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MEASUREMENT OF THE STRONG COUPLING CONSTANT USING τ DECAYS

The ALEPH Collaboration

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ABSTRACT

The strong coupling constant is determined from the leptonic branching ratios, the lifetime, and the invariant mass distribution of the hadronic final state of the τ lepton, using data accumulated at LEP with the ALEPH detector. The strong coupling constant measurement, $\alpha_s(m_\tau^2) = 0.330 \pm 0.046$, evolved to the Z mass, yields $\alpha_s(M_Z^2) = 0.118 \pm 0.005$. The error includes experimental and theoretical uncertainties, the latter evaluated in the framework of the Shifman, Vainshtein and Zakharov (SVZ) approach. The method allows the non-perturbative contribution to the hadronic decay rate to be determined to be $0.3 \pm 0.5\%$.

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1. Introduction

The R-ratio, defined in e^+e^- annihilation to be the ratio of the total hadronic cross section, $\sigma(e^+e^- \to \text{hadrons})$, to the leptonic cross section, $\sigma(e^+e^- \to \mu^+\mu^-)$, can be generalized to τ decay which involves W exchange rather than Z/γ exchange [1–2]. One defines the corresponding R_τ-ratio by

$$R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau \text{hadrons})}{\Gamma(\tau^- \to \nu_\tau l^- \bar{\nu}_l)} = \frac{\Gamma_h}{\Gamma_l}, \tag{1.1}$$

where the l lepton is taken to be massless.

The measurement of R_τ, or more precisely, the measurement of its sizeable deviation ($\sim 20\%$) from the parton level prediction $R_\tau = 3$, provides a means to probe the strong interaction and determine the strong coupling constant at low Q^2 [2]. A low Q^2 determination demonstrates the running of α_s by comparison with its value at high Q^2. The α_s determination from τ decays not only achieves this goal, but, once extrapolated to the Z mass, yields one of the most precise determinations of the strong coupling constant, since the α_s uncertainty scales with α_s^2.

The R_τ-ratio possesses the same advantages which make the more standard R-ratio so attractive, namely, it is an inclusive quantity and the QCD perturbative expansion is known up to the third order in α_s [3–4]. However, the validity of the QCD prediction appears questionable as at the mass scale characterizing R_τ, the tau mass, the strong coupling constant is large, roughly three times greater than the one which applies on the Z pole. As a result, one may expect a slowly converging or even diverging perturbative expansion and, in addition, as τ hadronic decays involve resonances, one may expect large non-perturbative contributions to cause severe problems for the α_s measurement.

However, it has been shown that although the α_s expansion becomes non-convergent for $\alpha_s(m_\tau^2) > 0.35$, this problem can be overcome since the ill-behaved part of the series can be identified and resummed to all orders [5–6]. In the case of non-perturbative effects, it has been shown [1–2] that, within the framework of the Operator Product Expansion applied in a context where non-perturbative effects are present [7] (hereafter refered to as the SVZ approach), the non-perturbative contributions are strongly suppressed due to the integration over the whole invariant mass spectrum of the hadronic final state which is involved in the calculation of R_τ. It must be emphasized that the reliability of the QCD prediction which is used in the present analysis (and hence, the reliability of the α_s determination from τ decays) depends on the applicability of the SVZ approach, which has not been established formally, yet.

This letter employs a method, proposed in ref. [8] , which makes use of the invariant mass-squared distribution of the hadronic final state in τ decays (hereafter denoted the s distribution) to determine $\alpha_s(m_\tau^2)$ without using previously estimated non-perturbative terms.

2. Overview of the method

The theoretical prediction for R_τ is obtained through an integration over the invariant mass-squared of the hadronic final state [1–2]

$$R_\tau = \frac{1}{\Gamma_l} \int_0^{m_\tau^2} ds \ \frac{d\Gamma_h}{ds} \quad , \tag{2.1}$$

where $d\Gamma_h/ds$ is a function which is not presently within the reach of the theory, but whose behaviour is nevertheless sufficiently well understood to allow a QCD prediction for the integral. This QCD prediction involves a perturbative part and a non-perturbative part; the latter relies on numerical estimates of a set of constants (the so-called condensates). The validity of the QCD prediction, if it holds for R_τ, must hold equally for quantities such as [8]

$$R_\tau^{kl} = \frac{1}{\Gamma_l} \int_0^{m_\tau^2} ds \ \left(1 - \frac{s}{m_\tau^2}\right)^k \ \left(\frac{s}{m_\tau^2}\right)^l \ \frac{d\Gamma_h}{ds} \quad , \tag{2.2}$$

provided that these moments do not probe the fine detail of the s distribution, i.e. provided both k and l remain small enough. The R_τ-ratio is just the special case, $R_\tau = R_\tau^{00}$. Due to the high correlations among these moments, the analysis is restricted to a small number of them; following ref. [8], the four moments R_τ^{10}, R_τ^{11}, R_τ^{12} and R_τ^{13} will be used in conjunction with R_τ. These generalized R_τ ratios provide a means to check experimentally the validity of the theoretical arguments invoked for the applicability of the QCD prediction at the τ mass scale. Earlier analyses which extract the non-perturbative constants from hadronic τ decays can be found in ref. [9].

Using the available values of the non-perturbative constants one can obtain two independent determinations of α_s.

• The first, from R_τ, uses the overall normalization of the s distribution which receives small corrections from the non-perturbative constants [1–2].

• The second, from the moments, is based on the shape of the s distribution and is more dependent upon the precise values of the non-perturbative constants.

Agreement between the two determinations supports the adequacy of the available evaluation of the non-perturbative constants. Combining R_τ and the moments into a global fit, one can determine simultaneously the strong coupling constant and the relevant non-perturbative constants; this latter approach will be the one adopted in this paper.

3. Data analysis

The analysis uses data collected at LEP by the ALEPH detector. A detailed description of the detector can be found in [10]. The event selection, the charged particle identification and the neutral particle reconstruction are the ones described in [11]. The present analysis makes use of the 8429 τ decay candidates used in the quasi-exclusive branching ratio determination of ref. [11] . This corresponds to the luminosity of 8 pb^{-1} collected in

1989-90. In this previous ALEPH analysis, each selected τ decay was classified either into one of the two leptonic classes e or μ or into one of six hadronic classes corresponding to the π, $\pi\pi^0$, $\pi 2\pi^0$, $\pi 3\pi^0$, 3π and $3\pi\pi^0$ final states. This classification was made on the basis of the number of charged particles, their type, the number of photons and the number of, π^0's reconstructed from photon pairs (no $\pi - K$ separation was attempted). The present analysis also uses the τ lifetime measurement of ref. [12] which was obtained with an additional luminosity of 10 pb^{-1} collected in 1991.

3.1. The R_τ *measurement*

The R_τ-ratio can be expressed as follows:

$$\text{R}_\tau = \frac{\Gamma_h}{\Gamma_l} = \frac{1 - B_e - B_\mu}{B_l} = \frac{1}{B_l} - f_e - f_\mu \quad , \tag{3.1}$$

where $f_e = 1$ and $f_\mu = 0.9726$ are phase space factors accounting for the non-zero mass ratios m_e/m_τ and m_μ/m_τ. Eq. (3.1) uses the validity of the completeness relation $B_{\text{total}} \equiv B_{\text{hadrons}} + B_e + B_\mu = 1$. Experimentally B_l can be obtained from the electronic and muonic branching ratios, B_e and B_μ, or from the τ lifetime, τ_τ. The leptonic branching ratio estimates of B_l are taken from ref. [11]: $B_l = B_e/f_e = 0.1809 \pm 0.0064$ and $B_l = B_\mu/f_\mu = 0.1784 \pm 0.0057$. The τ lifetime measurement $\tau_\tau = 294.7 \pm 6.2\ fs$ [12] yields $B_l = (\tau_\tau/\tau_\mu)(m_\tau/m_\mu)^5 = 0.1806 \pm 0.0038$ where the new world average of the τ mass [13–14] $m_\tau = 1.7771 \pm 0.0005$ GeV has been used. As the three estimates are in very good agreement they are combined to give $B_l = 0.1801 \pm 0.0028$, from which one obtains $\text{R}_\tau = 3.579 \pm 0.087$, with the quoted error including both statistical and systematic uncertainties.

3.2. The moment measurements

Because the B_{hadrons} branching ratio contributes implicitly in the previous determination of R_τ through the completeness relation, the overall normalization of the s distribution does not provide additional information, i.e., the s distribution must enter the moments analysis only through its shape. Kaons are not explicitly identified in the branching ratio analysis & hence the $\bar{u}d$- and $\bar{u}s$-quark final states cannot be dealt with separately. However, the smallness of the contribution of the Cabibbo suppressed $\bar{u}s$ final states allows for the decays involving kaons to be treated as a background to be subtracted from the data. In practice the analysis uses normalized moments, D_τ^{kl}, defined by

$$D_\tau^{kl} = \int_0^{m_\tau^2} ds \left(1 - \frac{s}{m_\tau^2}\right)^k \left(\frac{s}{m_\tau^2}\right)^l \frac{1}{\Gamma_{\bar{u}d}} \frac{d\Gamma_{\bar{u}d}}{ds} = \frac{\text{R}_\tau^{kl}(\bar{u}d)}{\text{R}_\tau^{00}(\bar{u}d)} \quad , \tag{3.2}$$

where $\Gamma_{\bar{u}d} = \Gamma(\tau \to \bar{u}d\nu_\tau)$ is the Cabibbo allowed decay width of the tau and $\text{R}_\tau^{kl}(\bar{u}d)$ are the Cabibbo allowed components of the moments. These normalized moments are determined from the corrected experimental s distribution. The bin width is chosen to be $\Delta s = 0.05\ (\text{GeV}/c^2)^2$ which is sufficiently small compared to the rather coarse structure of the distribution which is to be extracted. The raw s distribution is corrected for detector

Table 1
Number of events (N_{event}), branching ratios (B) and estimated τ background fractions (f_B) in the six classes of events.

	π	$\pi\pi^0$	$\pi 2\pi^0$	$\pi 3\pi^0$	3π	$3\pi\pi^0$
N_{event}	1092	1855	809	186	760	529
B(%)	11.9 ± 0.5	24.6 ± 1.0	9.85 ± 0.73	1.53 ± 0.45	8.93 ± 0.53	4.95 ± 0.50
f_B(%)	20.2	13.0	34.0	66.5	8.5	26.4

effects using a three-step procedure; it is first corrected for background contributions, then the detector resolution effects are unfolded and finally the slight s-dependence of the selection efficiency is accounted for. In order to perform these three operations the hadronic τ decays are resolved into the six classes of the quasi-exclusive branching ratio analysis quoted above as the detector effects are different in the various classes.

A Monte Carlo simulation is necessary for subtraction of the Cabbibo suppressed channels, the cross-channel contaminations, the migration of events across the s distribution and the efficiency correction. This is based on the event generator KORALZ [15] and the library of tau decays TAUOLA [16]. The invariant mass distribution in the 3π channel is corrected to match the data. The branching ratios that are not directly measured in ref. [11] are given the Particle Data Group values [13]. The generated events are then processed through a full simulation of the detector and through the ALEPH analysis chain. Several tests of the validity of the Monte Carlo simulation can be found in ref. [11].

The available number of events and the branching ratio values in the six classes are given in table 1. The quoted uncertainties account for the statistical errors and part of the systematic errors. The sources of systematics which may affect not only the branching ratio determinations but also the shape of the s distribution are dealt with separately below. The π class enters in the analysis only through its corresponding branching ratio since, neglecting radiative corrections, its contribution to the s distribution is just a delta function located at m_π^2. The three correction steps are described in turn below.

The overall non-τ background to the hadronic τ decays is negligible. The τ backgrounds, obtained from [11], are detailed in table 1. The raw s distribution, after τ background subtraction, is shown in fig. 1(a). To correct for the distortions of the s distribution which are induced by the various detector resolutions a probability matrix $P^c_{n\to m}$ is derived from the Monte Carlo simulation for each class of decay. This matrix provides the probability for a decay occuring in a given class c, produced with a hadronic invariant mass-squared in bin n and selected in the same class c to appear in bin m of the raw s distribution. The $P^c_{n\to m}$ matrix is then used to estimate, by iteration, the true distribution from the raw distribution. The last step in the correction procedure is to account for the slight dependence on s of the selection efficiency in order to define for each class of decay an efficiency corrected distribution which is normalized to unity. The fully corrected and normalized s distribution is then obtained from a weighted average of these distributions using the branching ratios of table 1. The resulting s distribution is shown in fig. 1(b). The D_τ^{kl} values obtained from the corrected distribution and their statistical errors are

Table 2
The D_τ^{kl} moments and their purely statistical errors σ[stat]. Also given are the total errors σ[exp] accounting for the statistical and systematic uncertainties (see text). The moment values obtained from the distribution of fig. 1(a) are quoted in the last row.

kl	10	11	12	13
D_τ^{kl}	0.7312	0.1536	0.0552	0.0244
σ[stat]	0.0054	0.0019	0.0012	0.0008
σ[exp]	0.0075	0.0022	0.0018	0.0012
D_τ^{kl}(raw)	0.7332	0.1486	0.0526	0.0230

given in table 2. For illustration, the D_τ^{kl}(raw) values obtained from the s distribution of fig. 1(a) are also quoted.

Other sources of systematics may induce changes to the shape of the invariant mass distributions and, in some cases, the branching ratio measurements. Both effects are taken into account. The sources which were considered are

(a) the limited Monte Carlo statistics used to correct the data,

(b) the calibration of the electromagnetic calorimeter, the photon reconstruction and the π^0 definition,

(c) the three charged pion reconstruction,

(d) the background subtraction,

(e) the unfolding procedure.

These sources of systematics contribute in a comparable way to the overall uncertainties. For example the systematic error on the first moment $D_\tau^{10} = 1 - \langle s/m_\tau^2 \rangle$ results from $0.0021(a) \oplus 0.0041(b) \oplus 0.0013(c) \oplus 0.0015(d) \oplus 0.0013(e) = 0.0052$ where the parantheses refer to the above sources. The total experimental errors, σ[exp], obtained by adding in quadrature the statistical and systematic uncertainties, are given in table 2.

4. The theoretical prediction

The theoretical prediction for the R_τ^{kl} moments takes the form

$$\mathrm{R}_\tau^{kl} \cong r(k,l)\,[1 + \delta_{\mathrm{pert}}(k,l) + \delta_{\mathrm{SVZ}}(k,l)] \quad , \tag{4.1}$$

where only the important terms have been included. The full expression is given in the Appendix. In eqn. (4.1):

• $r(k,l)$ are simple constants corresponding to the parton level predictions, their values are given in table 3.

• $\delta_{\mathrm{pert}}(k,l)$ correspond to the massless perturbative QCD predictions. Their values for $\alpha_s = 0.33$ are given in table 3. They are evaluated in terms of functions which are exactly known to the third order in α_s and incorporate a resummation of the leading terms of the series to all orders.

Table 3

Parton level predictions $r(k,l)$, perturbative QCD corrections $\delta_{\mathrm{pert}}(k,l)$ ($\alpha_s = 0.33$) and expected non-perturbative contributions $\delta_{\mathrm{SVZ}}(k,l)$ (cf. Appendix) for the R^{kl} moments.

k,l	0,0	1,0	1,1	1,2	1,3
$r(k,l)$	3	21/10	1/2	13/70	3/35
$\delta_{\mathrm{pert}}(k,l)$	0.20	0.18	0.08	0.08	0.08
$\delta_{\mathrm{SVZ}}(k,l)$	-0.01 ± 0.01	$0. \quad \pm 0.03$	-0.07 ± 0.09	0.03 ± 0.06	$0. \quad \pm 0.09$

Table 4

The theoretical uncertainties on R_τ and the D_τ^{kl} moments. The first row indicates the values Σ[th] one obtains when taking into account the uncertainties on the non-perturbative constants. The second row corresponds to the values σ[th] to be used in a global fit where the non-perturbative constants are let free to vary.

	R_τ	D_τ^{10}	D_τ^{11}	D_τ^{12}	D_τ^{13}
Σ[th]	0.043	0.0134	0.0133	0.0032	0.0019
σ[th]	0.033	0.0035	0.0028	0.0004	0.0002

- $\delta_{\mathrm{SVZ}}(k,l)$ are non-perturbative correction terms involving a set of constants determined from experiment; the values are taken from ref. [2]. Their total effect on R_τ and the moments are given in table 3. The leading contributions to $\delta_{\mathrm{SVZ}}(k,l)$ arise from three constants, termed $\langle\frac{\alpha_s}{\pi}GG\rangle$, $O(6)$ and $O(8)$ in the following (cf. Appendix).

The aim of the analysis is to perform a global fit for α_s and the three parameters describing $\delta_{\mathrm{SVZ}}(k,l)$ in order to reduce the theoretical uncertainties on the strong coupling constant determination.

The sources of theoretical systematics which have been considered and the definitions used to estimate their sizes are detailed in Appendix. When the non-perturbative constants are not allowed to vary in the fit but are taken from ref. [2], the resulting theoretical uncertainties are denoted Σ[th]. When these parameters are left free to vary, the much smaller theoretical uncertainties, denoted σ[th], are dominated by the missing higher order terms of the perturbative expansion. The two sets of theoretical systematics are given in table 4.

5. Fit results

Two fits are performed which use as overall covariance matrices the sum of the experimental and theoretical ones. Since R_τ is obtained from the leptonic branching ratios while the moments are obtained from the shape of the invariant mass-squared distribution of the hadronic final state, their experimental correlations are negligible. However, the moments are correlated amongst themselves.

Table 5
The overall uncertainties on R_τ and the D_τ^{kl} moments (diagonal elements) and the correlation coefficients (non-diagonal elements).

	R_τ	D_τ^{10}	D_τ^{11}	D_τ^{12}	D_τ^{13}
R_τ	0.09	-	-	-	-
D_τ^{10}	0.12	0.008	-	-	-
D_τ^{11}	−0.22	−0.76	0.004	-	-
D_τ^{12}	−0.04	−0.91	0.59	0.002	-
D_τ^{13}	−0.05	−0.93	0.55	0.97	0.001

5.1. Independent determinations of α_s

Using Σ[th] and the corresponding correlation coefficients for the theoretical uncertainties, R_τ and the D_τ^{kl} moments are used in conjunction with the estimates of the non-perturbative terms of ref. [2] to obtain two determinations of the strong coupling constant. The two resulting values are

$$R_\tau \;:\; \alpha_s(m_\tau^2) = 0.339 \pm 0.045\,, \tag{5.1a}$$
$$D_\tau^{kl} \;:\; \alpha_s(m_\tau^2) = 0.386 \pm 0.055\,. \tag{5.1b}$$

The fit of the second determination has a χ^2 of 4.5 for 3 degrees of freedom. The agreement observed between these two determinations of α_s provides a self-consistency check of the QCD prediction and, in particular, of the evaluation of non-perturbative effects.

5.2. Simultaneous determination of α_s and the non-perturbative constants

As stated in the introduction, the use of the moments of the s distribution combined with the measurement of R_τ allows a self contained analysis which does not rely on previous evaluations of the non-perturbative constants. The overall uncertainties obtained by adding in quadrature the experimental (σ[exp]) and theoretical (σ[th]) errors are given in table 5, together with the correlation coefficients. The resulting covariance matrix is dominated by experimental uncertainties, except for the correlations between R_τ and the moments which stem from theoretical uncertainties. Fitting simultaneously for α_s, and the $\langle \frac{\alpha_s}{\pi} GG \rangle$, $O(6)$ and $O(8)$ coefficients, one obtains,

$$\alpha_s(m_\tau^2) = 0.330 \pm 0.046\,, \tag{5.2}$$

and, for the non-perturbative constants, $\langle \frac{\alpha_s}{\pi} GG \rangle = 0.02 \pm 0.02$ (GeV/c^2)^4, $O(6) = -0.003 \pm 0.002$ (GeV/c^2)^6 and $O(8) = 0.003 \pm 0.002$ (GeV/c^2)^8, in agreement with the estimates of ref. [2] (cf. Appendix). The fit has a χ^2 of 1.7 for one degree of freedom. The contribution of the D_τ^{kl} moments to the α_s measurement can be appreciated from fig. 2 where the dependence of R_τ, D_τ^{10} and D_τ^{11} on α_s is shown.

Table 6
Errors (diagonal elements) and correlation coefficients (non-diagonal elements) obtained with the four-parameter fit for α_s, $\langle\frac{\alpha_s}{\pi}GG\rangle$, $O(6)$ and $O(8)$.

	α_s	$\langle\frac{\alpha_s}{\pi}GG\rangle$	$O(6)$	$O(8)$
α_s	0.046	-	-	-
$\langle\frac{\alpha_s}{\pi}GG\rangle$	−0.68	0.02	-	-
$O(6)$	0.48	−0.89	0.002	-
$O(8)$	−0.43	0.87	−0.98	0.002

The correlation coefficients between the four parameters are given in table 6. The limited number of observables and the high correlations between the D_τ^{kl} moments explain the large correlations observed between the determinations of the non-perturbative constants, however their overall contribution to R_τ is well defined. Taking into account these correlations, one obtains

$$\delta_{\mathrm{SVZ}}(0,0) = 0.003 \pm 0.005\,, \tag{5.3}$$

which is consistent with the previous estimate (cf. table 3). The same correlations forbid a complete self-consistency check of the theory. It is not possible to add another free parameter such as an additional $O(2)$ term in the fit. Such a term is absent in the SVZ approach but is not ruled out and is still controversial [17–19]. An $O(2)$ term would contribute directly to R_τ and more generally to the R_τ^{k0} moments. If one allows for such a term, the α_s determination then comes essentially from the three $R_\tau^{1,l\neq 0}$ moments and is much less precise.

In order to compare with high-Q^2 determinations of the strong coupling constant, the above α_s value is extrapolated to the Z mass using the formulae of ref. [20]. One obtains

$$\alpha_s(M_Z^2) = 0.118 \pm 0.005 \quad , \tag{5.4}$$

where the error includes an uncertainty of ± 0.001 due to the crossings of the c and b quark thresholds which affect the α_s evolution from the τ mass to the Z mass. This result is in good agreement with the most recent ALEPH measurement [21] $\alpha_s(M_Z^2) = 0.125 \pm 0.005$.

6. Conclusions

A determination of $\alpha_s(m_\tau^2)$ has been performed using the leptonic and hadronic τ decays measured by ALEPH. Within the framework provided by the SVZ approach the strong coupling constant measurement $\alpha_s(m_\tau^2) = 0.330 \pm 0.046$, evolved to the Z mass, yields $\alpha_s(M_Z^2) = 0.118 \pm 0.005$, where the error is dominated by experimental uncertainties. The non-perturbative contribution to R_τ is determined to be 0.3 ± 0.5 % .

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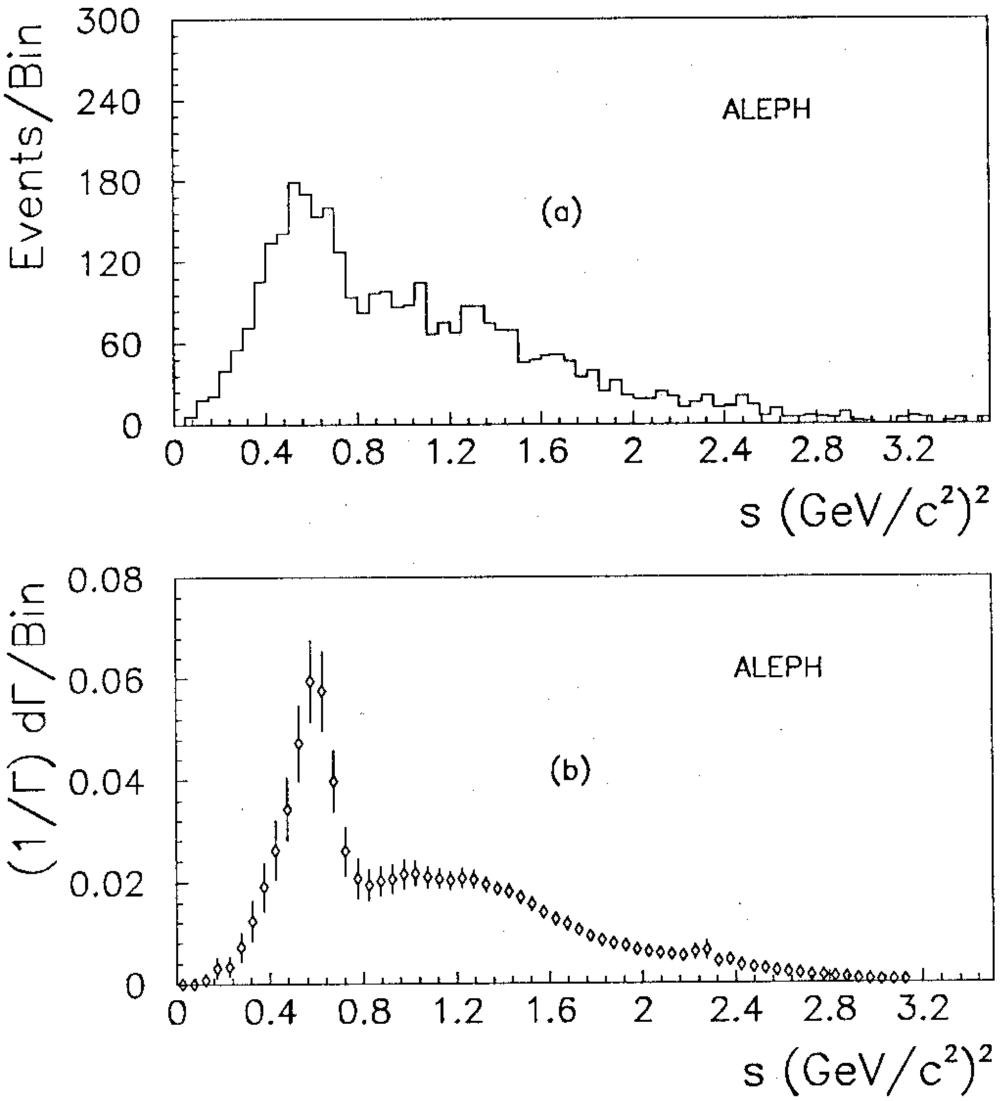

Figure 1 : (a) The raw mass-squared distribution of hadronic final sates, s, after τ background subtraction and (b) the fully corrected s distribution, normalized to unity. The bin width is 0.05 $(\mathrm{GeV}/c^2)^2$. The indicated errors account for the statistical uncertainties and most of the systematics (but not for the large point to point correlations). The delta functions corresponding to the π contribution are not shown.

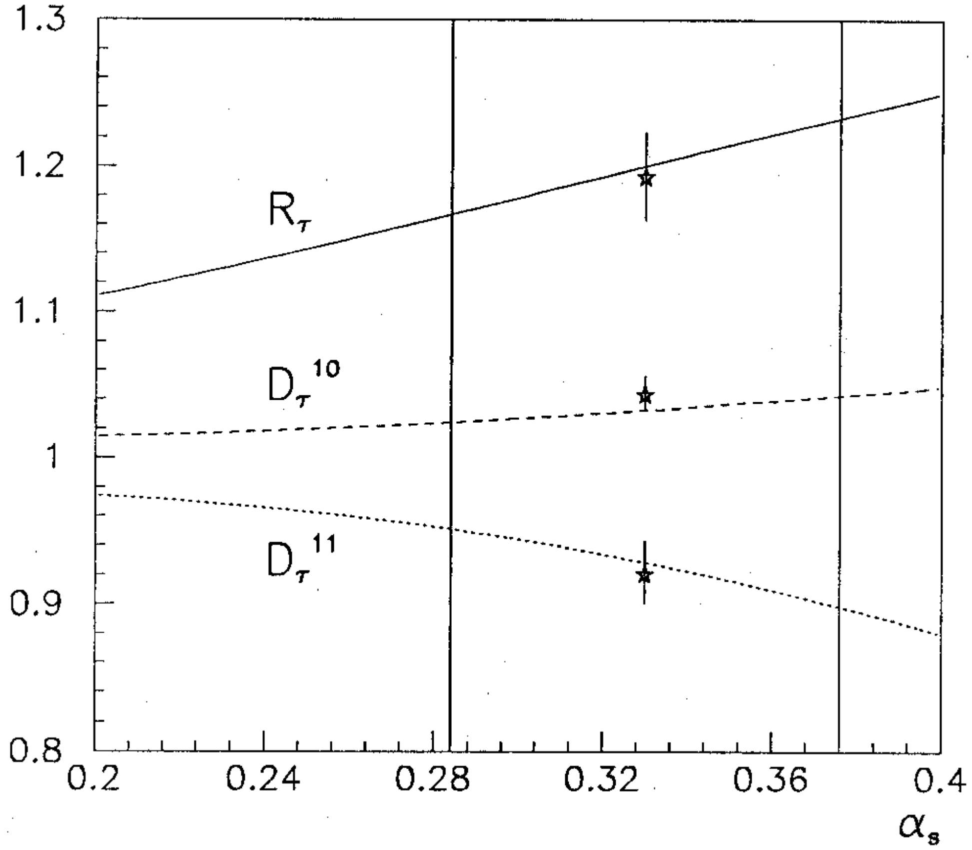

Figure 2 : The QCD predictions as a function of α_s for R_τ, D_τ^{10} and D_τ^{11} expressed in unit of their parton level predictions, 3, 7/10 and 1/6 respectively. The non-perturbative constants used in the computation are the fitted values. The data points represent the three measurements expressed in the same units. The error bars are obtained from table 5. The vertical lines delimit the one standard deviation interval corresponding to the four-parameter fit determination of α_s.

Appendix. The QCD prediction

These formulae and numerical values are taken from ref. [8], with a slight change of notation. The QCD prediction for the Cabibbo allowed components of the R_τ^{kl} moments is

$$R_\tau^{kl}(\bar{u}d) = |V_{ud}|^2 S_{\text{weak}}\, r(k,l)\left[1 + \delta_{\text{pert}}(k,l) + \delta_{m_q}(k,l) + \delta_{\text{SVZ}}(k,l)\right] \quad , \tag{6.1}$$

where

- $V_{ud} = 0.9753 \pm 0.0004$ [13].
- $S_{\text{weak}} = 1.0194$ is an electroweak correction [22]. Such a correction does not cancel out in the R_τ^{kl} ratios because the electric charges involved in the $\bar{u}d$ and $l\bar{\nu}$ final states are different.
- $r(k,l)$ corresponds to the parton level prediction (cf. table 3).
- $\delta_{\text{pert}}(k,l)$ is the massless perturbative QCD prediction. It results from the sum

$$\delta_{\text{pert}}(k,l) = \sum_{\nu=1}^{3} K_\nu\, A_{kl}^\nu \tag{6.2}$$

where A_{kl}^ν are functions of $\alpha_s(f^2 m_\tau^2)$, f being an arbitrary renormalization scale factor. These functions can be numerically calculated up to the contributions of the unknown coefficients ($\beta_{\nu \geq 4}$) of the Renormalization Group Equation which governs the running of the coupling constant. When expanded in α_s, the series start with α_s^ν. However, their α_s expansion is known not to converge [5] for $\alpha_s > 0.35$. The K_ν coefficients have been calculated up to $\nu = 3$; $K_1 = 1$, and for 3 flavours and $f = 1$, $K_2 = 1.64$ [3] and, in the $\overline{\text{MS}}$ renormalization scheme, $K_3 = 6.37$ [4].

- $\delta_{m_q}(k,l)$ accounts for quark mass corrections. Its contribution is negligible in the case of the D_τ^{kl} moments since only u and d quarks are involved while it gives a -1% correction to R_τ, due to the s quark mass. For the sake of completeness it has been accounted for in all cases.
- The last component $\delta_{\text{SVZ}}(k,l)$ accounts for non-perturbative effects. Neglecting α_s corrections, their contributions can be expressed as a sum

$$\delta_{\text{SVZ}}(k,l) = \sum_{D=2,4,6\ldots} \frac{c_D(k,l)}{m_\tau^D} O(D) \quad , \tag{6.3}$$

where D indicates the dimension in GeV of the non-perturbative terms $O(D)$ (the so-called condensates) which are obtained in principle from the matrix element in the vacuum of QCD operators [7]. A sample of $c_D(k,l)/m_\tau^D$ constants, obtained from ref. [8] , are given in table 7. When needed, the central values used for the non-perturbative constants are those quoted in ref. [2], which are derived from analyses based on various sets of data, including e^+e^- data and, to a lesser extent, τ data. The errors on the non-perturbative constants are conservatively taken as twice the ones quoted therein. The first term of the sum, $O(2)$, is absent in the SVZ approach since one cannot built operators of dimension GeV^2 from the QCD Lagrangian [7]. The perturbative expansion of the $D = 4$ coefficient is known up to $\mathcal{O}(\alpha_s^2)$ corrections [23]. Its zeroth-order contribution to R_τ cancels out but contributes to the R_τ^{10} and R_τ^{11} moments (cf. Table 7). $O(4)$ is expressed as a function of

Table 7
Numerical values of the $c_D(k,l)/m_\tau^D$ constants (in $(GeV/c^2)^{-D}$).

D	4	6	8	10
R_τ	0	-3.8	-0.8	0
R_τ^{10}	5.7	-5.4	-2.8	-0.4
R_τ^{11}	-23.8	-7.5	7.1	3.8
R_τ^{12}	0	20.2	6.4	-6.1
R_τ^{13}	0	0	-13.9	-4.4

terms involving quark masses, which are held fixed to the values given in ref. [2], and of the so-called gluon condensate $\langle \frac{\alpha_s}{\pi} GG \rangle = (0.02 \pm 0.02)$ $(\mathrm{GeV}/c^2)^4$ [2]. Accounting for its available $\mathcal{O}(\alpha_s^2)$ expression [8] , the resulting contribution of $O(4)$ to R_τ is $-0.3 \pm 0.3\%$ [2]. The $D = 6$ term is the largest non-perturbative term contributing to R_τ for which it yields a $-0.7 \pm 0.7\%$ correction [2] which results from $O(6) = (0.002 \pm 0.002)$ $(\mathrm{GeV}/c^2)^6$. The $D = 8$ term is the last one contributing directly to R_τ; it is expected to produce a correction much smaller than the one due to $D = 6$ and is neglected in ref. [2].

The sources of theoretical systematics which have been considered are

• The uncertainties on the numerical values entering into the evaluation of eq. (6.1). The systematics due to non-perturbative constants of dimensions $D \geq 8$ are evaluated using $O(D) = 10(0.4^D)$ $(\mathrm{GeV}/c^2)^D$. With this crude but conservative definition, the effects of the $O(D \geq 10)$ terms are negligible. The respective contributions to the theoretical systematics of the $O(D \leq 10)$ terms can be read-off from table 7. The largest systematics for the R_τ, R_τ^{10}, R_τ^{11}, R_τ^{12}, and R_τ^{13} ratios are due to the $O(6)$, $O(4)$, $O(4)$, $O(6)$ and $O(8)$ constants, respectively. The theoretical status of a potential contribution from a non-perturbative $O(2)$ term is not clear [17–19]. For the sake of clarity, no systematic uncertainties due to such a term have been included and, therefore, the validity of the present analysis relies on the applicability of the SVZ approach.

• The unknown K_4 coefficient. The corresponding systematics is evaluated using $K_4 = 2\,|K_3(K_3/K_2)| \simeq 50$.

• The unknown β_4 coefficient of the Renormalization Group Equation. The systematics evaluation is obtained using $\beta_4 = 2\,|\beta_3(\beta_3/\beta_2)| \simeq 100$.

• The renormalization scheme uncertainty. Since the perturbative expansion is known up to the third order, two effects must be considered; one arises from the ambiguity in the choice of the scale factor f, the other is due to the renormalization scheme dependence of the K_3 coefficient which can be evaluated by changing β_3 [8]. To avoid double-counting with the K_4 uncertainty, the corresponding systematics are evaluated by using the QCD prediction to the fourth order, with $\alpha_s(m_\tau^2) = 0.33$ and $K_4(f = 1)$ set to zero, while varying fm_τ from 1 to 2.5 GeV/c^2 and β_3 from 0 to twice its $\overline{\mathrm{MS}}$ value.

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