Average Charged Multiplicity of Events Containing Heavy Quarks in e^+e^- Annihilation

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We present the perturbative QCD results for heavy quark event multiplicities in e^+e^- annihilation, which provide that the difference δ_{Ql} between heavy and light quark event multiplicities at the same c.m.-system (c.m.s.) energy should be a calculable constant, independent of c.m.s. energy. Published data on heavy quark event multiplicities are presented in this light, and are consistent with the energy independence of the multiplicity difference. Averaging over c.m.s. energy, we find that $\delta_{bl}=4.3\pm0.9$ and $\delta_{cl}=2.3\pm1.0$ tracks, while the perturbative QCD expectations are 5.5 ± 0.8 and 1.7 ± 0.5 tracks, respectively.

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The perturbative (PT) approach to QCD jet physics is based on the modified leading logarithmic approximation (MLLA), which provides an analytical technique for the calculation of parton cascade development that includes summation of both double- and single-logarithmic terms for each power of the strong coupling parameter α_s [1-3]. The inclusion of the smaller, single-logarithmic contribution is found to be of crucial importance in providing an accurate quantitative description of the properties of multiparticle systems. In addition, the hypothesis of local parton-hadron duality (LPHD) [1,4], which is supported by experimental studies of multihadron production in QCD jets [5], suggests a close correspondence between the observable inclusive characteristics of hadron spectra and those calculated at the parton level by means of PT QCD. Thus, when combined with LPHD, the PT approach can possibly describe the gross features of hadronic systems, such as multiplicity distributions, the angular distribution of particle flows, inclusive energy spectra, etc., without invoking phenomenological fragmentation schemes. In this approach, nonperturbative effects are reduced to normalizing coefficients relating hadronic characteristics to partonic ones, which, according to LPHD, must be independent of both the hardness of the initiating partons and the energy range of the final-state particles.

Until now the main phenomenological successes of this approach were connected with the description of the inclusive characteristics of jets in e^+e^- annihilation, without distinction between the contributions of light and heavy primary quarks [6,7]. Prompted by these successes, and the recent availability of data on heavy quark jets, one would like to compare the PT predictions for heavy quark generated jets with existing data on heavy quark events. In this Letter, then, we shall present a comparison between the PT QCD description of particle multiplicity with data on the mean charged multiplicity of events containing heavy hadrons.

The physics of heavy quarks has always been considered a particularly good laboratory for detailed studies of QCD. The large quark mass $M_Q \gg \Lambda_{\text{QCD}}$ provides a natural cutoff, which keeps the relevant space-time region compact enough to avoid the truly strong, non-PT domain of strong interactions. In the case of e^+e^- annihilation at center-of-mass energies $W \gg M_Q \gg \Lambda_{\text{QCD}}$, one can hope for a good description of many inclusive properties of hadronic jets via PT QCD.

The results of the PT description of specific properties of particle distributions in heavy quark jets have been announced in a number of publications [6,8-10], but only recently has this subject been addressed more comprehensively [11]. It was demonstrated [6,9,11] that the difference in many properties of hadronic jets produced by heavy quarks (excluding the products of the weak decay of the heavy quark itself), from that of light (u,d,s) quarks, originates from the restriction of the phase space available to gluon radiation associated with the kinematic effects of the heavy quark mass.

Particle multiplicities were calculated [6,11] by convoluting the differential distribution of gluon (g) radiation from a massive quark (Q), with energy $E_Q \gg M_Q$, with the multiplicity distribution of subsequent gluon cascades. The angular radiation pattern of soft gluons, with energy $\omega \ll E_Q$ and emission angle $\Theta \ll 1$,

$$d\sigma_{Q\to Q+g} \sim \frac{\Theta^2 d\Theta^2}{(\Theta^2+\Theta_0^2)^2}\frac{d\omega}{\omega}\,, \tag{1}$$

where

$$\Theta_0 = M_Q/E_Q\,, \tag{2}$$

gives rise to a large double-logarithmic contribution for $\Theta > \Theta_0$ [3]:

$$d\sigma_{Q\to Q+g} \sim \frac{d\Theta^2}{\Theta^2}\frac{d\omega}{\omega} = d(\ln\Theta^2)d(\ln\omega)\,. \tag{3}$$

For $\Theta < \Theta_0$, however, the angular integration is no longer logarithmic, and the yield of particles in this region from (1) adds only a small, single-logarithmic $[O(\alpha_s^{1/2}N)]$ contribution to the total multiplicity N. This region of suppressed radiation in the forward direction is known as the "dead cone" [6]. On the other hand, for emission angles $\Theta \gg \Theta_0$, and for the internal structure of secondary gluon jets (due to the strict angle ordering of hadronic cascades), Eq. (1) yields completely identical behavior between light and heavy quark gluon radiation. This universality of the gluon radiation spectrum, up to a depopulation in a cone around Q of opening angle $\Theta \sim \Theta_0$, lies at the heart of comparisons between light and heavy quark jets within the framework of the PT approach.

A consequence of this suppression of forward gluon radiation is that the "companion" multiplicity $\Delta N(Q\bar{Q};W)$ of light hadrons accompanying the heavy quark, excluding the decay products of the on-shell heavy hadron, is less than the particle yield in a light quark jet at the same c.m.-system (c.m.s.) energy. Quantitatively [6,11],

$$\Delta N(Q\bar{Q};W) = N(q\bar{q};W) - N(q\bar{q};\sqrt{e}M_Q) + O(\alpha_s(M_Q^2)N(q\bar{q};M_Q)) , \qquad (4)$$

where $N(q\bar{q};W)$ is the mean total multiplicity in light quark events at c.m.s. energy W, and $e = \exp(1)$. It should be noted that this calculation improves an earlier calculation incorporating only double-logarithmic contributions [9], and which was thus limited in accuracy by higher-order corrections of $O([\alpha_s(W^2)]^{1/2}N(q\bar{q};W))$. The uncertainty from this correction is substantially larger than the $O(\alpha_s(M_Q^2)N(q\bar{q};M_Q))$ leading correction of the current approach, and is formally a function of c.m.s. energy.

The most important consequence of the current result stems from the fact that both the subtrahend and uncalculated higher-order correction in Eq. (4) are functions of the fixed mass scale M_Q, and thus independent of the c.m.s. energy W of the e^+e^- annihilation. Thus, it is a fundamental prediction of PT QCD that the difference

$$N(q\bar{q};W) - \Delta N(Q\bar{Q};W) \qquad (5)$$

is almost completely independent of the c.m.s. energy W; leading W-dependent corrections are of $O([\alpha_s(W^2)]^{1/2}M_Q^2/W^2)$ [12]. In addition, the extraction of $N(q\bar{q};\sqrt{e}M_Q)$ from existing low c.m.s. energy multiplicity data permits an estimate of the mean multiplicity difference (5) to $O(\alpha_s(M_Q^2)N(q\bar{q};M_Q))$ accuracy.

Viewed another way, it is QCD coherence, which consideration of the gluonic formation length shows to apply to the region $\Theta \lesssim \Theta_0 \equiv M_Q/E$, that provides for this relation between light and heavy quark multiplicities. The difference between light and heavy quark radiation in this forward region, where gluons radiated from heavy quarks cannot distinguish themselves quantum mechanically from the heavy parent quark, is roughly the integral of the light quark radiation spectrum out to Θ_0, which is dominated by the $N(q\bar{q};\sqrt{e}M_Q)$ term in Eq. (4).

It should be emphasized that, due to the observed steeply rising dependence of total multiplicity on $\ln(W)$, the current multiplicity picture is *not* consistent with the naively expected reduction of the energy scale [8,13]

$$\Delta N(Q\bar{Q};W) = N(q\bar{q};(1-\langle x_Q\rangle)W) , \qquad (6)$$

where $\langle x_Q\rangle = 2\langle E_Q\rangle/W$. A good description of the total multiplicity data is provided by the PT QCD-inspired multiplicity formula [1-3]

$$N(q\bar{q};W) \propto \alpha_s(W^2)^{1/4+10n_f/27b}\exp\left[\frac{\sqrt{96\pi}}{b}\alpha_s^{-1/2}(W^2)\right] , \qquad (7)$$

where n_f is the number of quark flavors and $b = 11 - 2n_f/3$. Applying this to Eq. (6), the difference (5) is not constant, but is asymptotically proportional to $N(W)$:

$$N(q\bar{q};W) - \Delta N(Q\bar{Q};W) \simeq \left[\frac{6\alpha_s(W^2)}{\pi}\right]^{1/2} \ln\frac{1}{1-\langle x_Q\rangle}N(q\bar{q};W) . \qquad (8)$$

With the recent addition to previous results [14-17] of a measurement at the Z^0 resonance by the Mark II Collaboration [18], the average total charged multiplicity $\bar{n}_b$ in e^+e^- annihilation to b quarks has been measured in a range of c.m.s. energy from $W = 29$ to 91 GeV. In addition, the average total charged multiplicity $\bar{n}_c$ for $e^+e^- \to c\bar{c}$ events has been measured by two experiments at $W = 29$ GeV [15,16], and at $W = 35$ GeV [19]. Combined with the world sample of total hadronic (all quark flavors) e^+e^- mean charged multiplicity measurements ($\bar{n}_{\text{had}}$) between $W = 1.5$ and 91 GeV [20], these data can be used to study the difference in charged particle yields between light (u,d,s) and heavy (c,b) quark production.

Table I shows the measured mean charged multiplicity of events containing heavy quarks at $W = 29$, 35, 42.1, and 90.9 GeV. Also shown is the total hadronic multiplicity at the same energies, derived from the corresponding multiplicity data in the c.m.s. energy region surrounding the heavy quark multiplicity point. Both heavy quark-associated and total hadronic multiplicities have been corrected for the effects of initial-state radiation (ISR), so that the quoted values correspond to the average charged multiplicity that would be observed at the given c.m.s. energy in the absence of ISR. Charged tracks from K_s^0 and Λ decays are included in the measured multiplicities.

The difference $\delta_{Ql} \equiv \bar{n}_Q - \bar{n}_l$ between measured heavy and light quark event total charged multiplicities can be written as

$$\delta_{Ql} = \frac{f_l\bar{n}_Q + f_b\bar{n}_b + f_c\bar{n}_c - \bar{n}_{\text{had}}}{f_l} , \qquad (9)$$

where $Q = c,b$, and the f_i are the standard model production fractions for light, c, and b quarks. For γ^* decays, $f_l = 0.55$, $f_c = 0.36$, and $f_b = 0.09$, while at the Z^0, $f_l = 0.61$, $f_c = 0.17$, and $f_b = 0.22$.

In order to calculate δ_{bl} from the measurements of $\bar{n}_b$ and $\bar{n}_{\text{had}}$, it is necessary to estimate $\bar{n}_c$ in the poorly measured region above $W = 29$ GeV. For this purpose, we

TABLE I. Measured mean charged multiplicities. For the total multiplicity $\bar{n}_{\rm had}$, the value is from an average of all experiments in the c.m.s. energy region surrounding the heavy quark event multiplicity point.

Experiment	$E_{\rm c.m.}$ (GeV)	$\bar{n}_{\rm had}$	$\bar{n}_c$	$\bar{n}_b$
DELCO [14]	29.0	12.41 ± 0.21		14.3 ± 1.2
Mark II [15]	29.0	12.41 ± 0.21	13.2 ± 1.0	16.1 ± 1.1
TPC [16]	29.0	12.41 ± 0.21	13.5 ± 0.9	16.7 ± 1.0
TASSO [17,19]	35.0	13.59 ± 0.30	15.0 ± 1.2	16.0 ± 1.5
TASSO [17]	42.1	14.85 ± 0.40		17.0 ± 2.0
Mark II [18]	90.9	20.94 ± 0.20		23.1 ± 1.9

have made the PT QCD-motivated assumption that $\bar{n}_c - \bar{n}_l$ is constant as a function of c.m.s. energy, which gives $\bar{n}_c = 14.7 \pm 0.7$ and 16.0 ± 0.7 tracks at $W = 35$ and 42.1 GeV, respectively. At these energies, the uncertainty in $\bar{n}_c$ is dominated by the ±0.7 track uncertainty in $\bar{n}_c$ at $W = 29$ GeV. Because of the large difference in c.m.s. energy between 90.9 and 29 GeV, in order to maintain model independence we have assumed only that $\bar{n}_l < \bar{n}_c < \bar{n}_b$, leading to a value $\bar{n}_c = 22.1 \pm 2.5$ tracks at $W = 90.9$ GeV. Because of the relatively small contribution of $Z^0 \to c\bar{c}$ (0.17 of $\sigma_{\rm had}$), and the large statistical error in $\bar{n}_b$ at 90.9 GeV, this constitutes only a small contribution to the uncertainty in δ_{bl}.

Combining these values for $\bar{n}_c$ with the $\bar{n}_{\rm had}$, $\bar{n}_c$, and $\bar{n}_b$ results in Table I yields the results for δ_{cl} and δ_{bl} exhibited in Table II. Also shown are averages for all experiments at $W = 29$ GeV, and for all c.m.s. energies combined, taking into account common systematic errors due to the uncertainty in the average $\bar{n}_c$, ISR corrections, the common use of lepton tagging to identify heavy quark events at $W = 29$ GeV, and the common use of displaced vertex information to identify b quark events at $W = 35$ and 42.1 GeV. The results for the individual measurements of δ_{bl} are also displayed in Fig. 1. To the available accuracy, the results are seen to be independent of energy, in marked contrast to the steeply rising total multiplicity data, and are thus consistent with the prediction discussed above. On the other hand, Fig. 1 in Ref. [18] exhibits a mild (1.1 standard deviation) disagreement between the energy dependence of the $Z \to b\bar{b}$ companion multiplicity and that of the naive expectation of Eq. (6).

TABLE II. Derived differences between heavy and light quark event mean multiplicities.

Experiment	$E_{\rm c.m.}$ (GeV)	δ_{cl}	δ_{bl}
DELCO [14]	29.0		2.9 ± 1.5
Mark II [15]	29.0	1.9 ± 1.7	5.0 ± 1.4
TPC [16]	29.0	2.4 ± 1.5	5.7 ± 1.3
Average	29.0	2.2 ± 1.2	4.7 ± 1.0
TASSO [17,19]	35.0	2.8 ± 2.1	3.6 ± 1.9
TASSO [17]	42.1		3.3 ± 2.5
Mark II [18]	90.9		3.3 ± 2.7
Average	All energies	2.3 ± 1.0	4.3 ± 0.9

Equation (4) predicts that the difference between the total light quark and companion heavy quark event multiplicities should be equal to the total light quark event multiplicity at $W = \sqrt{e}M_Q$. In terms of the heavy hadron decay multiplicity $\bar{n}_Q^{dk}$, this can be written as

$$\delta_{Ql} \equiv \bar{n}_Q - \bar{n}_l = \bar{n}_Q^{dk} - \bar{n}_l(\sqrt{e}M_Q) . \tag{10}$$

In order to estimate $\bar{n}_l(\sqrt{e}M_Q)$, we assume $\sqrt{e}M_c = \sqrt{e} \times 1.5 = 2.5$ GeV and $\sqrt{e}M_b = \sqrt{e} \times 4.8 = 7.9$ GeV, and use the measured total hadronic multiplicity for $2 < W < 3$ GeV [21,22] and $5.5 < W < 10$ GeV [22,23], respectively, to estimate $\bar{n}_{\rm had}$ at these c.m.s energies. At $W = 2.5$ GeV, below the charm threshold, $\bar{n}_{\rm had} = \bar{n}_l$, while for $W = 7.9$ GeV we assume the value of δ_{cl} measured at $W = 29$ GeV to correct $\bar{n}_{\rm had}$ for the effects of the 40% ad-

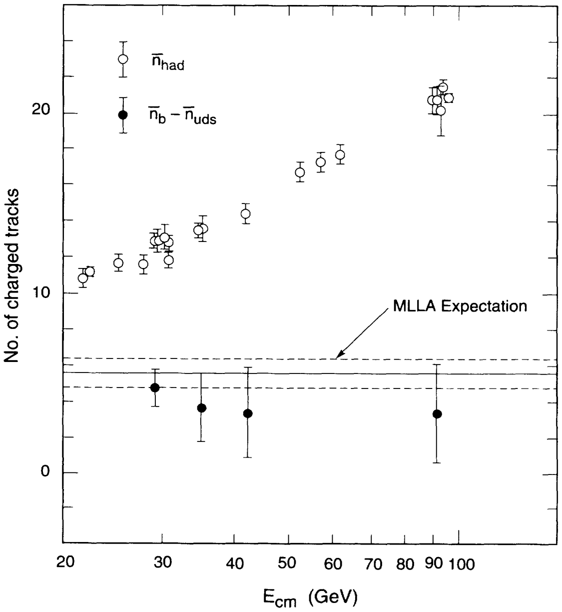

FIG. 1. Energy dependence of total multiplicity (open points) and the multiplicity difference between b and light quark production (solid points) in e^+e^- annihilation. MLLA predicts unambiguously that this multiplicity difference should be independent of energy. Also shown is the expected value of this multiplicity difference, given by lower energy multiplicity data in accordance with MLLA (see text). The 1 standard deviation range indicated by the dotted lines is dominated by the uncertainty in the light quark event multiplicity at $E_{\rm c.m.} = \sqrt{e}M_b$, and does not include a ~1 track uncertainty due to (energy-independent) higher-order corrections to MLLA. Citations for the total multiplicity data are compiled in Ref. [18].

mixture of $c\bar{c}$ events. This yields

$$\bar{n}_l(\sqrt{e}M_c) = 3.5 \pm 0.4 \text{ tracks}, \tag{11}$$

$$\bar{n}_l(\sqrt{e}M_b) = 5.5 \pm 0.7 \text{ tracks}. \tag{12}$$

For $W = \sqrt{e}M_b$, the quoted uncertainty includes a 0.5 track contribution from the uncertainty in the $c\bar{c}$ correction.

Assuming the values $\bar{n}_c^{dk} = 5.2 \pm 0.3$ [24] and $\bar{n}_b^{dk} = 11.0 \pm 0.2$ [18] yields

$$\bar{n}_c^{dk} - \bar{n}_l(\sqrt{e}M_c) = 1.7 \pm 0.5 \text{ tracks}, \tag{13}$$

$$\bar{n}_b^{dk} - \bar{n}_l(\sqrt{e}M_b) = 5.5 \pm 0.8 \text{ tracks}. \tag{14}$$

It should be noted that $O(\alpha_s \bar{n}_l(M_Q))$ terms neglected in Eq. (4) are expected to be of roughly the same size as the experimental uncertainties on these values. Comparing these values to the results for δ_{cl} and δ_{bl} in Table II, it is again seen that the experimental data are consistent with the predictions of PT QCD.

In conclusion, it has been seen that, to within the available accuracy, the observed mean multiplicities of events containing heavy hadrons are in good agreement with the predictions of PT QCD, and in mild disagreement with the naive relation (6). In particular, the data support the notion that the difference between the companion multiplicity in heavy quark events, and the total multiplicity in light quark events at the same c.m.s. energy, is independent of c.m.s. energy. This provides a fundamental check of the consistency of the PT approach, which predicts this result and provides that it should be independent of higher-order corrections. In addition, combined with the quantitative agreement between this multiplicity difference and the lower c.m.s. energy multiplicity data embodied in Eq. (10), this work supports the validity of LPHD as a phenomenological approach to modeling confinement.

Based on the result from Ref. [18], which was statistically limited, experiments currently running at 91 GeV at the CERN e^+e^- collider LEP and the SLAC Linear Collider should be able to measure $\bar{n}_b$ to ± 0.5 tracks or better. Combined with a measurement of $\bar{n}_c$ to ± 1.0 tracks, this would yield a measurement of δ_{bl} to $\sim \pm 0.8$ track, providing a much more stringent test of the PT predictions in the case of b quark production. Further reduction of the uncertainty in $\bar{n}_c$ to ± 0.5 track or better would allow measurements of δ_{cl} and δ_{bc} to $\sim \pm 0.7$ track, providing stringent tests of the PT predictions down to the M_c^2 scale. At this lower mass, the question of the relationship between LPHD and QCD confinement becomes particularly interesting.

Making use of the inclusive properties of heavy hadron decay to statistically remove the heavy hadron decay tracks, it should be possible to study more extensively the properties of radiated hadrons in heavy hadron events. It is expected that the gluonic radiation "dead cone" will appear as a depopulation in the region $2E_{\text{had}}/W > \Lambda_{\text{QCD}}/M_Q$, while the spectrum of soft hadrons with $2E_{\text{had}}/W \ll \Lambda_{\text{QCD}}/M_Q$ should be identical to that of light quark jets [6,10,11]. Finally, PT QCD predicts various aspects of the x_Q distribution itself [25], which can in principle be tested with an accurate measurement of $\langle x_Q \rangle$ and the x_Q spectrum.

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