



THE NATURE OF THE CONTINUUM LIMIT
IN STRONGLY COUPLED QUENCHED *QED*.

Maria-Paola Lombardo, John B. Kogut
Department of Physics
University of Illinois at Urbana-Champaign
1110 West Green Street, Urbana, IL 61801, U.S.A.
Aleksandar Kocić
Theory Division
C.E.R.N.
CH-1211 Geneva 23, Switzerland
K.C. Wang
School of Physics
University of New South Wales
Kensington, NSW 2203, Australia

Abstract

We review the results of large scale simulations[1] of noncompact quenched *QED* which use spectrum and Equation of State calculations to determine the theory's phase diagram, critical indices, and continuum limit. The resulting anomalous dimensions are in good agreement with Schwinger-Dyson solutions of the ladder graphs of conventional *QED* and they satisfy the hyperscaling relations expected of a relativistic renormalizable field theory. The spectroscopy results satisfy the constraints of the Goldstone mechanism and PCAC, and may be indicative of Technicolor versions of the Standard Model which are strongly coupled at short distances.

INTRODUCTION

What is the simplest, most elementary yet physical example of anomalous dimensions? Perhaps it is the relativistic hydrogen atom, a subject we all learn long before field theory. In the Dirac theory the ground state wave function behaves at short distances like $r^{(\gamma-1)}$ where $\gamma-1 = (1-Z^2\alpha^2)^{1/2}-1 = -Z^2\alpha^2/2+\dots$

The unscreened Coulomb attraction has made the wave function more singular than predicted by the Schrodinger equation. Considering scale transformations, we see that the interaction has changed the scaling dimension of the electron field and an anomalous dimension $\eta = \gamma - 1$ has appeared. The anomalous dimension vanishes in the Schrodinger description of the hydrogen atom because the kinetic energy $p^2/2m$ dominates the interaction e^2/r at short distances. The electron wave function has "canonical" or free field dimensions

*Partially supported by NSF-PHY 92-00148

To appear in the proceedings of the XXVI International Conference on High Energy Physics, Aug. 6-12, 1992 Dallas, Texas.

in this case. However, in the Dirac equation the relativistic kinetic energy is p which scales in the same fashion as the potential and the interaction can not be neglected at short distances. QED with massless fermions can be solved in the ladder "approximation" and much of the same physics emerges in a relativistic setting[2]. The appearance of anomalous dimensions which depend continuously on the bare charge is confirmed. In fact, the physics is very rich, complete with chiral symmetry breaking, a Goldstone mechanism, etc. Again, we are not dealing with a real field theory but the ladder approximation is a step in that direction and it is thought to be a good approximation to a class of Technicolor theories [3].

QED in the ladder approximation also poses a challenge to lattice gauge theory which should eventually solve strongly coupled QED including fermion vacuum polarization. Can lattice simulations of the quenched theory formulated on the lattice and simulated by first principle methods yield chiral symmetry breaking and anomalous scaling laws in agreement with the continuum formulation? In this report we shall answer this question in the affirmative, although the lattice approach has not mapped out the full parameter space of couplings available to the continuum formalism. We shall also see how lattice spectroscopy calculations can be used to find anomalous dimensions and uncover the non-triviality of this not-so-simple model.

THE MODEL

The lattice QED action S_{QED} in its non-compact formulation reads:

$$S = S_S + S_{SB} \quad (1)$$

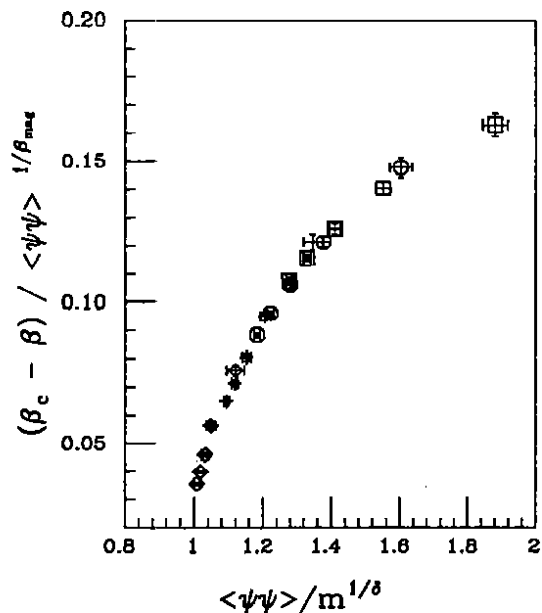


Figure 1. Chiral equation of state for $\beta_c = 0.257$, $\delta = 2.2$ and $\beta_{mag} = 0.833$

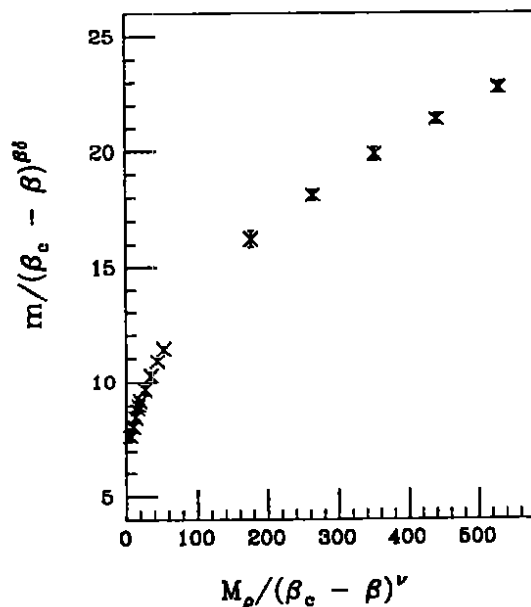


Figure 2. Equation of state for the ρ mass. M_ρ/t^ν is plotted versus $m/(\beta_c - \beta)^{\delta\beta_{mag}}$ for $\nu = 0.67$, $\beta_{mag} = 0.84$ and $\delta = 2.2$

where

$$S_S = 1/2e^2 \sum_{\mu,\nu,n} F_{\mu\nu}^2(n) + 1/2 \sum_{\mu,n} \bar{\psi}(n) \eta_\mu e^{i\theta_\mu(n)} \psi(n+\mu) + h.c. \quad (2)$$

$$S_{SB} = m\bar{\psi}\psi \quad (3)$$

θ_μ are the gauge variables - oriented, real numbers ranging from $-\infty$ to $+\infty$, $F_{\mu\nu}$ is the field strength tensor, ψ are the fermionic fields, η the staggered phases. S is thus controlled by two bare parameters, $\beta = 1/e^2$, e being the electromagnetic coupling, and the fermion mass m . By using staggered fermions S_S has, for any value of the lattice spacing a , a continuous chiral symmetry which is spontaneously broken at a finite (i.e. non zero) value of the coupling β_c . The mass term S_{SB} is an explicit symmetry breaking term which is required by technical difficulties connected with the chiral extrapolation. We thus sample the critical region, at small, but non-zero m . In this region ($\beta \rightarrow \beta_c, m \rightarrow 0$) the fundamental tools are the Equation of State (EOS) and the scaling laws: by exploiting them we will be able to get information about the singularities occurring at $\beta = \beta_c, m_g = 0$ by working at finite values of m and $\beta_c - \beta$. This is similar to the study of critical phenomena on finite systems: because of the finite size the system is always in the symmetric phase, thus in both cases (finite size/finite mass) we deal with single phase systems, and in both cases appropriate scaling laws tell us the physics of the phase transition.

THE EOS AND THE SCALING LAWS

The response of the system in the critical region to an external symmetry breaking field is expressed by universal functions of reduced variables: this is the general fact which allows the computation of the critical coupling, and

related exponents. In our case the symmetry breaking term is the mass term $m\bar{\psi}\psi$ in the lagrangian: the response functions we consider are the chiral condensate itself (the natural order parameter for the chiral symmetry), and the meson masses.

EOS for the chiral condensate

The EOS for the chiral condensate, in full analogy with a ferromagnetic system [4] (just replace m with an external magnetic field, and $\bar{\psi}\psi$ with the spontaneous magnetization), reads:

$$m / \langle \bar{\psi}\psi \rangle^\delta = f(t / \langle \bar{\psi}\psi \rangle^{1/\beta_{mag}}) \quad (4)$$

(Here and in the following $t = \beta_c - \beta$.) Its usage is demonstrated in Fig.1 where all our data are plotted and nicely fall on a universal curve when $\delta = 2.2$, $\beta_{mag} = 0.833 \beta_c = 0.257$. The universal behaviour is in principle unique to the correct parameters, but the errors on the data broaden the choice. Thus one has to check the persistence of the (apparent) universal behaviour when adjusting $\beta_{mag}, \delta, \beta_c$: the errors on these indices are determined by this procedure.

EOS for the masses

Once we assume that the critical behaviour of the system is controlled by only one macroscopic (diverging) scale length, the equation of state for the masses easily follows:

$$M_a = t^\nu G_a(m/t^{\delta\beta_{mag}}) \quad (5)$$

M_a is any meson mass (different of course from the Goldstone boson) whose reciprocal $1/M_a$ is to be identified with the correlation length of the system, times an irrelevant constant. In complete analogy with the EOS for the chiral condensate, we can determine β_c, δ and ν (Fig.2).

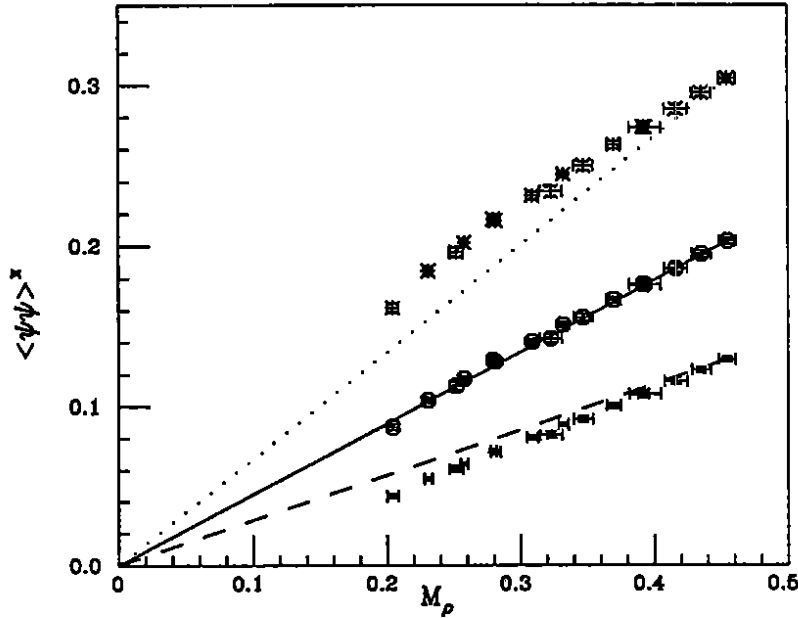


Figure 3. $\bar{\psi}\psi^x$ versus M_ρ

The check of the crucial hypothesis that the critical behaviour is controlled by one divergent length scale (apparently a hopeless task) is done a posteriori by verifying the hyperscaling relations among the critical exponents and indeed in this case proved to be correct (Table 1, to be discussed later).

The scaling laws

The EOS's for the chiral condensate and for the masses can be, in some sense, combined to give the following scaling law:

$$\langle \bar{\psi}\psi \rangle = C_a M_a^{\beta_{mag}/\nu} = C_a M_a^{d/2-1+\eta/2} \quad (6)$$

whose most noticeable characteristic is the dependence on just *one* parameter, the anomalous dimension η . So, eq.(6) offers the possibility of a simple test of trivial vs. non-trivial behaviour: $\bar{\psi}\psi^x$ ($x = \nu/\beta_{mag}$) plotted versus m gives a straight line (including the origin) for the correct value of x . Recall that in four dimensions $\eta = 0$ if the system is described by mean field behaviour, which thus corresponds

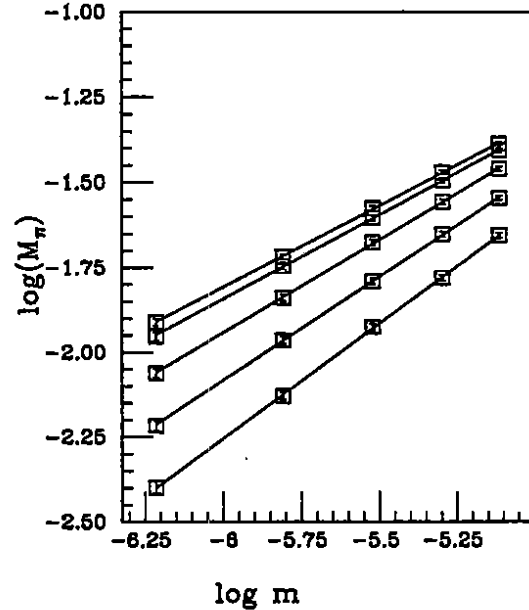


Figure 4. $\log M_\pi$ vs. $\log m$ for $\beta = (0.245, 0.250, 0.255, 0.260, 0.265)$ (top to bottom). The straight line superimposed are fits to the relations $M_\pi = Am^x$. In the strong coupling region x is consistent with the PCAC prediction of 0.5.

Table 1. Critical indices and hyperscaling relations

Index	Result from the simulation	Result from HS	MF
β_{mag}	0.86(3)	0.86(6)	0.5
$\gamma = \beta_{mag}(\delta - 1)$	1.00(5)		1.0
δ	2.2(1)	2.16	3.0
η	0.5(1)	0.5	0.0
ν	0.675	0.68(3)	0.5
$-4\nu + 2\beta_{mag} + \gamma$	0.1(1)	0.0	0.0
$(2 - \eta)\nu/\gamma$	1.1(1)	1.0	1.0
$(2\nu - \gamma)/(\nu\eta)$	1.1(1)	1.0	1.0
$(6 - \eta)/(2 + \eta) - \delta$	0.13(20)	0.0	0.0

to $x = 1$. In Fig. 3 we experiment with different x values: the x corresponding to the best fit (solid) coincides within errors with the ratio of β and ν from independent determinations (this method gives the most precise estimate of $\nu = 0.68(3)$); the dotted line, corresponding to the correct value of β_{mag} and to the mean field value for ν , demonstrates the sensitivity of the method to the exponents choice; $x = 1$ (dashed), hence $\eta = 0$, is clearly ruled out. η turns out to be $\simeq 0.5$, compatible with the result given by the hyperscaling relation between η and δ .

SPECTROSCOPY

The numerical values of the fermion and meson masses contribute to build a coherent scenario: the dynamical breaking of chiral symmetry should be associated with the appearance of a Goldstone boson, the approach to the continuum limit can be tested by looking at the scaling plots, which in turn give information on the level ordering.

The Goldstone character of the pion

In the strongly coupled, symmetry broken region, we expect the usual PCAC behaviour.

The square of the pion mass should be linear in the quark mass (plus second order corrections), eventually vanishing in the chiral limit. The pion decay constant, on the contrary, should stay finite in the same limit. The pion behaviour is shown in Fig. 4: the relation $m_\pi^2 \propto m_q$ is well verified in the strong coupling region, while deviations are observed at weak coupling. Figure 5 shows f_π^2 .

Level ordering

The mass ratios can be plotted one against the other: near the continuum limit the details of the lattice discretization are lost, and the resulting plots (scaling plots) are β independent. In a bit more formal fashion, we can derive this property (widely exploited in lattice QCD studies) by building up mass ratios from the EOS in Section 2 above. We sample here the results for σ/ρ vs. π/ρ for $\beta = (0.245, 0.250, 0.255)$: within large errors, all the data fall on the same plot (Fig. 6).

Moreover, we can get information on the level ordering in the chiral limit: from Figure 6, and the analogous ones for the other ratios, we found $0 = M_\pi < M_\sigma < 2M_f < M_\rho < M_{a1}$. As discussed at length in [5] $M_\sigma < 2M_f$ is a

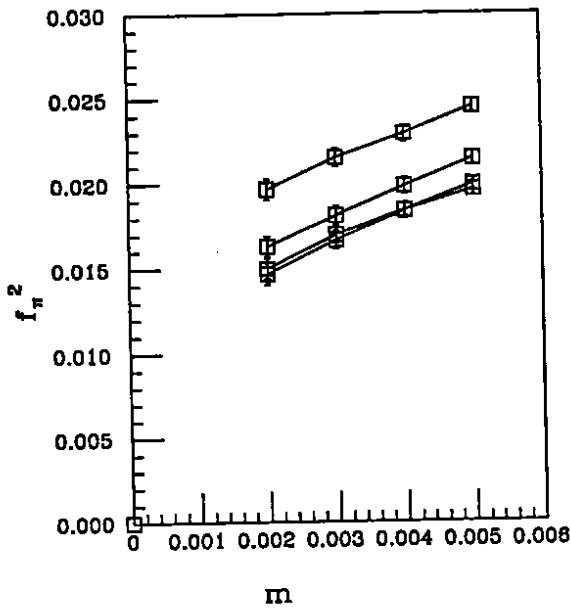


Figure 5. f_{π}^2 as a function of m for $\beta = (0.245, 0.250, 0.255, 0.260)$

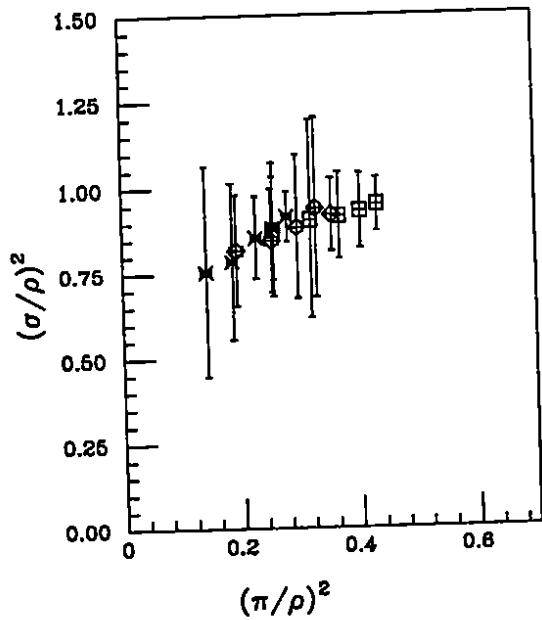


Figure 6. Scale invariant plot for $(\sigma/\rho)^2$ vs $(\pi/\rho)^2$. (Squares, diamonds, crosses) are $\beta = (0.255, 0.250, 0.245)$

peculiarity of non-trivial theories.

SUMMARY

We summarize in Table 1 the critical indices and the relations among them dictated by hyperscaling. In the first column we give the results from the simulation, in the second column the hyperscaling prediction assuming the other indices as input for the single index entries, in the third column the mean field prediction ($d=4$). These results, together with the ones from spectroscopy, support a picture of non-trivial critical behaviour, which is inferred both from the hyperscaling, and from the large anomalous dimension η .

REFERENCES

1. A. Kocić, J. B. Kogut, M. -P. Lombardo and K. C. Wang, "Spectroscopy, Scaling and Critical Indices in Strongly Coupled Quenched QED", CERN-TH.6542/92 ILL-(TH)-92-#12, (1992).
2. W. A. Bardeen, C. N. Leung and S. T. Love, "Aspects of dynamical symmetry breaking in gauge field theories", *Nucl.Phys.* B323, pp. 493-512, (1989).
3. M. E. Peskin, "Beyond the standard model", these Proceedings.
4. D. J. Amit, *Field theory, the renormalization group, and critical phenomena*, (McGraw-Hill, 1978).
5. A. Kocić, J. B. Kogut and M. -P. Lombardo, "Universal properties of chiral symmetry breaking", ILL-(TH)-92-#18, CERN-TH.6630/92, (1992).