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Naked singularities cannot be observed (an information-theoretic approach to the cosmic censorship conjecture)*

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Abstract

In this article we review some recent results obtained in the interplay between field theory in curved space-time and information theory, and explore some consequences. We shall explain how the Weyl tensor becomes responsible for the creation of quantum noise which, in turn, degrades a considerable fraction of the information which is optically transferred in a curved space-time. We conclude that causal connection between space-time regions, although being a necessary condition for transmitting information between them, is by no means sufficient. This is because this quantum noise might preclude the observation of very distant objects in the Universe and even closer ones if it becomes very intense. We argue that naked singularities are very powerful sources of noise and, therefore, cannot be observed optically . This leads to the intriguing possibility that the 'no-hair' theorem might be more general than was thought before.

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Suppose that we climb a mountain on a sunny day to observe a (supposedly) beautiful landscape. But, what a deception! The weather is considerably foggy, and we can just observe near-by objects in the scenery. During their long journey, those quanta which were supposed to convey to us information about more distant villages, mountains, etc., interacted with the medium (atmosphere) where they were scattered, absorbed and, later, re-emitted. The outcome of all these processes is that we can see a diffuse light, but no imprints (information) of the furthest objects in the scenery.

Well, what does a promenade in the mountains have to do with general relativity? This is what I will explain in the following. All the information about the events we detect in a telescope (optical, radio, infra-red, etc.) is conveyed by means of quanta. Furthermore, we all know that the vacuum state is not uniquely defined in a curved space-time [1]. What this means is that particles are continuously created as the Universe expands, and that those quanta coming from the outskirts of the Universe will also induce emission of other quanta (stimulated emission). Therefore, quanta coming from distant regions have undergone many interactions with the medium (vacuum-state) before they reach the telescope. Owing to all these interactions, the medium (vacuum) could be aptly termed 'quantum fog'. Therefore, after all, the situation is very similar to observing the landscape: very distant objects might well not be observable, and even closer ones if the 'quantum fog' is very intense. In order to give these vague ideas a precise meaning, we pose the question: How much information can be transmitted in a curved space-time? Recently we have been able to address this question in the framework of communication theory [2] and, in this essay, we shall briefly review the main results and explore some consequences.

For this purpose, let me introduce the (very) essentials of communication theory. As pointed out by Shannon [3], the communication process is completely specified by the joint probability, $p(m, n)$, of having m quanta detected at the observation site and n emitted at an event we wish to observe. From it we can compute the two *marginal* probability distributions: i) $p_i(n)$, that n quanta are emitted, by summing out m; ii) $p_o(m)$, that m quanta are detected, by summing out n. Let me emphasize that, from the informational-theoretic standpoint, the 'fog' is nothing but a pictorial representation of the noise, which is mathematically characterized by the *conditional* probability

$$p(m|n) \equiv \frac{p(m,n)}{p_i(n)} \tag{1}$$

of detecting m quanta, given that n were originally sent.

Shannon noted that the mutual entropy

$$H(o;i) \equiv \sum_m p(m,n) \ln \frac{p(m|n)}{p_o(m)} \tag{2}$$

quantifies the amount of information that, in principle, could be recovered from the output signal – even in the face of noise. Now, if we wish to know the maximum amount $H_{\max}$

of information that could be conveyed between the event and the observer, we have to maximize eq. (2). Shannon showed that the regime where information transmission is optimized can be reached (via some clever coding), but never be exceeded: any information sent in excess of $H_{\max}$ will inevitably *be degraded by noise* [3].

Setting the variational problem with the ansatz $p_o(m) = e^{-(\alpha+\beta m+B(m))}$, one shows that the mutual entropy is maximized whenever [2]

$$\sum_m B(m)p(m|n) = -\sum_m p(m|n)\ln p(m|n)\,, \tag{3}$$

in which case [2]

$$H_{\max} = \alpha + \beta\langle m\rangle\,. \tag{4}$$

In close analogy with statistical mechanics, α and β are determined via the normalization of probabilities and the knowledge of the mean number of quanta conditions. Observe that we still have to specify the gravitational noise $p(m|n)$. In a Friedman-Robertson-Walker (FRW) model, it was defined via the transition probability,

$$p(m|n) \equiv \left|{}_{\rm in}\langle 0_{-\vec{k}}, n_{\vec{k}}||m_{\vec{k}}, r_{-\vec{k}}\rangle_{\rm out}\right|^2\,. \tag{5}$$

Here $\vec{k}$ is the momentum of the observed quanta[1]. With the aid of the Bogolyubov transformation formalism [1], it was shown that the resulting conditional probability is a negative binomial distribution [2]:

$$p(m|n) = \binom{m}{n}(1-x)^{(n+1)}x^{(m-n)}\,, \tag{6}$$

where x is the noise intensity $x \equiv (|B|/|A|)^2 \leq 1$ (A and B are the Bogolyubov coefficients between 'in' and 'out' modes and satisfy $|A|^2 - |B|^2 = 1$). This distribution yields the following relation between the mean number of emitted and detected quanta

$$\langle m\rangle = \langle n\rangle + \frac{x}{1-x}\left(\langle n\rangle + 1\right), \tag{7}$$

which clearly entails both stimulated as well as spontaneous emission as discussed earlier.

Under the assumption that neither $\langle m\rangle$ nor $|B|$ are very large, we solved eq. (3) and, then, calculated $H_{\max}$ [eq. (4)] [2]:

$$H_{\max} = \ln(\langle m\rangle + 1) + \ln\left(\frac{\langle m\rangle + 1}{\langle m\rangle}\right) - \nu - \mu\langle m\rangle\,, \tag{8}$$

where

$$\nu = (4\ln 2)\,x^2 - x\ln x - (1-x)\ln(1-x) \tag{9}$$

and

$$\mu = (4\ln 2)\,x + \nu\,. \tag{10}$$

[1]Within a given 'number-of-particles budget', the most efficient way of conveying information is to emit all these particles towards the observer and none in the opposite direction.

This formula represents an upper bound on the amount of information that could be transferred, in principle, in a curved space-time. It is instructive to plot $H_{\max}$ against x for pulses containing, say, $\langle m \rangle = 5$ quanta. Observe in fig. 1 that the amount of information which is degraded by the noise increases very fast with x. Inspection of this figure shows an unphysical tail at $x \approx 0.8$ where $H_{\max} < 0$. This feature is a consequence of the adopted approximation. Indeed, we have argued [2] that $H_{\max}$ must vanish as $x \to 1$, because the quantity that measures the intensity of noise n emitted quanta are subjected to, $N(n) \equiv -\sum_m p(m|n) \ln p(m|n)$, blows up in this limit. This means that,as $x \to 1$, no information can be optically transferred ($H_{\max} \to 0$). For concreteness, let us assume that our information carriers are pions travelling through a FRW Universe with some anisotropy. The fact that we are considering scalar particles instead of photons (which is the way we really observe events in our Universe) is just a matter of convenience and does not change our conclusions. The pion field satisfies the equation.

$$\left[\Box - \xi R - m^2\right] \phi(x) = 0 \,. \tag{11}$$

The appropriate metric is

$$ds^2 = -dt^2 + a^2(t)(\delta_{ij} + h_{ij}(t))dx^i dx^j, \tag{12}$$

where it is assumed that $h_{ij}(t) << 1$. It is well known that in a conformally trivial configuration ($\xi = 1/6, h_{ij} = 0, m = 0$) the Bogolyubov coefficient $|B|$ between 'in' and 'out' modes vanishes. In this case no degradation of the information occurs, which confirms a long-standing conjecture that the source of the degradation of the information is related to parametric oscillation [4,5]. For conformally invariant fields ($m = 0, \xi = 1/6$) information jamming must be somehow related to Weyl's tensor, since it provides a natural measure of departure from conformal triviality. As a matter of fact, using the same technique as that considered by Birrel and Davies [1], one can show that, up to quadratic order in h_{ij}, the Bogolyubov coefficient can be expressed as the convolution

$$|B(\omega)|^2 = \int C^{\alpha\beta\gamma\delta}(\eta) C_{\alpha'\beta'\gamma'\delta'}(\eta') K^{\alpha'\beta'\gamma'\delta'}_{\alpha\beta\gamma\delta}(\omega, \eta, \eta') d\eta d\eta', \tag{13}$$

where K is the kernel

$$K^{\alpha\beta\gamma\delta}_{\alpha'\beta'\gamma'\delta'}(\eta, \eta') = \frac{1}{(4\omega)^2} e^{i\omega(\eta-\eta')} a^2(\eta) a^2(\eta') n^\alpha(\eta) n^\gamma(\eta) \hat{k}^\beta \hat{k}^\delta n_{\alpha'}(\eta') n_{\gamma'}(\eta') \hat{k}_{\beta'} \hat{k}_{\delta'} \tag{14}$$

where n^α is the orthonormal vector to the space-like hypersurfaces and $\hat{k}$ is the normal vector pointing in the line-of-sight direction (η is the conformal time). Of course, if the Weyl tensor grows, higher-order convolutions of Weyl tensors with this kernel must be considered.

Owing to the fact that an integral (in the conformal time) between the observation and emission instants is involved in eq. (13), the noise parameter depends on the 'separation' between the observer and the object to be observed. This means that the situation here is very akin to the ordinary fog: if during its journey towards us the pulse travelled through regions where the Weyl tensor was not negligible, then a considerable fraction of information about distant events is washed out and, the larger the distance, the more information is degraded. Furthermore, if the quanta had to travel through a region where the Weyl tensor is very large, then the optical observation of even closer events is precluded[2]. In the extreme case where the pulse travels through a region where the Weyl tensor diverges, the noise parameter $x \to 1$ and, consequently, $H_{\max} \to 0$.

This result has two important consequences. The first is related to the cosmic-censorship conjecture [6,7]. One of the most important questions in general relativity is whether the collapse of matter satisfying the standard energy conditions could yield as a final state a singularity which is not shielded by an event horizon; to put it another way, a singularity that could be seen by an observer at infinity. Such a horrible occurrence would create many theoretical paradoxes, such as the breakdown of predictability. Many years ago, Penrose conjectured that naked singularities shall not exist for physically reasonable situations. Unfortunately, this conjecture has been contradicted by many counter-examples [8]-[12]. I have sketched the situation in fig. 2. Suppose that a time-like singularity S develops at time t_0, and $\Sigma(t)$ represents a family of space-like hypersurfaces. Let us see what happens at point A. The singularity does not intersect the past light cone $I^-(A)$: it lies outside the observer's particle horizon and A cannot be influenced by S. The physics, as observed at A, is well-behaved. The situation is entirely different for an observer sitting at B: his past light cone $I^-(B)$ is intersected by the singularity ($I^-(B) \cap S \neq \emptyset$). Since the Cauchy problem is not well posed in the region $I^+(S)$, observers could actually notice a breakdown of predictability by detecting quanta coming from the singular region. Now, see how this horrible situation is cured by quantum effects: in the vicinity of the singularity, the Weyl tensor is very large (actually diverges) and a very intense quantum-noise is produced ($x \to 1$), in which case $H_{\max} \to 1$. Therefore, the singularity is hidden inside a very intense cloud of quantum fog and cannot be observed optically. So, it seems that we cannot learn anything about the singularity either or notice the breakdown of predictability. However, there are observables whose information is not conveyed by quanta and that are detected either via the Gauss theorem (say the electric charge, the angular momentum or the mass) or via a Bohm-Aharonov experiment (topological quantities). This fact raises the intriguing possibility that, after all, the 'no-hair' theorem [13,14], which was originally conceived for black holes, is more fundamental than earlier thought: it might well apply to naked singularities!! Regarding the cosmological singularity, in a semi-classical approach to quantum gravity, it would be natural to replace the Weyl tensors appearing in eq. (13) by the (quantum) correlation

[2]Eq. (13) was derived under the assumption that space-time is homogeneous. However, if the scale at which the Weyl tensor changes in space is much larger than the size of the pulse, eq. (13) should yield a good approximation for non-homogeneous space-time, provided the Weyl tensor is evaluated along the pulse world-line.

function $\langle C^{\alpha\beta\gamma\delta}(\eta)C_{\alpha\beta\gamma\delta}(\eta')\rangle$. As the initial singularity is approached, this correlation function ought to be large, since large (quantum) fluctuations away from the FRW geometry are expected. Therefore, if this semi-classical approach could be trusted, a very intense 'cloud' of noise is produced near the cosmological singularity and, consequently, no imprints of the initial conditions of the Universe can be observed.

A second application of this result is the time-arrow problem [7]. As we have already seen, information is degraded as the Weyl tensor grows. Although entropy and information are not equivalent entities, they are normally closely related and we even tend to regard entropy as the amount of missing information. Therefore, we expect that the issue of degradation of information during the cosmological evolution might provide some clue on the second law of thermodynamics. Indeed, it is hard to conceive that information could be upgraded as the entropy grows (or the other way around). If one could conceive a gedanken experiment supporting this intuitive feeling, this would confirm Penrose's [7] long-standing conjecture that the second law and the gravitational (clumpiness as measured through the Weyl tensor) time-arrows should point in the same direction.

These results are the latest developments of the very fruitful interplay between information theory and general relativity.

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References

[1] Birrel, N.D. and Davies, P.C. (1982), *Quantum Fields in Curved Spaces* (Cambridge University Press).

[2] Schiffer, M. (1991), preprint CERN TH-6310/91.

[3] Shannon, C. and Weaver, W. (1949), *The Mathematical Theory of Communication* (Univ. of Illinois Press).

[4] Hu, B.L. (1983), Phys. Lett. **A97**, 368.

[5] Hu, B. L. and Pavon, D. (1986), Phys. Lett. **B180**, 329.

[6] Penrose, R. (1969) Riv. Nuovo Cimento **1**, 252.

[7] Penrose, R. (1979), 'Singularities and Time-asymmetry' in *General Relativity: An Einstein Centenary Survey* eds: ,, (S. W. Hawking and W. Israel).Cambridge University Press

[8] Eardley, D.M. and Smarr, L. (1979), Phys. Rev. **D19**, 2239.

[9] Christodoulou, D. (1984), Commun. Math. Phys. **93**, 171.

[10] Newman, R.P.A.C. (1986), Class. Quant. Grav. **3**, 527.

[11] Hiscock, W. A. and Williams, L. G. (1986), Phys. Rev. **D34**, 2978.

[12] Ori, A. and Piran, T. (1987), Phys. Rev. Lett. **59**, 2137.

[13] Bekenstein, J.D. (1972), Phys. Rev. **D5**, 1239.

[14] Teitelboim, C. (1972), Phys. Rev. **D5**, 2941.

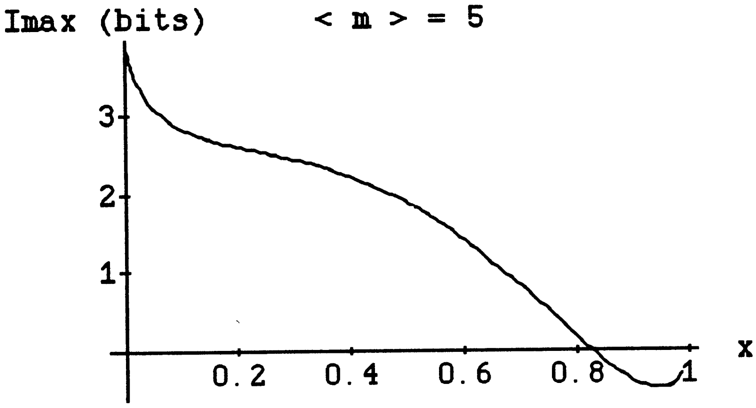

Figure 1: Here we plotted the maximum amount of information (in bits) that could be conveyed by pulses with a mean number of five photons. Observe that I_{max} decreases very fast with increasing x. Unfortunately, owing to our approximation, an unphysical negative tail develops at $x \approx 0.8$.

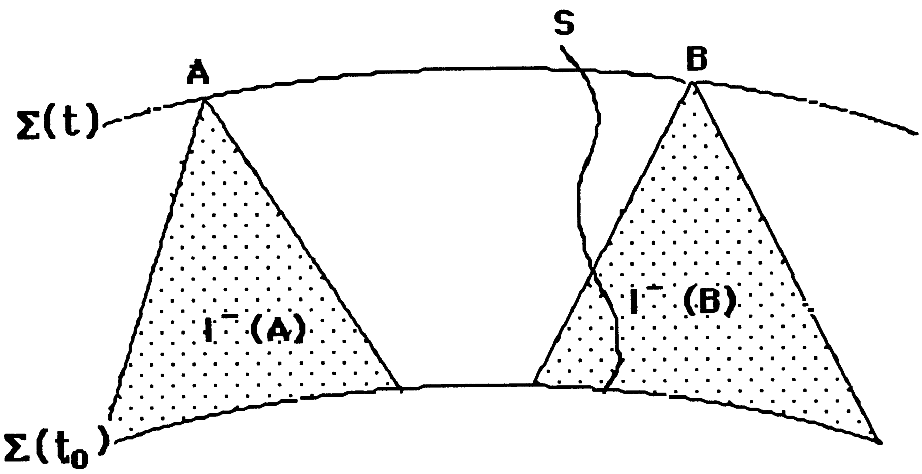

Figure 2: The observer A will not notice anything particularly strange because the singularity S is outside his particle horizon. The situation is rather different for B which, in principle, could observe the singularity and even notice a breakdown of predictability. The problem is resolved quantum mechanically: the singularity is surrounded by a 'cloud' of very intense noise, which prevents any information from being transferred from the singular region to an observer.