ALL ORDER RESULTS IN STRING THEORY

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ABSTRACT

A review is given of some results which can be proved to be valid to all orders in string theory. They include R-duality and β-duality, for toroidal compactifications and thermal strings, respectively, and the critical behaviour, that is, the Hagedorn temperature. Some remarks on non-perturbative effects are also included.

1. R-Duality and β-duality

A most remarkable property of strings is the so-called duality transformation which, in its simplest form, relates physical quantities computed for a toroidal compactification at some radius R, with the same quantities computed at another radius, $\alpha\prime/R$. A related (although slightly more complicated due to the different GSO projections one has to perform at finite temperature) symmetry exists between the free energy computed at a temperature β, and the same quantity computed at a temperature π^2/β.

This symmetry stems from the exchange of winding and momentum modes, and seems thus a very "stringy" property. Its physical meaning could be related with the "generalized string uncertainty principle", posited in [1]; in addition, it could be that this symmetry is spontaneously broken due to gaugino condensation, which is of the utmost importance from the phenomenological point of view (cf. [2] for a discussion of this possibility).

In [3] we succeeded in packing together all soliton contributions, (both for the free energy and for the simplest d-dimensional flat toroidal compactification) without background fields at genus g, in a theta function of order $2g$. By exploiting the well-known properties of the thetas, we were able to show that duality is an exact property of string perturbation theory; to be explicit, if we define the formal series:

$$F(\kappa, R) \equiv \sum_{g=0}^{\infty} \kappa^{2g-2} F_g(R) \tag{1}$$

$$F(\kappa, \beta) \equiv \sum_{g=0}^{\infty} \kappa^{2g-2} F_g(\beta) \tag{2}$$

then the duality transformations are:

$$F(\kappa, R) = F(\kappa^*, R^*) \tag{3}$$

$$\kappa^* \equiv \kappa \alpha\prime^{d/2}/R^d \tag{4}$$

$$R^* \equiv \alpha\prime/R \tag{5}$$

and

$$F(\kappa, \beta) = \pi^2/\beta^2 F(\kappa^*, \beta^*) \tag{6}$$

$$\kappa^* \equiv \kappa\pi/\beta \tag{7}$$

$$\beta^* \equiv \pi^2/\beta \tag{8}$$

2. Critical Behaviour

The density of states of any string theory grows exponentially with the energy; this fact alone implies that strings cannot be at equilibrium at temperatures greater than a critical one, called Hagedorn temperature. When interactions are included, however, most of the excited states are unstable, and the physical question becomes a quantitative one, as to whether they live enough as to persist between two successive interactions. To be precise, a state with width $\Gamma(m)$ has a probability $\exp -\beta\Gamma(m)$ of survive during one mean free time (of order β). This means that if we define

$$\tau \equiv \lim_{m\to\infty} m/\Gamma(m) \tag{9}$$

then, when $\tau = 0$, the interactions render the states so unstable that no critical temperature exists. On the other hand, if τ diverges, we expect that the critical temperature stays unchanged. The marginal case corresponds to $\tau = O(1)$; we expect then numerical modifications to the values of the critical temperature. Remarkably, the widths for closed string states have been estimated in [4] ,with the (numerical) result that $\tau = \infty$. This is in perfect concordance with our own results, to be found in the references [5] , which prove for the bosonic string, and strongly suggest for the heterotic string, that the critical temperature remains the same to all orders in string perturbation theory.

3. Non-perturbative results

We know that the predictions of string perturbation theory cannot be trusted in general, because the perturbative series is divergent and non even Borel summable [6]. Unfortunately, we do not even know whether there is a region (like the asymptotically free regime in QCD) in which these predictions give a good indication of the ingoing physics.

Once we have identified a symmetry of string perturbation theory, it is very important to check whether non-perturbative contributions are likely to break it or not.This research was undertaken for duality in [7], and further pursued for toy models in [8], but the results are inconclusive for the time being. We have been able to show, in particular, that it is possible to define the Ising model in a random lattice in such a way as to preserve Kramers-Warnier duality. This property is of course non-universal (it is already so in the simpler case of standard, non-random lattices), which means that we have to " fine tune " the potential to preserve duality. These results cannot be, unfortunately carried over the corresponding problem in string theory, although work on this is in progress.

Gross and Klebanov [7] claim that the discrete definitions which seem most natural not only break duality (through a Kosterlitz-Thouless phase transition), but also fail to give a correct description of some (apparently) well-established results of string perturbation theory. It is possible in most cases, however, to perform "ad hoc" modifications of the discrete action so as to preserve the perturbative symmetries.

More work is needed, however, before the physical meaning of these non perturbative effects can be unravelled.

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