}{(2\pi)^4} e^{-ipx} i \frac{\hat{p} + m_b}{p^2 - m_b^2} \quad (13)$$

$$= - \int_0^\infty \frac{d\alpha}{16\pi^2 \alpha^2} (m_b + i \frac{\hat{x}}{2\alpha}) e^{-m_b^2 \alpha + \frac{x^2}{4\alpha}}.$$

According to the analysis presented in [10] the expansion (12) should be valid in the range $0 \leq p^2 \leq m_b^2 - O(1\text{GeV}^2)$, i.e. sufficiently far below the physical states in the $b\bar{s}$ -channel.

The lowest lying charmonium levels evidently reside in this region, so that our results will be applicable to $B \rightarrow J/\psi K$.

Let us consider the first term in (12). The matrix element of the nonlocal operator is given by [5, 8]:

$$\langle K(q) | \bar{s}(x)\gamma_\mu\gamma_5 u(0) | 0 \rangle = -iq_\mu f_K \int_0^1 du e^{iuqx} \left(\varphi_K(u) + \frac{5\delta_K^2}{36} x^2 \varphi_{4K}(u) \right) \quad (14)$$

where $\varphi_K(u)$ is the K -meson light-cone wave function of the leading twist 2 and $\varphi_{4K}(u)$ represents one of the next-to-leading twist 4 wave functions. There are also other twist 4 terms in this matrix element specified in [8] which we do not show here explicitly since their numerical contribution to the final result is of the order of 1% as we have checked. All wave functions are normalized to unity and δ_K^2 is a dimensionful parameter given later.

The matrix element appearing in the second term of eq. (12) can be split in two matrix elements using the identity $\gamma_\mu\gamma_\rho = -i\sigma_{\mu\rho} + g_{\mu\rho}$. These matrix elements are determined by wave functions of twist 3:

$$\langle K(q) | \bar{s}(x)i\gamma_5 u(0) | 0 \rangle = \frac{f_K m_K^2}{m_u + m_s} \int_0^1 du e^{iuqx} \varphi_{pK}(u) \quad (15)$$

$$\langle K(q) | \bar{s}(x)\sigma_{\mu\nu}\gamma_5 u(0) | 0 \rangle = i(q_\mu x_\nu - q_\nu x_\mu) \frac{f_K m_K^2}{6(m_u + m_s)} \int_0^1 du e^{iuqx} \varphi_{\sigma K}(u). \quad (16)$$

Substituting the matrix elements (14) to (16) into (12), and integrating over x and the auxiliary parameter α we find the following expression for the coefficient of q_μ in (2) that is for the invariant amplitude F :

$$F_{QCD}(p^2, (p+q)^2) = -f_K m_b \int_0^1 du \left(\frac{\varphi_K(u)}{(p+qu)^2 - m_b^2} - \frac{10\delta_K^2 m_b^2 \varphi_{4K}(u)}{9((p+qu)^2 - m_b^2)^3} \right) \quad (17)$$

$$- \frac{f_K m_K^2}{m_u + m_s} \int_0^1 du \left[\frac{\varphi_{pK}(u)u}{(p+qu)^2 - m_b^2} + \frac{\varphi_{\sigma K}(u)}{6((p+qu)^2 - m_b^2)} \left(2 - \frac{p^2 + m_b^2}{(p+qu)^2 - m_b^2} \right) \right].$$

Borel transformation of (17) according to (8) then yields the QCD result anticipated in (10). Finally, the $B \rightarrow K$ transition form factor $f_K^+(p^2)$ is obtained from (11) and (17):

$$f_K^+(p^2) = \frac{f_K m_b^2}{2f_B m_B^2} \int_\Delta^1 \frac{du}{u} \exp\left[\frac{m_B^2}{M^2} - \frac{m_b^2 - p^2(1-u)}{uM^2} \right] \quad (18)$$

$$\left[\varphi_K(u) + \frac{\mu_K}{m_b} u \varphi_{pK}(u) + \frac{\mu_K}{6m_b} \varphi_{\sigma K}(u) \left(2 + \frac{m_b^2 + p^2}{uM^2} \right) - \frac{5\delta_K^2 m_b^2}{9M^4} \frac{\varphi_{4K}(u)}{u^2} \right].$$

where $\mu_K = m_K^2/(m_s + m_u)$ and $\Delta = (m_b^2 - p^2)/(s_0 - p^2)$.

In addition to the quark-antiquark wave functions considered above there are in principle also contributions from multi-particle wave functions. The most important corrections of this type are expected to arise from quark-gluon operators in the operator-product expansion of (1). A typical diagram where the gluon is emitted from the heavy quark is shown in Fig.1b. The leading contribution arises from the twist 3 operator:

$$\begin{aligned} \langle K(q) | \bar{s}(x)gG_{\mu\nu}(z)\sigma_{\rho\lambda}\gamma_5 u(0) | 0 \rangle = & if_{3K}[q_\mu(q_\rho g_{\lambda\nu} - q_\lambda g_{\rho\nu}) \\ & - q_\nu(q_\rho g_{\lambda\mu} - q_\lambda g_{\rho\mu})] \int \mathcal{D}\alpha_i \varphi_{3K}(\alpha_i) e^{iq(x\alpha_1 + z\alpha_3)} \end{aligned} \quad (19)$$

where $G_{\mu\nu}(z) = (\lambda^c/2)G_{\mu\nu}^c(z)$, λ^c and $G_{\mu\nu}^c$ being the usual colour matrices and the gluon field tensor, and $\mathcal{D}\alpha_i = d\alpha_1 d\alpha_2 d\alpha_3 \delta(\alpha_1 + \alpha_2 + \alpha_3 - 1)$. The three-particle wave function $\varphi_{3K}(\alpha_i) = \varphi_{3K}(\alpha_1, \alpha_2, \alpha_3)$ and the corresponding coupling constant f_{3K} are introduced and discussed in [5, 6].

The calculation of this gluonic correction is somewhat more involved than the previous one. Instead of the free propagator (13) we now need the b-quark propagator including the interaction with gluons in first order :

$$\langle 0 | T\{b(x)\bar{b}(0)\} | 0 \rangle_A = i\hat{S}_b^0(x) - ig \int dz \hat{S}_b^0(x-z) \gamma^\mu \frac{\lambda^c}{2} A_\mu^c(z) \hat{S}_b^0(z) \quad (20)$$

where $\hat{S}_b^0(x)$ is the free b-quark propagator defined in (13). In the fixed point gauge, $x^\mu A_\mu^c = 0$, the gluon field A_μ^c can be represented directly in terms of the field strength $G_{\mu\nu}^c$:

$$A_\mu^c(z) = z_\nu \int_0^1 u du G_{\nu\mu}^c(uz). \quad (21)$$

We substitute (20) into (1), use (19) and (21), and integrate over x and z . This yields the following expression for the quark-gluon contribution to the invariant amplitude F :

$$F_{QCD}^G(p^2, (p+q)^2) = 4f_{3K} \int_0^1 u du \int \mathcal{D}\alpha_i \frac{(pq)\varphi_{3K}(\alpha_i)}{((p+(\alpha_1+u\alpha_3)q)^2 - m_b^2)^2}. \quad (22)$$

Finally, after Borel transformation of (22) one arrives at the corresponding correction to f_K^+ :

$$\begin{aligned} f_K^{+G}(p^2) = & -\frac{f_{3K}m_b}{f_B m_B^2} \int_0^1 u du \int \mathcal{D}\alpha_i \Theta(\alpha_1 + u\alpha_3 - \Delta) \\ & \exp\left[\frac{m_B^2}{M^2} - \frac{m_b^2 - p^2(1 - \alpha_1 - u\alpha_3)}{(\alpha_1 + u\alpha_3)M^2}\right] \left[1 - \frac{m_b^2 - p^2}{(\alpha_1 + u\alpha_3)M^2}\right] \frac{\varphi_{3K}(\alpha_i)}{(\alpha_1 + u\alpha_3)^2}. \end{aligned} \quad (23)$$

Before concluding the QCD calculation of the $B \rightarrow K$ form factor $f_K^+(p^2)$ we want to make a few remarks on the remaining gluon corrections. One can distinguish three types of corrections. There are corrections where a gluon is emitted by one of the quark lines in Fig. 1a and absorbed in the meson wave function. One of these contributions is shown in Fig. 1b and calculated above. The others, when the gluon is emitted by a light quark line effectively give rise to three-particle corrections to the wave functions of twist 3 as it was shown in [8]. In the next section, we will study the numerical influence of these corrections on f_K^+ . Furthermore, there are corrections due to the interaction of the heavy virtual quark in the correlator (1) with the vacuum gluon condensate. They are expected to be of order of $\langle G_{\mu\nu}G_{\mu\nu} \rangle / m_b^4$ and, therefore negligible for the b-quark. Corresponding effects for the light quarks are considered to be included in the light-cone wave functions. Finally, there are additional perturbative $O(\alpha_s)$ corrections from gluon exchange between the light and heavy quarks. These corrections are beyond the scope of the present paper. However, we will approximately take into account the important $O(\alpha_s)$ correction to the pseudoscalar vertex $\bar{b}\gamma_5 u$ shown in Fig. 1c following the procedure advocated in [4].

In order to obtain the corresponding expressions for the $B \rightarrow \pi$ form factor f_π^+ from (18) and (23), one only needs to replace f_K by f_π , μ_K by μ_π , φ_K by φ_π , etc. It is important to note that twist 3 contributions to f_K^+ and f_π^+ are suppressed by a factor μ_K/m_b and μ_π/m_b , respectively. Therefore, one expects the form factors to be dominated by the contributions from the twist 2 wave functions. This dominance may in fact be exploited to improve our knowledge of these universal wave functions by comparing the predictions which will be presented in section 5 with experimental data.

4 Choice of parameters and wave functions

Having the necessary formulae at hand we next explain our choice of the relevant parameters and wave functions. Let us first consider the parameters which characterize the B -meson channel that is $m_B = 5.28 GeV$, f_B , m_b and s_0 . We note that what was actually calculated in (18) is a sum rule for the product of two amplitudes, $f_K^+ f_B$. On the other hand, the B -meson decay constant f_B is determined independently by a well-known QCD sum rule for the two-point correlator of $\bar{b}\gamma_5 u$ -currents. As it was stressed in [4], the consistent and convenient way to deal with the above parameters is to take their values from this two-point sum rule, but neglect $O(\alpha_s)$ radiative corrections in f_B since these corrections tend to be cancelled by the corresponding radiative corrections to the sum rule (18) depicted in Fig. 1c. In other words, instead of substituting the physical value of f_B in (18) we may take \tilde{f}_B defined as the square root of the two-point sum rule for f_B^2 without radiative gluon corrections, and at the same time drop the correction from Fig. 1c diagram. From a numerical analysis of the sum rule for f_B^2 we find several self-consistent sets of parameters: $\tilde{f}_B = 135 MeV$, $m_b = 4.7 GeV$, $s_0 = 35 GeV^2$ (I), $\tilde{f}_B = 160 MeV$, $m_b = 4.6 GeV$, $s_0 = 37 GeV^2$ (II), and $\tilde{f}_B = 115 MeV$, $m_b = 4.8 GeV$ and $s_0 = 33 GeV^2$ (III). As our nominal choice we take set (I) which is in accordance with the values used in the conventional QCD sum rule approach of [4]. However, we have checked that sets (II) and (III) lead to almost the same results.

A second set of parameters is connected with the light mesons π or K . These are $f_\pi = 133 MeV$, $f_K = 160 MeV$, $\mu_\pi = m_\pi^2/(m_u + m_d)$ and $\mu_K = m_K^2/(m_s + m_d)$. With $m_u + m_d \simeq 11 MeV$ [11] one has $\mu_\pi \simeq 1.6 GeV$ (at a normalization scale of order $1 GeV$). To estimate μ_K we use the familiar PCAC relation for pseudo-Goldstone bosons to get

$$\frac{\mu_K}{\mu_\pi} = \frac{(\langle \bar{u}u \rangle + \langle \bar{s}s \rangle) f_\pi^2}{(\langle \bar{u}u \rangle + \langle \bar{d}d \rangle) f_K^2} \simeq 0.62. \quad (24)$$

Here, we assumed $\langle \bar{u}u \rangle : \langle \bar{d}d \rangle : \langle \bar{s}s \rangle \simeq 1 : 1 : 0.8$ as suggested by QCD sum rules for strange hadrons (see e.g. [12]). With $\mu_\pi \simeq 1.6 GeV$ one thus obtains $\mu_K \simeq 1.0 GeV$. On the other hand, simple $SU(3)$ and $SU(6)$ symmetry arguments suggest $\mu_K \simeq \mu_\pi$, i.e. a considerably larger value. Interestingly, the latter choice corresponds to $m_s + m_u \simeq 150 MeV$, whereas the use of (24) implies a heavier s-quark. Fortunately, the uncertainty in μ_K only affects the twist 3 contribution to the $B \rightarrow K$ form factor, and therefore does not preclude reasonably accurate predictions.

Finally, for the parameters δ_π^2 and $\varphi_{3\pi}$ which appear in the coefficients of the higher twist wave functions $\varphi_{4\pi}$ and $\varphi_{3\pi}$, respectively, we take the values given in the literature [5, 6]: $\delta_\pi^2 \simeq 0.2 GeV^2$ and $f_{3\pi} \simeq 0.0035 GeV^2$. Since these contributions are small, our results are quite insensitive to possible differences between π and K in this respect and we simply put $\delta_K^2 \simeq \delta_\pi^2$ and $\varphi_{3K} \simeq \varphi_{3\pi}$.

Turning now to the light cone wave functions themselves, we recall the asymptotic expressions which are completely determined by perturbative QCD and well-known [5]. In the SU(3) limit, one has ($H = K, \pi$)

$$\varphi_H = 6u(1 - u) \quad (25)$$

$$\varphi_{pH} = 1, \quad \varphi_{\sigma H} = 6u(1 - u) \quad (26)$$

$$\varphi_{4H} = 30u^2(1 - u)^2 \quad (27)$$

$$\varphi_{3H} = 360\alpha_1\alpha_2\alpha_3^2. \quad (28)$$

Clearly, the asymptotic wave functions should be renormalized to the characteristic scale of the process under consideration. This brings nonasymptotic effects into play which change the shape of the above wave functions, but preserve their normalization to unity. Moreover, SU(3)-breaking effects give rise to asymmetries in the K-meson wave functions under $u \leftrightarrow 1 - u$ reflecting the asymmetry in the quark masses m_s and $m_{u,d}$. Since in the Breit frame the light-cone wave functions represent distributions of the fraction of the K -meson momentum carried by the constituent quarks, one would expect the s -quark to carry more momentum on average than the light quarks.

Being of nonperturbative origin, these effects are difficult to evaluate. Fortunately, since the form factor $f^+(p^2)$ only depends on integrals over the light-cone wave functions, it is not necessary to know their actual shape very precisely. Moreover, the Borel mass parameter $M^2 \geq O(M_B^2 - m_b^2)$ provides a reasonably high renormalization scale. Therefore, one can expect the asymptotic wave functions to yield a quite reliable estimate. Nevertheless, in order to be on the safe side, we have investigated nonasymptotic effects for the leading-twist wave functions φ_π and φ_K . For that purpose we have used the model suggested in [5] which is based on an expansion over orthogonal Gegenbauer polynomials with coefficients determined by means of QCD sum rules for the 2-point correlators of π and K currents. The explicit expressions are given below :

$$\varphi_K = 6u(1 - u)\left\{1 + A_K^+[(2u - 1)^2 - \frac{1}{5}] + 5b(2u - 1)[1 + A_K^-[(2u - 1)^2 - \frac{3}{7}]]\right\} \quad (29)$$

$$\varphi_\pi = 6u(1 - u)[1 + A_\pi^+[(2u - 1)^2 - \frac{1}{5}]]. \quad (30)$$

Taking for the parameters in (29) and (30) the values fitted at the normalization scale $\mu \simeq 500 MeV$ [5, 9] and renormalizing these values to the scale $\mu \simeq M_B^2 - m_b^2$ one finds $A_K^+ = 1.8$, $A_K^- = 1.2$, $A_\pi^+ = 3.0$ and $b \simeq 0.1$. Note that the parameter b incorporates the difference between the average s - and u -quark momenta. We shall study numerically the change in f^+ due to changes in the shape of φ_π and φ_K . For the higher-twist wave functions φ_p , φ_σ , φ_4 the asymptotic expressions should provide sufficiently reliable estimates of the subleading contributions.

Concluding the discussion of parameters and wave functions it is important to note that at the level of accuracy adopted here, there are at least three sources of SU(3)-breaking effects which may cause differences between the $B \rightarrow \pi$ and $B \rightarrow K$ form factors: the difference between f_π and f_K , the difference between μ_π and μ_K , and the difference between φ_π and φ_K . Further refinement in this respect is possible, but this is far beyond the goal of our present study.

5 Numerical results

Before giving numerical predictions on the form factors $f^+(p^2)$ we must first determine the range of values for the Borel parameter M^2 for which the sum rules (18) can be expected to yield reliable results. The lower limit of this range is determined by the requirement on the terms proportional to M^{-2n} , $n > 1$ to remain subdominant. In (18) this concerns in particular the twist-4 contribution from φ_4 , which increases rapidly at small M^2 analogously to higher order power corrections in conventional QCD sum rules. The upper limit of the allowed interval in M^2 is determined by demanding the higher resonance and continuum contribution not to grow too large. We have checked numerically for $0 \leq p^2 \leq 20\text{GeV}^2$ and $10 \leq M^2 \leq 15\text{GeV}^2$ that (a) the contribution from φ_4 is less than 10% , (b) the higher states contribute less than 30% , and (c) the resulting values of $f^+(p^2)$ are practically independent of the Borel parameter at $p^2 \leq 10\text{GeV}^2$ and vary only slightly with M^2 at $p^2 \geq 10\text{GeV}^2$. This is illustrated in Fig. 2 for $f_K^+(p^2)$. The breakdown of the stability as p^2 approaches the region $m_b^2 - O(1\text{GeV}^2)$ is expected. From the fiducial window in Fig. 2a we can read off the QCD prediction for the $B \rightarrow K$ form factor at zero momentum transfer:

$$f_K^+(0) = 0.32. \quad (31)$$

The variation with M^2 is about 0.005. The p^2 -dependence of this form factor is plotted in Fig. 3.

While the twist 4 contribution from φ_{4K} do not exceed 10% as required and the gluonic correction from φ_{3K} is found to be smaller than 2%, the twist 3 wave functions contribute at the level of 20 to 40% and are therefore important. They are shown separately in Fig. 2.

The stability features and the hierarchy in the contributions from the wave functions of different twist are essentially the same when going from the K -meson to the π -meson. For the $B \rightarrow \pi$ form factor at zero momentum transfer we find

$$f_\pi^+(0) = 0.29. \quad (32)$$

Here, the result varies with M^2 within the fiducial range by 0.01. The p^2 dependence of this form factor is shown in Fig. 4.

It is of course important to investigate further sources of theoretical uncertainties. Therefore, we have carefully examined the sensitivity of $f_K^+(p^2)$ and $f_\pi^+(p^2)$ to reasonable changes of the parameters and wave functions discussed in the previous section. As far as the B -channel parameters f_B , m_b and s_0 are concerned, we stress again that they are interrelated through sum rules for two-point correlators. Hence, they should not be changed independently. Numerically, for the three consistent sets of values given in section 4 , the resulting form factor vary by less than 5% despite the sizable variation of the individual parameters. Obviously, f_B enters only in the total normalization of the form factors and drops out in the ratio $f_\pi^+(p^2)/f_K^+(p^2)$, whereas m_b also sets the main scale for the p^2 -dependence.

In contrast, the sensitivity to the parameters μ_π and μ_K entering the coefficients of the twist 3 contributions is quite considerable. This causes no problem for $f_\pi^+(p^2)$ since μ_π is known with good accuracy. However, this is not the case for μ_K as pointed out in section 4 where we have obtained values varying from 1.0 to 1.6 GeV. The corresponding uncertainty in $f_K^+(p^2)$ amounts to 20% at large p^2 as illustrated in Fig. 5a.

Finally, we have studied the influence of the shape of the leading-twist wave functions φ_π and φ_K which give the dominant contributions. In Fig. 5b, $f_K^+(p^2)$ is compared for two

different choices for φ_K : (a) the asymmetric form (29) used in all numerical illustration presented so far, and (b) the asymptotic form given in (25). The analogous comparison for $f_\pi^+(p^2)$ is shown in Fig. 5c. As can be seen, the nonasymptotic corrections damp the increase of the form factor with p^2 . However, this effect is much more pronounced in $B \rightarrow \pi$ than in $B \rightarrow K$. On the other hand, the absolute value of $f_\pi^+(p^2)$ remains almost unchanged, whereas the value of $f_K^+(p^2)$ grows by roughly 0.1 at $p^2 \leq 10 GeV^2$.

Some corrections are also expected from nonasymptotic effects in the twist 3 wave functions, including SU(3)-breaking effects. As far as we know, the latter have not been considered so far. In the SU(3)-limit, nonasymptotic corrections to the wave functions φ_p and φ_σ are investigated in [8]. It is shown that they are determined by the wave function φ_{3K} given in (19) above. As already pointed out, these corrections can be associated with gluons emitted from the light quark lines in the Fig. 1a and absorbed in the meson wave function. Using explicit formulae from [8], we illustrate the effect on f_K^+ and f_π^+ in Fig. 5b and c. Clearly, in order to eliminate the uncertainties illustrated in Fig. 5 a better understanding of the higher twist wave functions is required.

6 Conclusions

Summarizing our investigations, in Figs. 3 and 4 we compare our predictions on $f_K^+(p^2)$ and $f_\pi^+(p^2)$ with the results of other calculations [1, 4, 9]. Within the uncertainties there is satisfactory agreement. To be more definite, at zero momentum transfer we find $f_K^+(0) = 0.26 \div 0.37$ and $f_\pi^+(0) = 0.24 \div 0.29$ where the ranges indicate our estimate of theoretical uncertainties on the basis of Fig. 5. We would like to emphasize in particular the coincidence with the result $f_\pi^+(0) = 0.24 \pm 0.025$ obtained in [4] from an alternative QCD sum rule approach in which the large-distance effects are parametrized in terms of vacuum condensates rather than by wave function on the light-cone.

Also the p^2 -dependence of the form factors is rather similar in the different approaches. Note, however, that in the quark-model [1] the form factors are assumed to have a simple pole-behaviour:

$$f^+(p^2) = \frac{f^+(0)}{1 - p^2/m_*^2} \quad (33)$$

with $m_* = 5.3 GeV$ in the case of f_π^+ and $m_* = 5.43 GeV$ for f_K^+ as expected in the spirit of vector dominance. The authors of [4] have also presented their calculated result for f_π^+ in the form (33) and obtained $m_* = 5.2 \pm 0.05 GeV$. In comparison to that we find a slightly steeper p^2 -dependence corresponding roughly to $m_* \simeq 5.0 GeV$.

Furthermore, it is obvious from Figs. 3 to 5 that the method put forward here is not yet precise enough to deal with SU(3)-breaking effects. In other words, we are not in a position to clearly distinguish $f_K^+(p^2)$ from $f_\pi^+(p^2)$, apart from the tendency to get a slightly smaller value for $f_\pi^+(0)$ than for $f_K^+(0)$. Improvements in this direction are possible, but require some further study.

On the other hand, we are able to present a quite accurate prediction for the value of the form factor f_K^+ at the J/ψ mass, to wit

$$f_K^+(m_{J/\psi}^2) = 0.50 \div 0.60. \quad (34)$$

This result is important, since it is obtained from a well-defined and, as we have shown, rather stable calculation. It puts investigations of the $B \rightarrow J/\psi K$ decay mode, in particular, in the

context of possible searches for CP -violation, on a more reliable basis.

Last but not least, the use of light-cone wave functions simplifies the calculation of weak matrix elements considerably in comparison to the conventional QCD sum rule approach. This may become crucial in more complicated problems such as calculations of exclusive nonleptonic B-decays beyond the factorization approximation.

7 Acknowledgements

We are grateful to V.M. Braun for very useful discussions and suggestions. The help of F. Cuyper in computer calculations is greatly acknowledged. V.B. thanks the Max-Planck Institute for hospitality and financial support during the initial phase of this work. A.K. gratefully acknowledges financial support from the Alexander von Humboldt Foundation.

References

- [1] M. Bauer, B. Stech, M. Wirbel, Z. Phys. C 34 (1987) 103.
- [2] C.A. Dominguez, N. Paver, Z. Phys C 41 (1988) 217.
- [3] A.A. Ovchinnikov, Sov. J. Nucl. Phys. 50 (1989) 519.
- [4] P. Ball, V.M. Braun, H.G. Dosch, Phys. Lett. B 273 (1991) 316.
- [5] V.L. Chernyak, A.R. Zhitnitsky, Phys. Rep. 112 (1984) 173.
- [6] A.S. Gorsky, Sov. J. Nucl. Phys. 41 (1985) 1008; *ibid.* 45 (1987) 512.
- [7] I.I. Balitsky, V.M. Braun, A.V. Kolesnichenko, Nucl. Phys. B 312 (1989) 509.
- [8] V.M. Braun, I.B. Filyanov, Z. Phys. C 44 (1989) 157 ; *ibid.* 48 (1990) 239.
- [9] V.L. Chernyak, I.R. Zhitnitsky, Nucl. Phys. B345 (1990) 137.
- [10] P. Ball, V.M. Braun, H.G. Dosch, Phys. Rev. D 44 (1991) 3567.
- [11] J. Gasser, H. Leutwyler, Phys. Reports 87 (1982) 77.
- [12] S.Narison, QCD Spectral sum rules (World Scientific, Singapore, 1989).

Figure Captions

Figure 1: QCD diagrams contributing to the correlation function (1). Solid lines represent quarks, dashed lines gluons, wavy lines are external currents, and the blobs denote K -meson wave functions on the light-cone.

Figure 2 : Form factor $f_K^+(p^2)$ as a function of the Borel mass squared M^2 at various values of the momentum transfer p^2 . The solid curves depicts the total sum rule results, while the dashed curves show the twist 3 contribution alone. The arrows indicate the fiducial interval in M^2 .

Figure 3 : Form factor $f_K^+(p^2)$ of the $B \rightarrow K$ transitions at $M^2 = 10\text{GeV}^2$ (upper solid curve) and $M^2 = 15\text{GeV}^2$ (lower solid curve) for the nominal choice of parameters and wave functions specified in section 4. The dash-dotted curve shows the quark model prediction given in [1] .

Figure 4 : Form factor $f_\pi^+(p^2)$ of $B \rightarrow \pi$ transitions at $M^2 = 10\text{GeV}^2$ (upper solid curve) and $M^2 = 15\text{GeV}^2$ (lower solid curve) for the nominal choice of parameters and wave functions as in Fig. 3. The quark model prediction from [1] (dash-dotted curve) and the QCD sum rule result from [4] (dashed curve) are shown for comparison. The arrow indicates the result of a QCD calculation [9] similar to ours at zero momentum transfer.

Figure 5 : Sensitivity of the form factors to parameters and wave functions:(a) f_K^+ for $\mu_K = 1.0\text{GeV}$ (solid curve) and 1.6GeV (dashed curve); (b) f_K^+ for the nonasymptotic (solid curve) and asymptotic (dashed curve) twist 2 wave function φ_K , and for nonasymptotic corrections included in the twist 3 wave functions φ_p and φ_σ (dash-dotted curve); (c) the same as (b) for f_π^+ .