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CONDITIONS ON SUPERSYMMETRY SOFT-BREAKING TERMS FROM GUTs

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Abstract

We study the effect of integrating out the heavy fields in a supersymmetric GUT which does not contain small mass parameters in the limit of exact supersymmetry. The trilinear (A) and bilinear (B) coefficients of the supersymmetry soft-breaking terms of the low-energy effective theory are related in a simple and model-independent way to those of the underlying theory. From these relations, we obtain the bound $|B| \geq 2$, which, together with the requirements of stability of the potential and electroweak symmetry breaking, imposes severe constraints on the space of allowed supersymmetric parameters. In models based on supergravity with a flat Kähler metric, we obtain $B = 2$, instead of the relation $B = A - 1$ usually used in phenomenological applications. The low-energy theory contains also a supersymmetric mass term μ for the two Higgs doublets, which is of the order of the supersymmetry-breaking scale.

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Low-energy supersymmetric theories of fundamental interactions contain, in addition to the parameters of the Standard Model (SM), a set of new unknown parameters, which can only be the subject of theoretical speculation until we are able to derive them from experimental data. These parameters appear in the supersymmetry soft-breaking terms and are directly related to the mechanism of supergravity breakdown. Their typical mass scale is expected to be $\mathcal{O}(M_W)$, if supersymmetry is to solve the naturalness problem of the SM.

Even more obscure is the origin of the μ-term, the mixing-mass of the two Higgs doublet superfields of the Minimal Supersymmetric Standard Model (MSSM). This mass term is invariant under supersymmetry and therefore it seems unrelated to the weak scale, although it is phenomenologically required to be $\mathcal{O}(M_W)$. It has been previously proposed that a term $\mu \sim \mathcal{O}(M_W)$ can be accommodated in models with an extra gauge-singlet superfield [1] or generated as a supergravity-breaking effect in theories with non-trivial Kähler metrics [2], or induced by higher-dimension operators [3].

Here we want to concentrate on models where the solution of the μ-problem lies at some Grand Unified Theory (GUT) scale. This is of particular interest after measurements at LEP have revealed the remarkable property of the MSSM that gauge couplings unify at a scale $M_X \sim 10^{16}$ GeV. We will therefore consider theories with a GUT threshold at M_X and assume that, at that scale, $\mu = 0$ in the absence of supersymmetry-breaking effects. This is dictated by a naturalness criterion, since, if non-vanishing, μ should be of the order of the only available scales, M_X or M_{Pl}. The relation $\mu = 0$ can be the result of some symmetry at the GUT level (as, for example, in the pseudo-Goldstone bosons model of ref. [4]), of the specific field content of the heavy theory (as in the "missing partner" models [5]) or of a mere fine-tuning of the parameters (as in the ordinary supersymmetric SU(5) model [6]). Whatever the reason is, very little information on the structure of the GUT theory is needed in order to derive the form of the soft-breaking terms of the effective low-energy theory, as was shown in ref. [7]. By extending the analysis of ref. [7], we connect with simple relations the supersymmetry-breaking parameters of the GUT with the low-energy ones and with the induced μ-term. From these relations, some constraints on the low-energy supersymmetry-breaking parameters can also be derived.

In the limit $M_{Pl} \to \infty$ after supergravity breaking, the theory at the GUT scale is defined by a softly broken supersymmetric Lagrangian. In particular, the potential for the complex scalar fields z is [8]:

$$V(z^*, z) = \left|\frac{\partial f}{\partial z}\right|^2 + m^2 \left|z\right|^2 + m(A_X f^{(3)} + B_X f^{(2)} + \text{h.c.}) + \frac{1}{2}\sum_k D_k^2, \tag{1}$$

where $D_k = z^\dagger T_{(k)} z$, $T_{(k)}$ are the gauge group generators, A_X, B_X, m are the soft-breaking parameters, $f^{(2)}$ and $f^{(3)}$ are the terms in the superpotential respectively bilinear and trilinear in the fields z. For simplicity, we assume that the superpotential does not contain terms linear in the fields, and thus $f = f^{(2)} + f^{(3)}$. If the underlying supergravity theory has a flat Kähler metric, then [8]:

$$B_X = A_X - 1. \tag{2}$$

For general supergravity couplings, eq. (2) does not hold, and m, A_X and B_X are no longer universal but become matrices. This is usually the case in theories derived from superstrings [9]. Motivated by the observed suppression of flavor-changing neutral currents, we will assume universal couplings in eq. (1), allowing only for $B_X \neq A_X - 1$. We will briefly comment later on the general case.

At M_X the GUT is spontaneously broken to the Standard Model. We define $\bar{z} \sim \mathcal{O}(M_X)$ to be the vacuum expectation values of the fields z in the limit of exact supersymmetry. Since supersymmetry is not spontaneously broken at the scale M_X:

$$\frac{\partial f}{\partial z}(\bar{z}) = D_k(\bar{z}) = 0. \tag{3}$$

Since we are assuming that all mass parameters entering the superpotential f are $\mathcal{O}(M_X)$, we can distinguish the fields z^a appearing in the potential in three classes. In the limit of exact supersymmetry, z^A are complex scalars with mass $\mathcal{O}(M_X)$, z^α are massless complex scalars with $\langle z \rangle = 0$ (which corresponds to the low-energy fields), and z^K are real scalars belonging to the massive vector supermultiplets of the broken generators. We now want to integrate out the heavy fields z^A and z^K in order to obtain the low-energy effective theory, following the same procedure as in ref. [7][1]. Using a series expansion in m/M_X to solve the equations of motion for the heavy fields, we obtain:

$$\begin{aligned} z^A &= \bar{z}^A + \Phi^A - (f^{-1})^{AB}\left[m\left(B_X^* + \frac{1}{B_X - A_X}\right)\Phi_B^* \right. \\ &\quad \left. + \frac{1}{2} f_{BCD}\Phi^C\Phi^D + \frac{1}{2} f_{B\alpha\beta} z^\alpha z^\beta\right] + \mathcal{O}\left(\frac{m^3}{M_X^2}\right), \end{aligned} \tag{4}$$

$$z^K = -\frac{1}{2}(\mathcal{M}^{-1})^{KL} D_L(\Phi^A, z^\alpha) + \mathcal{O}\left(\frac{m^3}{M_X^2}\right), \tag{5}$$

$$\Phi^A \equiv m(B_X - A_X)^*(f^{-1})^{AB}\bar{z}_B^*, \qquad \mathcal{M}_{KL} \equiv \bar{z}^\dagger\{T_{(K)}, T_{(L)}\}\bar{z}, \tag{6}$$

where $f_{AB} = \frac{\partial^2 f}{\partial z^A \partial z^B}|_{z=\bar{z}}$, $f_{abc} = \frac{\partial^3 f}{\partial z^a \partial z^b \partial z^c}|_{z=\bar{z}}$, etc.

Plugging these expressions back in eq. (1), we obtain the potential of the low-energy effective theory

$$V_{eff} = \left|\frac{\partial f_{eff}}{\partial z^\alpha}\right|^2 + m^2|z_\alpha|^2 + m(A f_{eff}^{(3)} + B f_{eff}^{(2)} + \text{h.c.}) + \frac{1}{2}\sum_{k'} D_{k'}^2, \tag{7}$$

where the index k' runs over the unbroken generators and

$$f_{eff}^{(3)} = \frac{1}{6} f_{\alpha\beta\gamma} z^\alpha z^\beta z^\gamma, \quad f_{eff}^{(2)} = \mu_{\alpha\beta} z^\alpha z^\beta, \quad f_{eff} = f_{eff}^{(3)} + f_{eff}^{(2)}, \tag{8}$$

$$A = A_X, \tag{9}$$

$$B = A_X - B_X + \frac{1}{(A_X - B_X)^*}, \tag{10}$$

$$\mu_{\alpha\beta} = m(A_X - B_X)^* C_{\alpha\beta}, \quad C_{\alpha\beta} \equiv -\frac{1}{2} f_{\alpha\beta A}(f^{-1})^{AB}\bar{z}_B^*. \tag{11}$$

[1]We have tacitly assumed $M_X \ll M_{Pl}$. This is just to simplify the calculation, but our results are valid also for $M_X \sim M_{Pl}$, since, as shown in ref. [7], the renormalizable interactions of the low-energy effective theory do not contain any power of M_X/M_{Pl}.

Therefore, the potential of the low-energy theory, eq. (7), has the same form as eq. (1), with the soft-breaking parameters related by eqs. (9) and (10) and an induced μ-term as in eq. (11). Notice that the case of a flat Kähler metric, eq. (2), corresponds to

$$B = 2. \tag{12}$$

For generic A_X and B_X we obtain from eq. (10):

$$|B| \geq 2. \tag{13}$$

The constraints in eqs. (12) and (13) apply to the low-energy B parameter and are relevant to phenomenological applications.

All the dependence on the details of the GUT model is contained in the parameter $C_{\alpha\beta}$ in eq. (11). There is a class of theories in which $C_{\alpha\beta}$ turns out to be independent of the details of the model: the supersymmetric GUT with Higgs as pseudo-Goldstone bosons [4].

In these models, the Higgs sector is globally invariant under a group G_{gl} larger than the GUT gauge group G_{loc}. At M_X, G_{loc} is broken to H_{loc} (the SM gauge group) and G_{gl} to H_{gl}. The Goldstone bosons corresponding to the broken generators of G_{gl}/H_{gl}, not contained in G_{loc}/H_{loc} and therefore not eaten by the Higgs mechanism, are physical particles belonging to massless chiral supersymmetric multiplets. After inclusion of supersymmetry soft-breaking terms, these particles, interpreted as the low-energy Higgs bosons, acquire masses $\mathcal{O}(m)$. The simplest model of this kind is based on G_{loc} =SU(5) and G_{loc} =SU(6) [4], but models based on G_{loc} =SO(10) [10] or larger gauge groups with an automatic G_{gl} invariance [11] have also been proposed. We want now to compute $C_{\alpha\beta}$ in this class of models.

The invariance of the superpotential under G_{gl} implies:

$$\frac{\partial f}{\partial z^a} T^a_{(i)b} z^b = 0, \tag{14}$$

where $T^{(i)}$ are the generators of G_{gl}. By differentiating eq. (14) at $z = \bar{z}$, we get

$$f_{ac} T^a_{(i)b} \bar{z}^b + f_a T^a_{(i)c} = 0. \tag{15}$$

Since supersymmetry is not spontaneously broken at the scale M_X ($f_a = 0$), any non-vanishing combination $T^a_{(i)b}\bar{z}^b$ corresponds to a massless (in the limit of exact supersymmetry) mode. We can now construct the orthonormal Goldstone states G^α as follows:

$$G^\alpha = U^{*\alpha}_a (z^a - \bar{z}^a), \tag{16}$$

$$U^a_\alpha \equiv N_{\alpha i} T^a_{(i)b} \bar{z}^b, \qquad N_{\alpha i} \equiv \frac{V_{\alpha i}}{\sqrt{\pi_\alpha}}, \tag{17}$$

where π_α and $V_{\alpha i}$ are respectively the non-zero eigenvalues and associated orthonormal eigenvectors of the matrix $\Pi_{ij} \equiv \bar{z}^*_a T^a_{(i)b} T^b_{(j)c} \bar{z}^c$. These definitions ensure that

$$U^{*\alpha}_a U^a_\beta = \delta^\alpha_\beta. \tag{18}$$

By differentiating eq. (14) with respect to z^c and z^d at $z = \bar{z}$, and then multiplying the result by $U^c_\alpha N_{\beta i}(f^{-1})^{DE}\bar{z}^*_E$, we obtain from the definition of $C_{\alpha\beta}$, eq. (11):

$$C_{\alpha\beta} = \frac{1}{2}\delta_{\alpha\beta}, \tag{19}$$

a result which is independent of the details of the GUT model.

In the case of the MSSM, $f^{(2)}_{eff}$ consists of only one term, the two Higgs doublet mixing mass $\mu H_1 H_2$, and therefore there is only one C parameter. With the definition

$$H_1 = \frac{1}{\sqrt{2}}(z_1 + iz_2), \quad H_2 = \frac{1}{\sqrt{2}}(z_1 - iz_2), \tag{20}$$

we obtain, from eq. (19), $C = 1$ and from eq. (11):

$$\mu = m(A_X - B_X)^*. \tag{21}$$

This coincides with the relation $\mu = m$ obtained in ref. [4] in the case of a flat Kähler metric.

If the low-energy Higgs doublets are not Goldstone bosons of some global symmetry spontaneously broken at M_X, the parameter C will generally depend on unknown couplings of the GUT. As an example consider the simplest GUT, SU(5), with Higgs superpotential

$$f = \frac{M_\Sigma}{2}\mathrm{Tr}\Sigma^2 + \frac{M_H}{2}H_1H_2 + \frac{\lambda}{3}\mathrm{Tr}\Sigma^3 + \frac{\alpha}{3}H_1\Sigma H_2, \tag{22}$$

where H_1 ($\bar{\mathbf{5}}$) and H_2 ($\mathbf{5}$) contain the low-energy Higgs doublets and Σ ($\mathbf{24}$) spontaneously breaks SU(5) into the SM:

$$\langle\Sigma\rangle = \frac{M_\Sigma}{\lambda}\begin{pmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{pmatrix}. \tag{23}$$

The condition that $\mu = 0$ at M_X, in the absence of supersymmetry breaking, is achieved by a fine-tuning of the parameters:

$$\frac{2\alpha}{\lambda} = \frac{M_H}{M_\Sigma}. \tag{24}$$

The parameter C can be directly computed from its definition in eq. (11) and is equal to:

$$C = -\frac{\alpha}{2\lambda} = -\frac{M_H}{4M_\Sigma}. \tag{25}$$

In the context of this minimal SU(5) model, measurements of the low-energy parameters can give information on the coupling constants of the GUT.

We want to stress that the relations (9)–(11) follow only from our assumption that the low-energy fields, in the limit of exact supersymmetry, are exactly massless at the scale M_X. However if the GUT couplings do not induce a μ-term, in other words if $C = 0$, then eqs. (10) and (11) do not contain any information. This is the case, for instance, of the GUT models with "missing partners" [5], where no single term of the superpotential contains both H_1 and H_2, and therefore $C = 0$ at tree level. It is possible that loop corrections induce a $C \neq 0$ or that the origin of the μ-term in these models is completely unrelated to the GUT scale.

If the soft-breaking masses in the second term of eq. (1) are not universal, a new model-dependent coefficient enters in eq. (10), generally invalidating the constraint of eq. (13). However if deviations from universality are small, as seems to be required by the strong observed suppression of flavor-changing neutral current processes, our conclusion should not be drastically modified. A non-universality of the A-term is important only if $H_1 H_2$ is coupled to superheavy fields in more than one term.

In the case of the MSSM, we can derive further constraints from eqs. (9)–(11). Since the D-terms for the neutral components of the Higgs doublets vanish in the direction $|H_1| = |H_2|$, the stability of the potential, eq. (7), requires:

$$m^2 + |\mu|^2 \geq |Bm\mu|. \tag{26}$$

From eqs. (9)–(11), this implies

$$\begin{array}{ll} \left|\frac{\mu}{m}\right| \leq \sqrt{|C|} \leq \frac{1}{|A_X - B_X|} & \text{if } \ |C| < 1 \\ \frac{1}{|A_X - B_X|} \leq \sqrt{|C|} \leq \left|\frac{\mu}{m}\right| & \text{if } \ |C| > 1 \end{array} \tag{27}$$

and any value of $A_X - B_X$ and μ/m is allowed for $|C| = 1$.

One of the most attractive aspects of the MSSM is that the renormalization of the parameters of the theory from M_X to low energies induces the breaking of the electroweak symmetry. We can therefore investigate the constraints imposed by SU(2)×U(1) breaking with the additional relations among the parameters that we have found here. For a fixed value of the top-quark Yukawa coupling constant, we run the Renormalization Group Equations (RGEs) from M_X to the low-energy scale and study the region of the soft-breaking parameters where: *i*) the potential is bounded from below at any energy between the weak scale and M_X; *ii*) electroweak symmetry is spontaneously broken; *iii*) the mass spectrum of the supersymmetric particles satisfies the present experimental bounds. We have also corrected the potential with the dominant one-loop contribution coming from the top–stop sector, and we have verified the stability of our results under variations of the scale where we stop the running of the RGE. Figure 1 shows the allowed regions for the parameters A and μ/m for three choices of B: $B = 2$ (flat Kähler metric), $B = 5$, and $B = (1 + |\mu/m|^2)/(\mu/m)$ (Higgs as pseudo-Goldstone bosons). Figure 2 shows the allowed regions for the parameters B and μ/m for $A = 0$ and $A = 5$. The constraints of eq. (13) and eq. (26) are also shown in fig. 2; notice that they respectively correspond to the cases of a flat Kähler metric and of Higgs as pseudo-Goldstone bosons. In figs. 1a and 2a the top-quark Yukawa coupling constant is chosen such that $m_t = \sin\beta \cdot 140$ GeV and in figs. 1b and 2b $m_t = \sin\beta \cdot 180$ GeV, where $\tan\beta = v_2/v_1$, the ratio of the two vacuum expectation values; in the MSSM, sinβ is forced to satisfy $1/\sqrt{2} < \sin\beta < 1$. These figures show the strong existing constraints, especially for $m_t = \sin\beta \cdot 180$ GeV, where the top-quark Yukawa is close to its infrared fixed-point.

In conclusion, we have shown that in theories where $\mu = 0$ at M_X, in the limit of exact supersymmetry, the GUT couplings together with the supersymmetry-breaking effects can, and generally will, induce a non-vanishing μ-term, which turns out to be $\mathcal{O}(M_W)$. The supersymmetry-breaking parameters of the low-energy effective theory are simply related to those of the GUT by eqs. (9)–(11). We obtain the model-independent bound $|B| \geq 2$, and $B = 2$ in the case of flat Kähler metrics. All the dependence of the GUT model in the low-energy theory is contained in a single parameter C, which can be experimentally measured, if supersymmetry is discovered. In the class of models where the Higgs are pseudo-Goldstone bosons, $C = 1$, but in general C depends on coupling constants of the GUT. The stability of the potential and the electroweak symmetry breaking can be used to further constrain the soft-breaking parameters.

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Figure captions

Fig. 1. The regions of parameter space A–μ/m allowed by the consistency conditions *i)–iii)* described in the text, for $B = 2$ (flat Kähler metric), $B = 5$, and $|B| = |\mu/m| + |m/\mu|$ (PGB – hypothesis of Higgs as pseudo-Goldstone bosons). The top-quark Yukawa coupling constant is chosen such that $m_t = \sin\beta \cdot 140$ GeV (1a) and $m_t = \sin\beta \cdot 180$ GeV (1b).

Fig. 2. The regions of parameter space B–μ/m allowed by the consistency conditions *i)–iii)* described in the text, for $A = 0$ and $A = 5$. The constraints $|B| \geq 2$ of eq. (13) and $|B| \leq |m/\mu| + |\mu/m|$ of eq. (26) are also shown. The top-quark Yukawa coupling constant is chosen such that $m_t = \sin\beta \cdot 140$ GeV (2a) and $m_t = \sin\beta \cdot 180$ GeV (2b).