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Measurements of total production cross sections for
 $\pi^+ + \text{C}$, $\pi^+ + \text{Al}$, $K^+ + \text{C}$, and $K^+ + \text{Al}$ at 60 GeV/c and
 $\pi^+ + \text{C}$ and $\pi^+ + \text{Al}$ at 31 GeV/c

The NA61/SHINE Collaboration

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58 This paper presents several measurements of total production cross sections and total inelastic cross sections
59 for the following reactions: π^+ +C, π^+ +Al, K^+ +C, K^+ +Al at 60 GeV/c, π^+ +C and π^+ +Al at 31 GeV/c.
60 The measurements were made using the NA61/SHINE spectrometer at the CERN SPS. Comparisons
61 with previous measurements are given and good agreement is seen. These interaction cross sections
62 measurements are a key ingredient for neutrino flux prediction from the reinteractions of secondary hadrons
63 in current and future accelerator-based long-baseline neutrino experiments.

64 1 Introduction

65 The NA61 or SPS Heavy Ion and Neutrino Experiment (SHINE) [1] at the CERN Super Proton Synchrotron
66 (SPS) has a broad physics program that includes heavy ion physics, cosmic ray physics, and neutrino
67 physics. Long-baseline neutrino beams are typically initiated by high-energy protons that strike a long
68 target, yielding hadrons that can decay to neutrinos or can reinteract in the target or in the aluminum
69 focussing horns, potentially producing additional neutrino-yielding hadrons. NA61/SHINE has already
70 been very successful at measuring the yields of secondary hadrons generated by 31 GeV/c protons on carbon
71 targets [2, 3] for the Tokai-to-Kamioka (T2K) long-baseline neutrino oscillation experiment [4]. Data at
72 higher energies are now being collected to benefit other neutrino experiments, particularly MINERνA [5],
73 NOνA [6] that use the current NuMI neutrino beamline at Fermilab, and the proposed DUNE experiment [7]
74 which will use the planned LBNF beamline. The NuMI beamline is initiated by 120 GeV/c protons on a
75 carbon target, while LBNF will use 60-120 GeV/c protons on a carbon or beryllium target.

76 During the fall of 2015, NA61/SHINE recorded interactions of positively charged protons, pions, and kaons
77 on thin carbon and aluminum targets. In the case of pions, interactions were recorded at beam momenta of
78 31 GeV/c and 60 GeV/c. Kaons were recorded with a beam momentum of 60 GeV/c only, and protons at
79 31 GeV/c only. The NA61/SHINE vertex magnets were not operational during this period. Therefore, final
80 state particles could not be identified and spectral measurements could not be extracted from this data run.
81 As a result of this setup, data-taking was optimized for making measurements of the total production and
82 total inelastic cross sections for each interaction.

83 The total cross section of hadron-nucleus interactions σ_{tot} can be defined in terms of the inelastic σ_{inel} and
84 coherent elastic σ_{el} cross sections:

$$\sigma_{\text{tot}} = \sigma_{\text{inel}} + \sigma_{\text{el}}. \quad (1)$$

85 The inelastic cross section σ_{inel} is defined as the sum of all processes due to strong interactions except
86 coherent nuclear elastic scattering. The production processes are defined as those in which new hadrons are
87 produced. The inelastic processes additionally include interactions which only result in the disintegration of
88 the target nucleus (quasi-elastic interactions). Taking into account quasi-elastic scattering as a subset of the
89 inelastic scattering process, one can define the production cross section σ_{prod} in terms of the quasi-elastic
90 cross section σ_{qe} as:

$$\sigma_{\text{prod}} = \sigma_{\text{inel}} - \sigma_{\text{qe}}. \quad (2)$$

91 This paper is organized as follows: Section 2 describes the experimental apparatus. Section 3 presents the
92 event selection to ensure the quality of the measurements. Section 4 presents the procedure for measuring
93 σ_{inel} and σ_{prod} cross sections. Section 5 describes the corrections to the raw trigger probability. Section 6
94 discusses systematic uncertainties. The final results and discussion are presented in Sections 7 and 8.

95 2 Experimental setup, Beams, and Data Collected

96 NA61/SHINE receives a secondary hadron beam from the 400 GeV/c SPS proton beam. The primary
97 proton beam strikes a beryllium target 535 m upstream generating the secondary beam. A magnet system
98 is then used to select the desired beam momentum. Unwanted positrons and electrons are absorbed by two
99 4 mm lead absorbers.

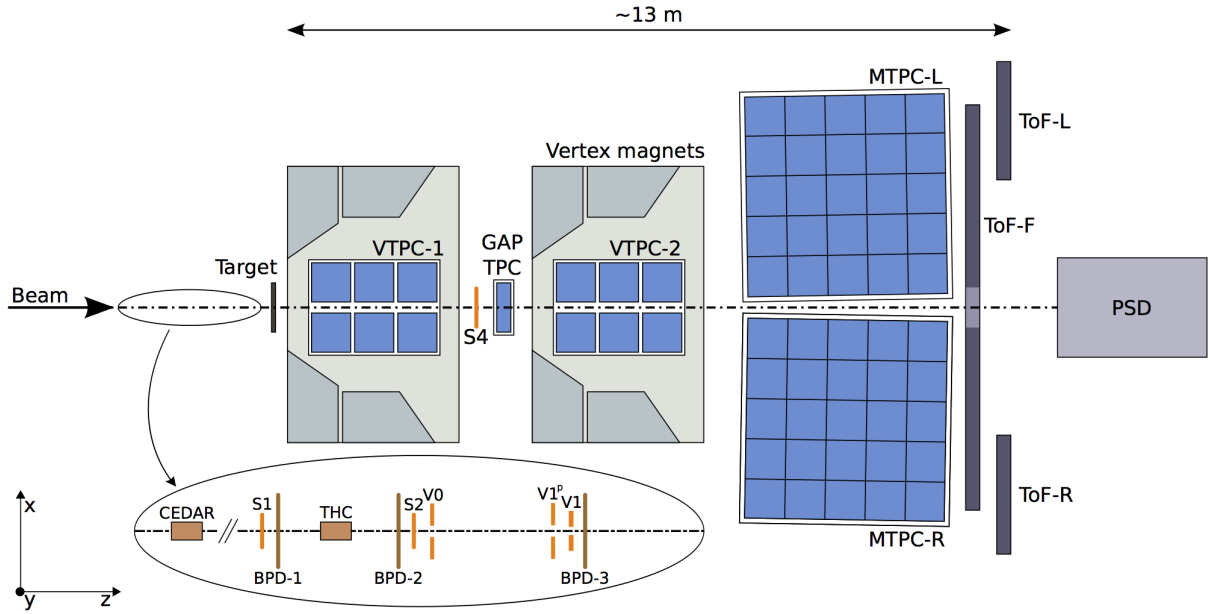


Figure 1: The schematic top-view layout of the NA61/SHINE experiment in the configuration used during the 2015 data-taking. The TOF-F was not installed for the data collected for this analysis.

100 The NA61/SHINE detector [1] is shown in Figure 1. In standard operation, it comprises four large Time
 101 Projection Chambers (TPCs) and a Time of Flight (ToF) system allowing NA61/SHINE to make spectral
 102 measurements of produced hadrons. Two of the TPCs, Vertex TPC 1 (VTPC-1) and Vertex TPC 2 (VTPC-2),
 103 are contained within superconducting magnets, capable of generating a combined maximum bending power
 104 of 9 T·m. However these magnets were not operational during the 2015 run presented here. Downstream
 105 of the VTPCs are the Main TPC Left (MTPC-L) and Main TPC Right (MTPC-R). Additionally, a smaller
 106 TPC, the Gap TPC (GTPC), is positioned along the beam axis between the two VTPCs. The forward
 107 Time-of-Flight (ToF-F) was not installed in 2015, but the two side ToF-Left and ToF-Right walls were
 108 present. The Projectile Spectator Detector (PSD), a forward hadron calorimeter, sits downstream of the
 109 ToF system.

110 The most critical systems for the analyses of the 2015 data presented here are the trigger system and the
 111 Beam Position Detectors (BPDs). The NA61/SHINE trigger system uses two scintillator counters (S1
 112 and S2) to trigger on beam particles. The S1 counter provides the start time for all counters. Three veto
 113 scintillation counters ($V0$, $V1$ and $V1^p$) each with a 1 cm diameter hole are used to remove divergent beam
 114 particles upstream of the target. The S4 scintillator with a 1 cm radius sits downstream of the target and
 115 is used to determine whether or not an interaction has occurred. A Cherenkov Differential Counter with
 116 Achromatic Ring Focus (CEDAR) [8, 9] and a threshold Cherenkov counter (THC) select beam particles of
 117 the desired species. The CEDAR focusses the Cherenkov ring from a beam particle onto a ring of 8 PMTs.
 118 The pressure is adjusted so that only particles of the desired species will trigger the PMTs, and typically a
 119 coincidence of at least 6 PMTs is required to tag a particle for the trigger. Pressure scans of the CEDARs
 120 are shown in Figure 2. For these 2015 data at 31 GeV/c the beam was composed of approximately 87%
 121 pions, 11% protons, and 2% kaons. At 60 GeV/c the beam was composed of approximately 74% pions,
 122 23% protons, and 3% kaons.

123 The beam particles are selected by defining the beam trigger (T_{beam}) as the coincidence of $S1 \wedge S2 \wedge \overline{V0} \wedge$

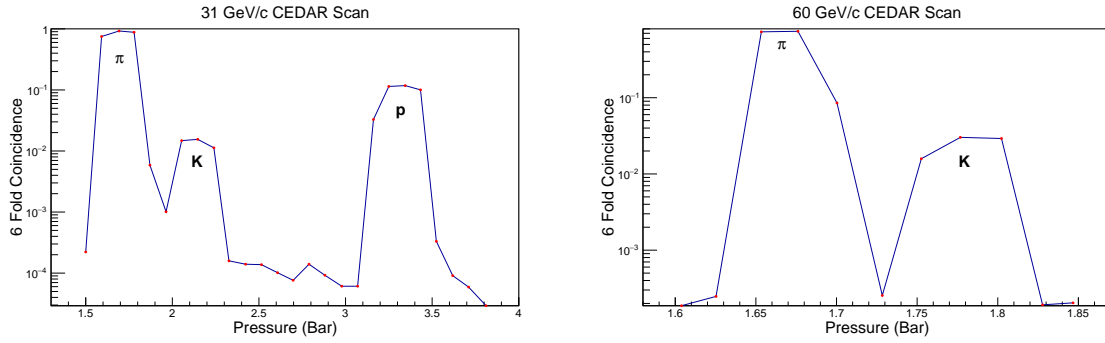


Figure 2: CEDAR pressure scans for the 31 GeV/c beam (*left*) and the 60 GeV/c beam (*right*). The vertical axis shows the fraction of beam particles that fires at least 6 of the 8 CEDAR PMTs.

124 $\overline{V1} \wedge \overline{V1^p} \wedge \overline{CEDAR} \wedge \overline{THC}$. The interaction trigger (T_{int}) is defined by the coincidence of $T_{\text{beam}} \wedge \overline{S4}$
 125 to select beam particles which have interacted with the target. A correction factor will be discussed in
 126 detail in Section 5.1 to correct for interactions that hit the S4. Three BPDs, which are proportional wire
 127 chambers, are located 30.39 m, 9.09 m, and 0.89 m upstream of the target and determine the location of the
 128 incident beam particle to an accuracy of $\sim 100 \mu\text{m}$.

129 For these 2015 data, the interactions of p , π^+ , and K^+ beams were measured on thin carbon and aluminum
 130 targets. The carbon target was composed of graphite of density $\rho = 1.84 \text{ g/cm}^3$ with dimensions of 25 mm
 131 (W) x 25 mm (H) x 20 mm (L), corresponding to roughly 4% of a proton-nuclear interaction length. The
 132 aluminum target has a density of $\rho = 2.70 \text{ g/cm}^3$ with dimensions of 25 mm (W) x 25 mm (H) x 14.8 mm
 133 (L), corresponding to roughly 3.6% of a proton-nuclear interaction length.

134 3 Analysis Procedure

135 3.1 Event selection

136 Several cuts were applied to events to ensure the purity of the measurement and to control the systematic
 137 effects caused by beam divergence. First, the so-called WFA (Wave Form Analyzer) cut was used to
 138 remove events in which multiple beam particles pass through the beam line in a small time frame. The
 139 WFA determines the timing of beam particles that pass through the S1 scintillator. If another beam particle
 140 passes through the beam line close in time to the triggered beam particle, it could cause a false trigger in
 141 the S4 scintillator. In order to mitigate this effect, a conservative cut of $\pm 2 \mu\text{s}$ was applied to the time
 142 window to ensure that only one particle is allowed to pass through the S1 in a $4 \mu\text{s}$ time window around the
 143 selected beam particle.

144 The trajectories of the incoming beam particles are measured by three BPDs, located along the beamline
 145 upstream of the target as shown in Figure 1. The measurements from the BPDs are especially important
 146 for estimating the effects of beam divergence on the cross section measurements. To understand these
 147 effects, tracks are fitted to the reconstructed BPD clusters, and the tracks are extrapolated to the S4 plane.
 148 The so-called ‘‘Good BPD’’ cut requires that the event includes a cluster in the most-downstream BPD
 149 and that a track was successfully fit to the BPDs. Figures 3 and 4 show the resulting BPD extrapolation
 150 to the S4 plane for the interactions studied. It can be seen from these figures that the 31 GeV/c beams

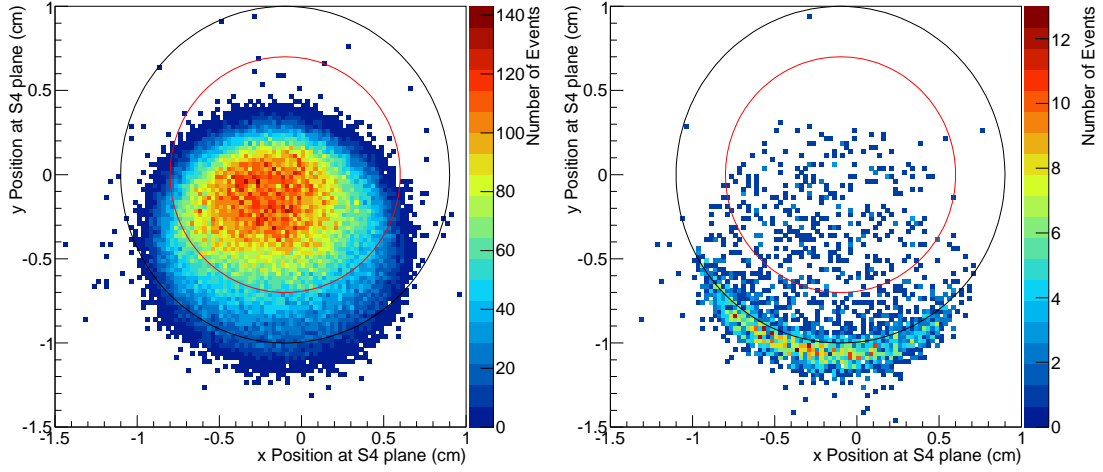


Figure 3: Positions of BPD tracks extrapolated to the S4 plane in Target Removed data runs from the π^+ + C at 31 GeV/c dataset. The measured S4 position is shown as a black circle and the BPD radius cut is shown as a red circle in both figures. (*Left*) Events taken by the beam trigger. (*Right*) Events taken by the interaction trigger.

151 were much wider than the 60 GeV/c beams. From these figures, it is also evident that the V1 veto counter
 152 (which is close to the most downstream BPD) and S4 were not well-aligned. The beam was wide enough
 153 that a significant fraction of the beam particles have trajectories missing the S4. This leads to an apparent
 154 interaction rate higher than the actual interaction rate. To reduce this effect, a radial cut was applied to the
 155 BPD tracks extrapolated to the S4, and this is indicated by the red circles on Figures 3 and 4.

156 The number of events after the described selection cuts for the interactions: 60 GeV/c K^+ and π^+ and
 157 31 GeV/c π^+ with C and Al targets (Target Inserted) and with the targets removed (Target Removed) are
 158 shown in Tables [1 - 3].

Interaction	π^+ + C		π^+ + Al	
	Inserted	Removed	Inserted	Removed
Total	593,176	195,492	534,813	234,302
WFA	591,414	194,969	531,785	233,056
Good BPD	547,297	180,315	491,019	215,181
Radial cut	437,373	142,790	367,240	158,872

Table 1: Event selection table for π^+ + C and π^+ + Al at 31 GeV/c.

159 4 Interaction trigger cross sections

160 In general, the probability of a beam particle interaction inside of a thin target is proportional to the thickness
 161 L of the target and the number density of the target nuclei n . Thus, the interaction probability P can be
 162 defined by taking into account the thin target approximation and by defining the interaction cross section σ

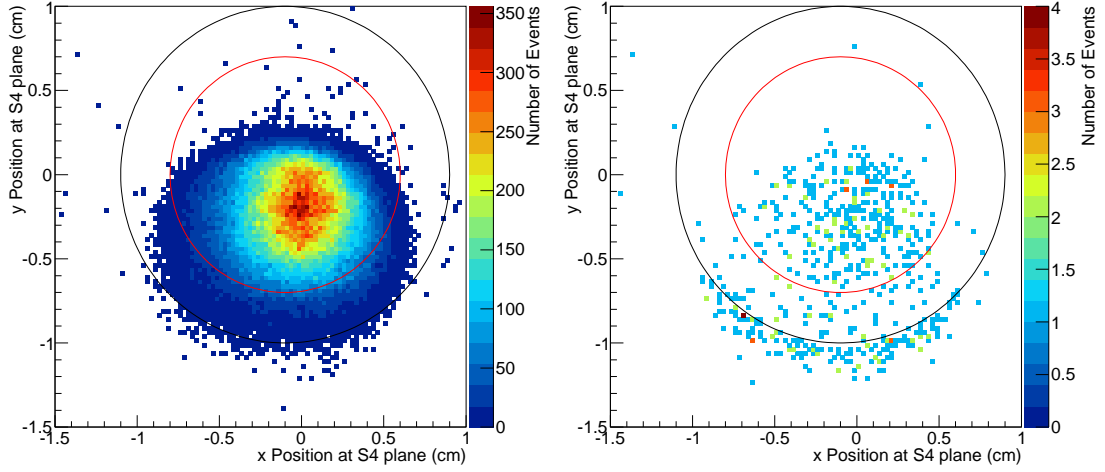


Figure 4: Positions of BPD tracks extrapolated to the S4 plane in Target Removed data runs from the π^+ + C at 60 GeV/c dataset. The measured S4 position is shown as a black circle and the BPD radius cut is shown as a red circle in both figures. (*Left*) Events taken by the beam trigger. (*Right*) Events taken by the interaction trigger.

Interaction	π^+ + C		π^+ + Al	
	Inserted	Removed	Inserted	Removed
Total	528,086	246,902	458,800	285,721
WFA	513,449	240,438	447,793	279,031
Good BPD	479,199	224,512	417,369	260,163
Radial cut	462,912	217,080	405,379	252,237

Table 2: Event selection table for π^+ + C and π^+ + Al at 60 GeV/c.

Interaction	K^+ + C		K^+ + Al	
	Inserted	Removed	Inserted	Removed
Total	505,525	239,145	338,987	155,796
WFA	503,110	238,024	337,309	155,035
Good BPD	465,832	220,703	312,418	143,502
Radial cut	462,544	218,946	310,482	142,625

Table 3: Event selection table for K^+ + C and K^+ + Al at 60 GeV/c.

163 as:

$$P = \frac{\text{Number of events}}{\text{Number of beam particles}} = n \cdot L \cdot \sigma. \quad (3)$$

164 The density of nuclei n can be calculated in terms of N_A , ρ , and A , which are Avogadro's number, the
 165 material density, and the atomic number, respectively.

166 The counts of beam and interaction triggers as described in Sec. 2 can be used to estimate the trigger

167 probability as follows:

$$P_{\text{Tint}} = \frac{N(T_{\text{beam}} \wedge T_{\text{int}})}{N(T_{\text{beam}})}, \quad (4)$$

168 where $N(T_{\text{beam}})$ is the number of beam events passing the event selection cuts and $N(T_{\text{beam}} \wedge T_{\text{int}})$ is the
 169 number of selected beam events which also have an interaction trigger. In order to correct for events where
 170 the beam particle interacts outside of the target, data were also taken with the target removed from the
 171 beam (Target Removed). Figure 5 shows an example of the trigger interaction probabilities for each run
 172 for the $\pi^+ + \text{C}$ at 60 GeV/c dataset. Table 4 gives the total trigger interaction probabilities for the data
 173 sets used in this paper for both the Target Inserted and Target Removed data. The kaon target removed
 174 interaction probabilities are larger than those for pions due to the fact that $\sim 1\%$ of the beam kaons will
 175 decay between BPD 3 and S4.

176 Taking into account the trigger probabilities with the target inserted (I) and the target removed (R), $P_{\text{Tint}}^{\text{I}}$
 177 and $P_{\text{Tint}}^{\text{R}}$, the interaction probability P_{int} can be obtained:

$$P_{\text{int}} = \frac{P_{\text{Tint}}^{\text{I}} - P_{\text{Tint}}^{\text{R}}}{1 - P_{\text{Tint}}^{\text{R}}}. \quad (5)$$

178 Equation 3 leads to the definition of the trigger cross section σ_{trig} , by using P_{int} and the effective target
 179 length L_{eff} , which accounts for the exponential beam attenuation:

$$\sigma_{\text{trig}} = \frac{A}{\rho L_{\text{eff}} N_{\text{A}}} \cdot P_{\text{int}}. \quad (6)$$

180 The effective target length can be calculated using the absorption length,

$$L_{\text{eff}} = \lambda_{\text{abs}}(1 - e^{-L/\lambda_{\text{abs}}}), \quad (7)$$

181 with

$$\lambda_{\text{abs}} = A/(\rho N_{\text{A}} \sigma_{\text{trig}}). \quad (8)$$

182 By simplifying Equations 6, 7, and 8, one can obtain σ_{trig} as

$$\sigma_{\text{trig}} = -\frac{A}{\rho L N_{\text{A}}} \ln(1 - P_{\text{int}}). \quad (9)$$

183 5 Correction factors

184 5.1 S4 trigger correction factors

185 The trigger cross section contains the interactions where the resulting particles miss the S4 scintillator
 186 counter that is downstream of the target. But even when there has been a production or quasi-elastic
 187 interaction in the target, there is a possibility that a forward-going particle will strike the S4 counter.
 188 Moreover, not all elastically scattered beam particles strike the S4. Corrections must be applied to the

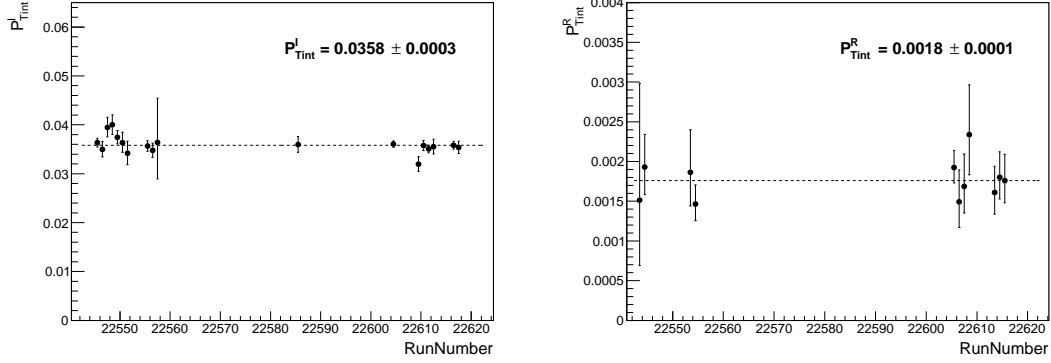


Figure 5: Trigger interaction probabilities for $\pi^+ + C$ at 60 GeV/c dataset. (*Left*) Target Inserted dataset. (*Right*) Target Removed dataset.

Interaction	p (GeV/c)	P_{Tint}^I	P_{Tint}^R
$\pi^+ + C$	31	0.0407 ± 0.0003	0.0025 ± 0.0001
$\pi^+ + Al$	31	0.0391 ± 0.0003	0.0029 ± 0.0001
$\pi^+ + C$	60	0.0358 ± 0.0003	0.0018 ± 0.0001
$\pi^+ + Al$	60	0.0320 ± 0.0003	0.0018 ± 0.0001
$K^+ + C$	60	0.0394 ± 0.0003	0.0103 ± 0.0002
$K^+ + Al$	60	0.0373 ± 0.0004	0.0103 ± 0.0003

Table 4: Trigger Interaction probabilities in data. For each configuration, the observed probabilities for Target Inserted and Target Removed data are given.

189 trigger cross section to account for these effects. Combining Equations 1 and 2, the trigger cross section
 190 can be related to the production cross section through Monte Carlo (MC) correction factors as follows:

$$\sigma_{trig} = \sigma_{prod} \cdot f_{prod} + \sigma_{qe} \cdot f_{qe} + \sigma_{el} \cdot f_{el} , \quad (10)$$

191 where f_{prod} , f_{qe} , and f_{el} are the fractions of production, quasi-elastic, and elastic events that miss the S4
 192 counter. σ_{qe} and σ_{el} are also estimated from Monte Carlo. Equation 10 can be rewritten to obtain σ_{prod}
 193 and σ_{inel} as:

$$\sigma_{prod} = \frac{1}{f_{prod}} (\sigma_{trig} - \sigma_{qe} \cdot f_{qe} - \sigma_{el} \cdot f_{el}) \quad (11)$$

194 and

$$\sigma_{inel} = \frac{1}{f_{inel}} (\sigma_{trig} - \sigma_{el} \cdot f_{el}). \quad (12)$$

195 A GEANT4 detector simulation [10, 11, 12] was used to estimate the MC correction factors discussed
 196 above. The FTFP_BERT physics list with GEANT4 version of 10.2.p03 was used to estimate correction
 197 factors as presented in Table 5.

Interaction	p (GeV/c)	Monte Carlo Correction Factors					
		σ_{el} (mb)	f_{el}	σ_{qe} (mb)	f_{qe}	f_{prod}	f_{inel}
$\pi^+ + \text{C}$	31	55.5	0.734	18.8	0.946	0.989	0.985
$\pi^+ + \text{Al}$	31	114.5	0.745	29.7	0.949	0.990	0.987
$\pi^+ + \text{C}$	60	54.0	0.289	16.4	0.811	0.967	0.952
$\pi^+ + \text{Al}$	60	110.0	0.232	25.7	0.814	0.969	0.956
$K^+ + \text{C}$	60	18.1	0.323	14.5	0.821	0.990	0.975
$K^+ + \text{Al}$	60	44.6	0.183	23.5	0.821	0.990	0.997

Table 5: Monte Carlo correction factors.

198 5.2 Beam composition correction factors

199 In the case of π^+ beams, a correction must also be applied for the beam composition. This is because the
200 CEDAR and threshold Cherenkov detectors do not have the power to completely discriminate positrons
201 from pions at 31 GeV/c and 60 GeV/c as shown in [8, 9]. This problem is worse for 60 GeV/c. Fortunately,
202 it was possible to estimate the amount of positron contamination with special maximum field runs and
203 with the PSD. During the neutrino data-taking in 2016, a special maximum field data run was taken. The
204 magnets were set to the 9-m field setting such that the 60 GeV/c beam was bent into the MTPC-L. Both
205 PSD and dE/dx data were recorded. Data were also taken with upstream lead absorbers in and out of the
206 beam leading to different levels of positron contamination.

207 The PSD is usually used as a hadron calorimeter for heavy ion interactions, but it can also be used to help
208 discriminate between low mass hadrons and positrons. The electromagnetic radiation length of positrons
209 is much smaller than the hadronic radiation length of pions at 31 GeV/c and 60 GeV/c. Therefore, the
210 positrons tend to deposit all of their energy in the first two sections (longitudinal layers) of the PSD, while
211 pions penetrate deeper into the PSD calorimeter. By only selecting beam particles that penetrate deep
212 into the PSD, a pure pion sample is obtained. This sample is used to determine the parameters of the pion
213 dE/dx gaussian distribution μ_π and σ_π .

214 To determine the positron and π^+ compositions of the beam, a sum of two gaussians is fit to the dE/dx
215 data. The distance between the positron and pion means and the ratio of the positron and pion spread are
216 determined from a Bethe Bloch model. Therefore, only the amplitudes of the pion and positron distributions
217 are allowed to float. The positron contamination was determined to be $2\% \pm 2\%$ for the 60 GeV/c beam.
218 Figure 6 shows the resulting fit to the maximum field data.

219 Finally, the effect of the positrons on the trigger cross section must be estimated. The same GEANT4 MC
220 simulation is used to determine this effect. Positrons were simulated with a carbon target, an aluminum
221 target and with the targets removed to determine the $P_{\text{Tint}}^{\text{I}}$ and $P_{\text{Tint}}^{\text{R}}$ rates. In the case of 60 GeV/c pion
222 interactions, a correction is applied to the measured values of $P_{\text{Tint}}^{\text{I}}$ and $P_{\text{Tint}}^{\text{R}}$:

$$P_{\text{Tint}}^{\text{corr}} = (P_{\text{Tint}} - P_e \cdot f_e) / f_\pi \quad (\text{Target I, R}), \quad (13)$$

223 where $f_e = 0.02$ and $f_\pi = 0.98$. The resulting corrections applied to σ_{prod} (σ_{inel}) are $+2.2\%$ ($+2.1\%$) for
224 $\pi^+ + \text{C}$ at 60 GeV/c and $+1.8\%$ (1.7%) for $\pi^+ + \text{Al}$ at 60 GeV/c.

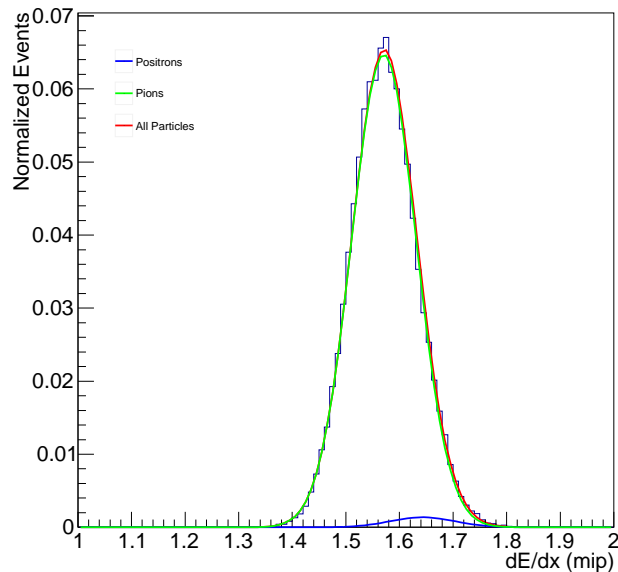


Figure 6: The binned data shows the dE/dx distribution of the maximum field dataset for the 60 GeV/c π^+ beam. Overlaid is the sum of gaussians fit to the histogram as well as the individual π^+ and e^+ components. From this fit, the positron contribution was estimated to be 2%.

225 In the case of 31 GeV/c, the potential for positron contamination was reduced by requiring that the CEDAR
 226 had a more stringent 7-fold coincidence signal. No special data run was undertaken with the 31 GeV/c
 227 beam to measure the positron contamination, so no correction is applied. But this contamination will be
 228 taken into account later as an asymmetric systematic uncertainty.

229 For the pion beams at both 31 GeV/c and 60 GeV/c, a small number of muons are also present in the beam
 230 due to the decays of pions upstream of the target, and the CEDAR cannot completely distinguish these
 231 from pions. Many of these muons will emerge at an angle and will strike the veto counters, but simulations
 232 at both momenta show the muon fraction that will pass the veto counters and trigger our beam counters is
 233 about $1.5 \pm 0.5\%$ of the pion beam. A correction for the muon component of the beam is applied to the
 234 31 GeV/c, and 60 GeV/c pion beam interactions.

235 For the kaon beam, any kaons that decay upstream of the CEDAR will not satisfy the beam selection and
 236 will not be selected as good beam particles. Only kaon decays downstream of the CEDAR where the
 237 decay products are headed towards the S4 will pass the beam selection and "Good BPD" cut. It has been
 238 estimated that only 0.1% of the CEDAR-tagged kaons will decay with decay products that pass these cuts.
 239 Therefore no correction is applied for kaon decays in the beamline.

240 6 Systematic uncertainties

241 6.1 Target density uncertainty

242 The uncertainty on the target density affects the calculation of the trigger cross section as shown in Equation 9.
243 The density uncertainty for each target is estimated by calculating the standard deviation of the target
244 densities determined from measurements of the mass and dimensions of the machined target samples. A
245 0.65% uncertainty on the density of carbon and a 0.29% uncertainty on the density of aluminum were
246 used. The uncertainties on the densities are then propagated to the uncertainties on the cross section results
247 for all of the interactions studied. The target density uncertainties are included in the breakdowns in the
248 systematic uncertainties for the production and inelastic cross sections presented in Tables 6 and 7.

249 6.2 Out-of-target interactions

250 As shown in Equation 5, the measured interaction rates are corrected for interactions occurring outside of
251 the target by measuring the trigger rates with the target both inserted and removed. Switching between
252 target “I” and “R” is achieved by moving the target holder out of the path of the beam. To look for possible
253 additional systematic effects, during the 2015 data-taking two special runs were undertaken as a cross-check.
254 These data were taken with the target holder in the “I” position and with the target holder in the “R” position,
255 but with no target attached. The data were taken with 31 GeV/c and 60 GeV/c π^+ . With no additional
256 out-of-target effects, the target holder data (both the “I” and “R” runs) should exhibit the same trigger
257 probability as the target removed data.

258 In the case of the 31 GeV/c target holder data, there was no significant difference between the trigger
259 probability of the empty target holder data and the target removed data. However, in the case of the
260 60 GeV/c data run, a high trigger probability in the target holder “I” run was observed. These out-of-target
261 interactions may be related to the beam conditions during those runs. An asymmetric uncertainty was
262 assigned for the 60 GeV/c interactions. These uncertainties are included in the breakdowns of the systematic
263 uncertainties for the production and inelastic cross sections presented in Tables 6 and 7.

264 6.3 S4 size uncertainty

265 Another systematic uncertainty comes from the uncertainty in the size of the S4 scintillator. The diameter
266 of the S4 was measured with calipers to be $D_{S4} = 20.06 \pm 0.40$ mm.

267 In order to propagate this uncertainty to σ_{inel} and σ_{prod} , two additional MC simulation samples with the S4
268 diameter modified to be 2.04 and 1.96 cm were generated. After obtaining the new S4 correction factors
269 f_{inel} , f_{prod} , f_{qe} , and f_{el} , σ_{inel} and σ_{prod} were recalculated. The maximum and minimum values of σ_{inel}
270 and σ_{prod} obtained from these MC simulation samples are taken as the upper and lower limits on the S4
271 size uncertainty. Uncertainties related to the S4 size are included in the breakdowns of the systematic
272 uncertainties for the production and inelastic cross sections presented in Tables 6 and 7.

273 6.4 S4 efficiency

274 The uncertainty on the S4 scintillator efficiency was estimated using Target Removed data. GTPC tracks
275 are extrapolated to the S4 plane and matched with beam tracks which pass the “Good BPD” requirement.
276 Then, the S4 inefficiency was obtained by calculating the trigger probability as defined in Eq. (4) for events
277 which have matched tracks. Previous NA61/SHINE analyses have found that S4 inefficiency is negligibly
278 small [13] and this analysis also found no S4 inefficiency. The S4 inefficiency is concluded to be less than
279 0.1% and neither an uncertainty nor a correction relating to the S4 scintillator efficiency is applied to the
280 results.

281 6.5 Beam composition uncertainty

282 As was mentioned in Section 5.2, for interactions with the 60 GeV/c π^+ beam, a correction was applied to
283 reflect the small amount of positrons in the beam. To be conservative, 100% of this correction is assumed
284 as a systematic uncertainty. For π^+ at 31 GeV/c, no correction is applied, but an uncertainty is reported
285 accounting for a 1% positron contamination. This results in an asymmetric uncertainty of $[+1.9, -0.0]$ mb
286 for $\pi^+ + C$ at 31 GeV/c and $[+2.7, -0.0]$ mb for $\pi^+ + Al$ at 31 GeV/c.

287 As was also mentioned in Section 5.2, the muon fraction in the pion beam is estimated to be 1.5% for both
288 the 31 GeV/c and 60 GeV/c π^+ beams and a correction was applied. An uncertainty of 0.5% is applied to
289 this correction.

290 The CEDAR counter has a high purity of identifying kaons using a 6-fold coincidence. Kaons are well-
291 separated from pions and protons. The lower limit on the purity of the kaon beam has been calculated to be
292 99.4% according to the CEDAR gas pressure scan data. The estimated systematic error from this source is
293 applied to the total systematic uncertainty.

294 Uncertainties related to uncertainty in the beam composition are summarized in the breakdowns of the
295 systematic uncertainties for the production and inelastic cross sections presented in Tables 6 and 7.

296 6.6 Model uncertainties

297 The S4 correction factors f_{prod} , f_{inel} , f_{el} and f_{qe} as well as the cross sections σ_{qe} and σ_{el} were estimated with
298 GEANT4 MC simulations using the FTFP_BERT physics list. In order to estimate the model uncertainties
299 associated with these correction factors, the correction factors were recalculated with three additional
300 physics lists: QBBC, QGSP_BERT and FTF_BIC. These physics lists use different underlying physics
301 models in GEANT4’s internal calculation of rates for different interaction processes. Using these additional
302 physics lists, the model dependency on the total cross section measurements was studied. For each physics
303 list, σ_{inel} and σ_{prod} is recalculated with the new correction factors. The maximum and minimum values of
304 σ_{inel} and σ_{prod} from the four physics lists are taken as the upper and lower limits to the model uncertainties
305 in the total cross section results.

306 These model uncertainties are presented along with the systematic uncertainties associated with the
307 production and inelastic cross sections in Tables 6 and 7.

Interaction	p (GeV/ c)	Systematic uncertainties for σ_{prod} (mb)					Total Syst. Uncer.	Model Uncer.
		Density	Out-of- target	S4 Size	Beam Purity	MC Stat.		
$\pi^+ + \text{C}$	31	± 1.4	–	$\pm_{0.7}^{0.9}$	$\pm_{1.1}^{2.3}$	± 0.3	$\pm_{2.0}^{2.8}$	$\pm_{0.4}^{1.1}$
$\pi^+ + \text{Al}$	31	± 1.2	–	$\pm_{1.8}^{1.8}$	$\pm_{2.2}^{3.5}$	± 0.6	$\pm_{3.1}^{4.2}$	$\pm_{0.6}^{3.9}$
$\pi^+ + \text{C}$	60	± 1.3	$\pm_{1.2}^{0.0}$	$\pm_{1.3}^{1.4}$	$\pm_{3.8}^{4.0}$	± 0.3	$\pm_{4.4}^{4.4}$	$\pm_{1.4}^{0.4}$
$\pi^+ + \text{Al}$	60	± 1.1	$\pm_{4.3}^{0.0}$	$\pm_{2.8}^{2.4}$	$\pm_{6.1}^{6.4}$	± 0.6	$\pm_{8.1}^{6.9}$	$\pm_{0.7}^{0.8}$
$K^+ + \text{C}$	60	± 0.8	± 0.6	$\pm_{0.3}^{0.3}$	$\pm_{0.3}^{0.3}$	± 0.1	$\pm_{1.1}^{1.1}$	$\pm_{2.9}^{0.2}$
$K^+ + \text{Al}$	60	± 1.1	± 1.2	$\pm_{0.5}^{0.5}$	$\pm_{0.5}^{0.5}$	± 0.1	$\pm_{1.8}^{1.8}$	$\pm_{4.1}^{0.1}$

Table 6: Breakdown of systematic uncertainties for production cross section measurements with the NA61/SHINE data.

Interaction	p (GeV/ c)	Systematic uncertainties for σ_{inel} (mb)					Total Syst. Uncer.	Model Uncer.
		Density	Out-of- target	S4 Size	Beam Purity	MC Stat.		
$\pi^+ + \text{C}$	31	± 1.4	–	$\pm_{0.7}^{0.9}$	$\pm_{1.1}^{2.3}$	± 0.3	$\pm_{2.0}^{2.8}$	$\pm_{0.4}^{1.2}$
$\pi^+ + \text{Al}$	31	± 1.2	–	$\pm_{1.8}^{1.8}$	$\pm_{2.2}^{3.6}$	± 0.6	$\pm_{3.2}^{4.2}$	$\pm_{0.6}^{4.0}$
$\pi^+ + \text{C}$	60	± 1.3	$\pm_{1.3}^{0.0}$	$\pm_{1.2}^{1.4}$	$\pm_{4.0}^{4.1}$	± 0.3	$\pm_{4.6}^{4.5}$	$\pm_{3.9}^{0.3}$
$\pi^+ + \text{Al}$	60	± 1.1	$\pm_{4.3}^{0.0}$	$\pm_{2.8}^{2.5}$	$\pm_{6.2}^{6.4}$	± 0.6	$\pm_{8.1}^{7.0}$	$\pm_{0.8}^{1.1}$
$K^+ + \text{C}$	60	± 0.8	± 0.6	$\pm_{0.4}^{0.3}$	$\pm_{0.3}^{0.3}$	± 0.1	$\pm_{1.1}^{1.1}$	$\pm_{2.3}^{0.1}$
$K^+ + \text{Al}$	60	± 1.1	± 1.2	$\pm_{0.5}^{0.6}$	$\pm_{0.5}^{0.5}$	± 0.1	$\pm_{1.8}^{1.8}$	$\pm_{3.1}^{0.1}$

Table 7: Breakdown of systematic uncertainties for inelastic cross section measurements with the NA61/SHINE data.

308 7 Results

309 Several production cross sections have been measured in this analysis: $\pi^+ + \text{C}$ ($\pi^+ + \text{Al}$) at 31 GeV/ c is
310 found to be 158.3 mb (310.4 mb), $\pi^+ + \text{C}$ ($\pi^+ + \text{Al}$) at 60 GeV/ c is found to be 171.6 mb (321.0 mb), and
311 $K^+ + \text{C}$ ($K^+ + \text{Al}$) at 60 GeV/ c is found to be 144.5 mb (284.0 mb), respectively. Statistical, systematic,
312 and physics model uncertainties are estimated separately and are summarized in Table 8. π^+ and K^+ at
313 60 GeV/ c measurements are compared with the results of Carrol et al. [14] as shown in Figure 7.

314 Several inelastic cross sections have also been determined in this analysis: $\pi^+ + \text{C}$ ($\pi^+ + \text{Al}$) at 31 GeV/ c
315 is found to be 177.0 mb (340.0 mb), $\pi^+ + \text{C}$ ($\pi^+ + \text{Al}$) at 60 GeV/ c is found to be 188.2 mb (347.0 mb), and
316 $K^+ + \text{C}$ ($K^+ + \text{Al}$) at 60 GeV/ c is found to be 159.0 mb (307.5 mb), respectively. Statistical, systematic, and
317 physics model uncertainties are estimated separately and are summarized in Table 9. These measurements
318 are compared with the results of Denisov et al. [15] as shown in Figure 8.

319 Additionally, a short data run of interactions of 31 GeV/ c protons with carbon was analyzed as a cross-check
320 with the previous higher statistics NA61/SHINE total cross section results from the 2009 T2K data run [2].
321 The total production (total inelastic) cross section was found to be 229.8 ± 4.4 mb (259.9 ± 4.5 mb) (statistical
322 uncertainty only). These are consistent with the 2009 result of 230.7 mb (258.4 mb).

Interaction	p (GeV/c)	Production cross section (mb)				
		σ_{prod}	Δ_{stat}	Δ_{syst}	Δ_{model}	Δ_{total}
$\pi^+ + \text{C}$	31	158.3	± 2.0	± 2.8 ± 2.0	± 1.1 ± 0.4	± 3.6 ± 2.9
$\pi^+ + \text{Al}$	31	310.4	± 4.3	± 4.2 ± 3.1	± 3.9 ± 0.6	± 7.2 ± 5.3
$\pi^+ + \text{C}$	60	171.6	± 1.7	± 4.4 ± 4.4	± 0.4 ± 1.4	± 4.7 ± 4.9
$\pi^+ + \text{Al}$	60	321.0	± 4.0	± 6.9 ± 8.1	± 0.8 ± 0.7	± 8.0 ± 9.1
$K^+ + \text{C}$	60	144.5	± 2.0	± 1.1 ± 1.1	± 0.2 ± 2.9	± 2.3 ± 3.7
$K^+ + \text{Al}$	60	284.0	± 5.1	± 1.8 ± 1.8	± 0.1 ± 4.1	± 5.4 ± 6.8

Table 8: Production cross section measurements with the NA61/SHINE data. The central value as well as the statistical (Δ_{stat}), systematic (Δ_{syst}), and model (Δ_{model}) uncertainties are shown. The total uncertainty (Δ_{total}) is the sum of the statistical, systematic, and model uncertainties in quadrature.

Interaction	p (GeV/c)	Inelastic cross section (mb)				
		σ_{inel}	Δ_{stat}	Δ_{syst}	Δ_{model}	Δ_{total}
$\pi^+ + \text{C}$	31	177.0	± 2.0	± 2.8 ± 2.0	± 1.2 ± 0.4	± 3.6 ± 2.9
$\pi^+ + \text{Al}$	31	340.0	± 4.4	± 4.2 ± 3.2	± 4.0 ± 0.6	± 7.3 ± 5.5
$\pi^+ + \text{C}$	60	188.2	± 1.8	± 4.5 ± 4.6	± 0.3 ± 3.9	± 4.9 ± 6.3
$\pi^+ + \text{Al}$	60	347.0	± 4.1	± 7.0 ± 8.1	± 1.1 ± 0.8	± 8.2 ± 9.1
$K^+ + \text{C}$	60	159.0	± 2.1	± 1.1 ± 1.1	± 0.1 ± 2.3	± 2.4 ± 3.3
$K^+ + \text{Al}$	60	307.5	± 5.1	± 1.8 ± 1.8	± 0.1 ± 3.1	± 5.4 ± 6.2

Table 9: Inelastic cross section measurements with the NA61/SHINE data. The central value as well as the statistical (Δ_{stat}), systematic (Δ_{syst}), and model (Δ_{model}) uncertainties are shown. The total uncertainty (Δ_{total}) is the sum of the statistical, systematic, and model uncertainties in quadrature.

323 8 Summary

324 In summary, the production and inelastic cross sections of π^+ and K^+ on carbon and aluminum targets have
325 been measured with the NA61/SHINE experiment. The production cross section with π^+ beams at 31 GeV/c
326 was measured for the first time with a precision of about 2%. At 60 GeV/c the measured production cross
327 sections are comparable to previous results for π^+ and K^+ and the precision was improved to about 3%
328 and 2%, respectively. Inelastic cross section measurements with π^+ and K^+ beams at 60 GeV/c were
329 measured for first time with precisions of about 3% and 2%, respectively. For the inelastic production cross
330 section for π^+ at 31 GeV/c reasonable agreement with a previous measurement was found. Especially for
331 π^+ beams, the measurements here are limited by positron contamination in the beam and steps will be
332 taken in future data-taking to better limit this uncertainty.

333 The current uncertainties on the neutrino fluxes in the NuMI neutrino beam at Fermilab from the MINER ν A
334 collaboration [16] rely on measurements of the inelastic cross section (which is termed the ‘‘absorption’’
335 cross section in the MINER ν A paper). For $\pi^+ + \text{C}$ and $\pi^+ + \text{Al}$ they assumed an uncertainty of 5%, while
336 for the $K^+ + \text{C}$ and $K^+ + \text{Al}$ cross sections they assumed a 10-30% uncertainty, which is significantly larger
337 than the systematic uncertainties determined in this paper. Thus this data will greatly reduce the uncertainty
338 on the neutrino flux prediction in NuMI due to kaon interactions.

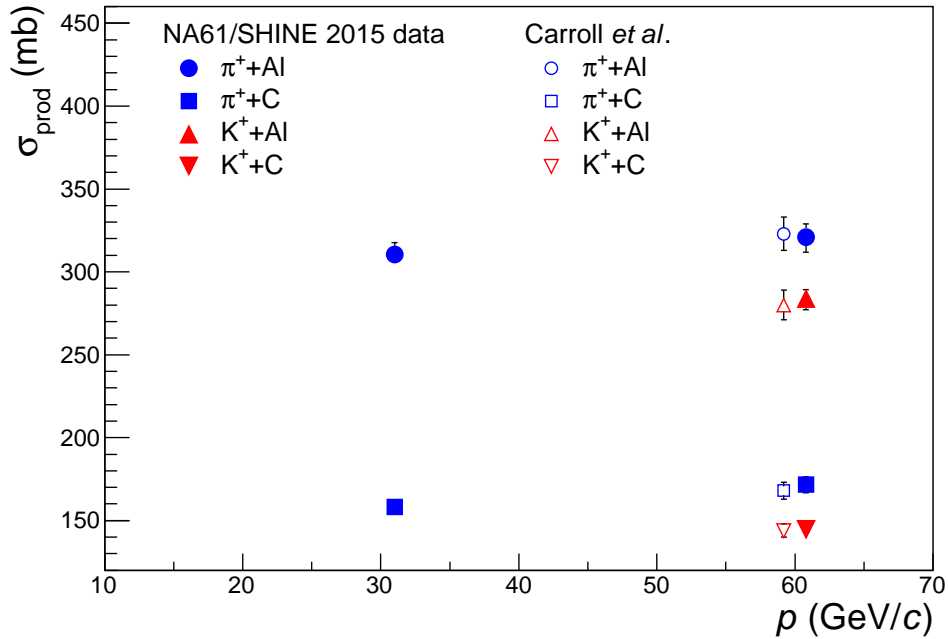


Figure 7: Summary of production cross section measurements. The results are compared to previous results obtained with a beam momentum of 60 GeV/c by Carroll et al. [14].

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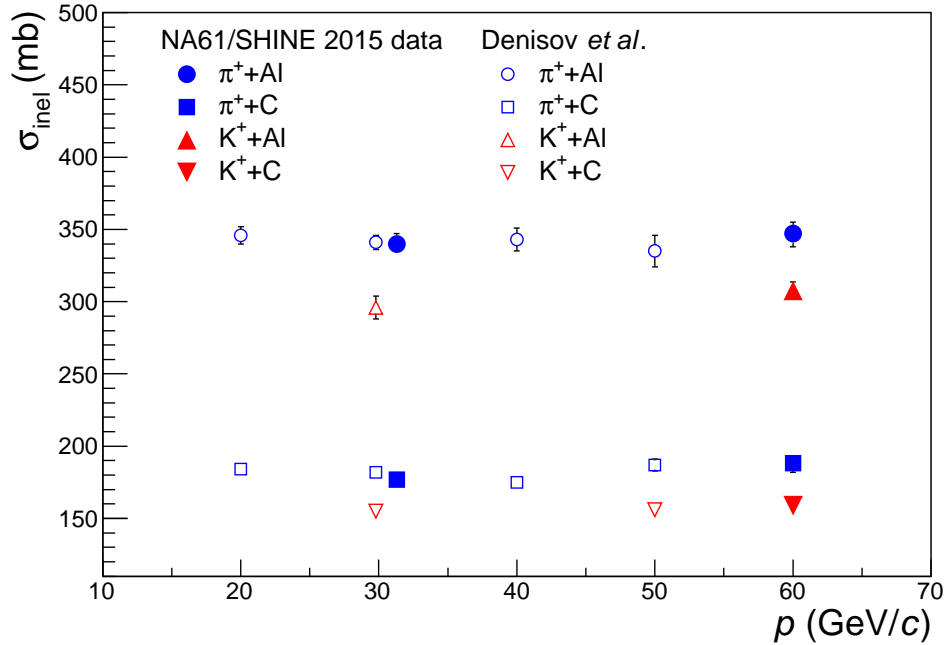


Figure 8: Summary of inelastic cross section measurements. The results are compared to previous results by Denisov et al. [15].

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