COUPLING AND DISPERSION CORRECTION FOR THE TOLERANCE STUDY IN FCC-EE

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Abstract

The FCC-ee study is investigating the design of a 100 km e+/e- circular collider for precision measurements and rare decay observations in the range of 90 to 350 GeV center of mass energy with luminosities in the order of $10^{35} cm^{-2} s^{-1}$. In order to reach such performances, an extreme focusing of the beam is required in the interaction regions with a low vertical beta function of 2 mm at the IP. Moreover, the FCC-ee physics program requires very low emittances never achieved in a collider with 1.3 nm for ϵ_x and 2 pm for ϵ_y at 175 GeV, reducing the coupling ratio to around 2/1000. With such requirements, any field errors and sources of coupling will introduce spurious vertical dispersion which degrades emittances, limiting the luminosity of the machine. This study describes the status of the tolerance study and the impact of errors that will affect the vertical emittance. In order to preserve the FCC-ee performances, in particular ϵ_y, a challenging correction scheme based on dispersion free steering and linear coupling correction is proposed to keep the coupling and the vertical emittance as low as possible.

INTRODUCTION

Electron-positron circular colliders profit from small vertical beam size due to vertical emittances close to the quantum excitation. The FCC-ee machine is foreseen to run at 4 different energies in order to perform precision measurements of the Z and W resonance and the Higgs and top. In order to produce a high luminosity, an extreme focusing of the beam is required in the interaction regions with a low vertical beta function of 2 mm at the IP. The baseline foresees very low emittance never achieved in a collider with 1.3 nm for ϵ_x and 2 pm for ϵ_y at 175 GeV, bringing down the coupling ratio to 2/1000. The main parameters are presented in Tab. 1. With such performances, the chromaticities reach several hundred units and the high beta functions in the interaction regions cause the machine to be very sensitive to lattice errors, resulting in large distorsion of the vertical dispersion. As a consequence, the vertical emittance will be enlarged, since

$$\epsilon_y = \left(\frac{dp}{p}\right)^2 (\gamma D^2 + 2\alpha D D' + \beta D'^2) \qquad (1)$$

where D is the vertical dispersion, D' the dispersion derivative with s, $\frac{dp}{p}$ the momentum spread, γ, β, α are the lattice parameters. This article present the status of the tolerance of the FCC-ee lattice to errors such as magnet misalignements, rolled angles, which are the main cause of vertical dispersion and emittance blowup. The main challenge is to establish an optics correction methodology suitable for large machines with such challenging beam parameters baseline such as FCC-ee.

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FCC-EE RACETRACK LAYOUT

The FCC-ee machine is foreseen to run at 4 different energies and in term of tolerance, the biggest challenge comes from the 175 GeV case, due the 8 GeV energy loss per turn by synchrotron radiation, as shown in Tab. 1. A constraint being that the lepton and hadron collider layout should fit together, several lattice scenarios with different interaction regions and sextupole layouts are under study (See [1] [2]). This paper will show mainly results about a racetrack lattice, with 2 RF sections and a LEP-like interaction region, where the final doublet quadrupoles focus the beam down to 2 mm β_y^*. The IR and arc optics are shown in Fig. 1. In this lattice, the chromaticity is corrected with sextupoles in the arcs including a matching from arc to IP of the Montague functions, whereas another lattice option provides as well a local chromaticity correction at the IPs [2]. The two layouts are shown in Fig. 2 for a lattice with the LEP-like IR and chromaticity correction in the arcs and Fig. 3 for a lattice with a local chromaticty correction at the IPs.

Table 1: Baseline Beam Parameters in FCC-ee

Beam Energy (GeV)	**120**	**175**
Beam current (mA)	30	6.6
Bunch/beam	780	81
Bunch population (10^{11})	0.8	1.7
Horizontal ϵ (nm)	0.61	1.3
Vertical ϵ (nm)	0.0012	0.0025
Momentum compaction (10^{-5})	0.7	0.7
Hor. β^* at the IP (mm)	1000	1000
Vert. β^* at the IP (mm)	2	2
Energy loss/turn (GeV)	1.67	7.55
Total RF Voltage (GV)	3	10

For FCC-ee, the so-called sawtooth effect is particulary important at 175 GeV: with 8 GeV of energy loss per turn, the off-momentum particles are following the dispersive orbit until they reach the next RF section. This effect causes an orbit distorsion of about 1.5 millimeter, which is very problematic when the beam goes off center through the strong sextupoles. Two options are foreseen to alleviate this problem: the FCC-ee lattice can be tapered either fully or partially. In the fully tapered option, every magnet strength (dipole, quadrupole, sextupole) is adapted to the current energy loss. In the partially tapered option - or sectorwise version- the machine

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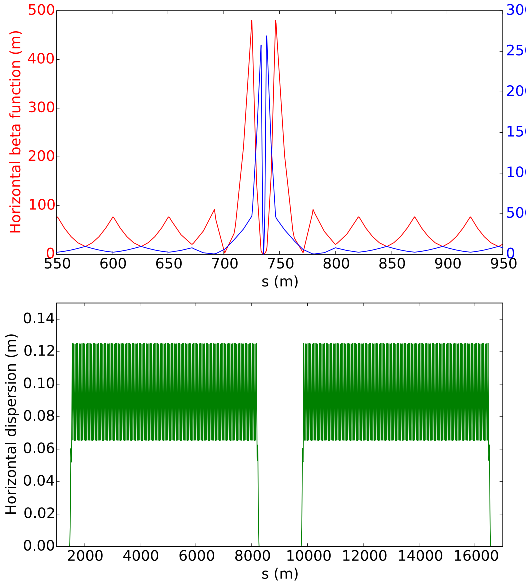

Figure 1: Beta functions (upper figure) in the IR and in the arcs and horizontal dispersion (lower figure).

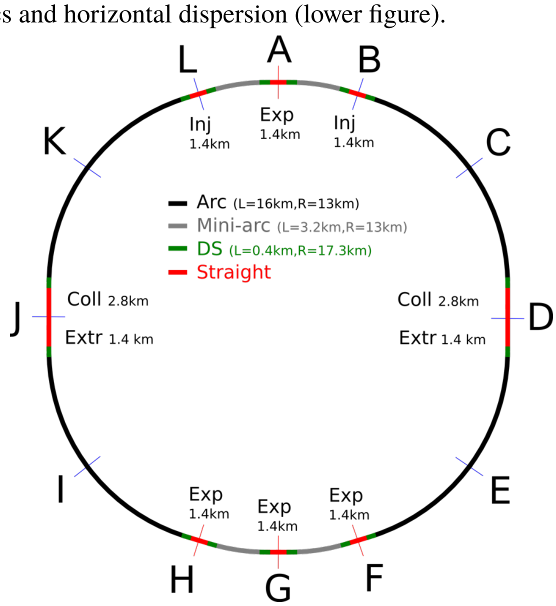

Figure 2: Racetrack layout with chromaticty correction in the arcs [1].

provides a tapering to the dipoles only, leaving therefore a remaining horizontal orbit as shown in Fig. 4 [3].

Therefore, with targeted emittances of the order of nm and pm, FCC-ee is a collider with foreseen performances of light sources (ESRF, SLS).

AMPLIFICATION FACTOR BY ERROR TYPE

In this section, amplification factors on the orbit and/or the vertical dispersion are computed by errors type. For emittance tuning purposes, any source of vertical dispersion and coupling has to be identified and should be corrected as much as possible. Let consider the most important errors to consider.

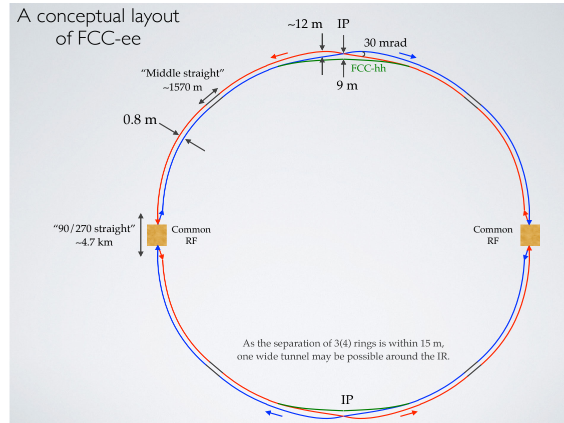

Figure 3: Racetrack layout with local chromaticty correction at the IR [2].

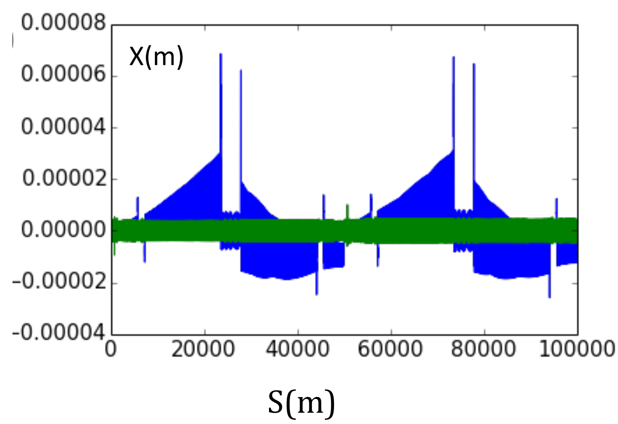

Figure 4: In green, typical horizontal orbit remainding after correction without synchrotron radiation, in blue [3]

A vertical offset Δy in the quadrupole provides a dipolar kick since [4],

$$B_x = k(y + \Delta y) = ky + k\Delta y \tag{2}$$

with k the normalised quadrupole strength. The constant term $k\Delta y$ provides a vertical dipole component and therefore vertical dispersion. Sextupole offsets produce coupling and vertical dipole kick since,

$$\begin{aligned} B_x &= kxy + kx\Delta y \\ B_y &= k(x^2 - y^2) - 2k\Delta y - (\Delta y^2) \end{aligned} \tag{3}$$

Quadrupole roll angles produce a skew strength, generating betatron coupling and transfering horizontal emittance to vertical emittance. The resulting vertical dispersion change due to a skew strength componant is

$$\Delta D_y = -(\Delta J_w) D_x \frac{\sqrt{\beta_y \beta_{y0}}}{2sin(\pi Q)} cos(\pi Q - |\phi_{y0} - \phi_y|) \tag{4}$$

where J_w is the skew strength, D_x is the horizontal dispersion, β_y and β_{y0} are respectively vertical beta function at the measurement point and at the location of the skew

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Table 2: Amplification Factor by Error Type on Vertical Closed Orbit (Vert. CO) and Vertical Dispersion Dy

Error type	Vert. CO	Dy
Quad. Vert. displ. (2 μm)	300	$1.0e6$
Roll quad (10 μrad)		25 (0.2mm RMS)
Sextupole V. displ.(1 μm)	«1	80

quadrupole component. ϕ and ϕ_0 are the phase advance at the measurement point and at the skew quadrupole.

Applying 2 μm vertical displacement gaussian distributed truncated at 3 sigma amplifies the vertical orbit by 300 and gives a vertical orbit amplitude of 0.3mm RMS. The most problematic consequece is the impact on the vertical dispersion, which is amplified by a factor 10^6, scaling values of the vertical dispersion to 2 m RMS in the arcs and 20 m at the IPs. Therefore, the vertical quadrupole misalignments have to be treated very carefully.

CORRECTION METHODS

So far, MADX has been used in combinaison with Python to apply the different correction methods of the vertical dispersion and of the betatron coupling. All the optics corrections are done without RF activated and energy loss by synchrotron radiation in the magnets, that approach being valide only for fully or sector-wise tapered machine. The EMIT command from MADX allows to finally compute the equilibrium emittances after correction: EMIT [5] is based on the Chao formalism and takes into account the energy loss via synchrotron radiation in dipoles, quadrupoles, sextupoles etc. This paper mainly concentrate transverse displacements of the quadrupoles and their tilt angles.

The following method has been used:

1. Applying alignment errors in the lattice, gaussian distributed around the ring and truncated at 2 sigma.
2. Rough orbit corrections without sextupoles in the lattice.
3. Local vertical dispersion correction at the IPs without sextupoles in the lattice.
4. Dispersion Free Steering with orbit correctors without sextupoles in the lattice.
5. Correction of the chromaticity.
6. Local vertical dispersion correction at the IPs with sextupoles in the lattice.
7. Dispersion Free Steering with orbit correctors with sextupoles in the lattice.
8. Coupling correction and vertical dispersion correction with skew quadrupoles.
9. Tapering
10. Optics and tune matching.
11. RF cavity voltage switch on in the lattice, energy loss via synchrotron radiation
12. Emittance computation with EMIT.

In principle, an orbit correction after introducing errors helps to reduce the vertical dispersion due vertical dipolar kicks. However, in presence of BPM reading errors, the orbit correction might produce higher vertical dispersion. The next section will present the effect of BPM errors in FCC-ee and justify to use the Dispersion Free Steering correction method rather than an orbit correction to reduce the vertical dispersion.

BPM ERROR TOLERANCE

In order to be consistent with the layout of the FCC-hh project, a racetrack layout ([1], [2]) was adopted. With only 2 RF sections, the magnets need either a fully or a sectorwise tapering at 175 GeV [3]. The BPM tolerance was evaluated at 175 GeV for both options by introducing BPM reading errors, the "wrong" orbits are then corrected with an orbit correction algorithm (MICADO+SVD), creating then vertical dispersion, and the equilibrium vertical emittance is evaluated. The results are showns in Fig. 5 and Fig. 6.

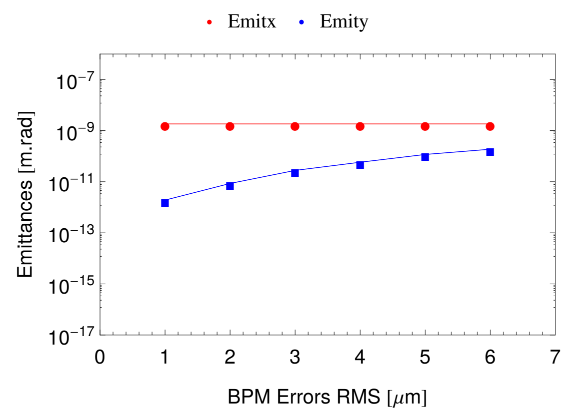

Figure 5: Resulting RMS vertical dispersion with the presence of BPM error readings.

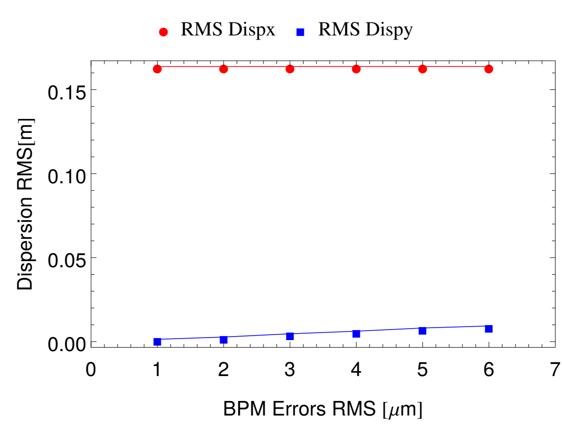

Figure 6: Resulting emittances with the presence of BPM error readings.

Relying only on orbit corrections to reduce the vertical dispersion increases of the vertical emittance when BPM

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errors are taken into account. LEP and light sources used to minimize ϵ_y by rather correcting the vertical dispersion than the orbit via a method called Dispersion Free Steering (DFS). This method allows to overcome the problem of BPM errors and put more weight on the vertical dispersion correction.

DISPERSION FREE STEERING

Dispersion Free Steering (DFS) is an efficient method which was used in LEP to minimize the vertical emittance [6]. Response matrices for orbits and dispersions are established with correctors with and without sextupoles, and the following system has to then to be solved [6].

$$\begin{pmatrix} (1-\alpha)\vec{y} \\ \alpha \vec{D}_y \end{pmatrix} + \begin{pmatrix} (1-\alpha)\mathbf{A} \\ \alpha \mathbf{B} \end{pmatrix} \vec{\theta} = 0 \quad (5)$$

where $\mathbf{A}, \mathbf{B}$ are the response matrices of the orbit and the dispersion due to a corrector kick, θ is the corrector strength, α is a weight. When α is 0, the correction is only on the orbit. With $\alpha = 1$, the correction is purely dispersive. A singular value decomposition (SVD) is then applied

$$T = UWV^t \quad (6)$$

where W is a diagonal matrix, composed by the singular values w_i, on which a cut-off has to be applied to optimize the efficiency of the correction. More singular values means more local correction but more noise, while less singular values will put the emphasis onto more global correction: a compromise has to be found between noise and local correction, in particular in a machine with large distortion of the orbit and of the dispersion which would then necessitate local correction at the IPs. As a first approach, a pure dispersion correction was used on a 2μm vertical displacement in the quadrupoles of the lattice, with random gaussian distribution cut at 3 sigma. The response matrices with and without sextupoles are very large, with a dimension of (2006x2006), and a scan of the number of singular values taken into account was performed in order to identify a minimum in emittance. Depending to the seed, 5 or 6 μm are enough to reach the foreseen vertical emittance without DFS. This tight tolerance also comes from the high vertical dispersion at the IPs, and any errors in the quadrupoles at the final focus are amplified.

In order to reduce the vertical emittance, the vertical dispersion at the IPs has to be locally corrected and treated separatly from the arcs. Four correctors around the IPs are used to create a vertical dispersion dump in order to minimize D_y at places where the β_y is the largest. Combining the DFS method with a local vertical dispersion correction at the IPs after a rough orbit correction allows to decrease the vertical dispersion from some centimeters to 10^{-5}m RMS, as shown in Fig.7.

Combining the Dispersion Free Steering first without sextupoles, then with sextupoles with the local vertical dispersion correction at the IPs increase the tolerance limit for quadrupole misalignments to 20/30 μm instead of 5, Fig. 8 and Fig. 9.

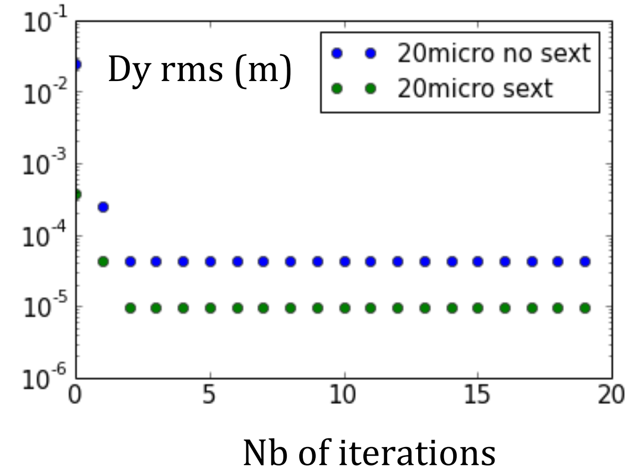

Figure 7: RMS vertical dispersion for several iterations of Dispersion Free Steering first without sextupole and then with sextupoles.

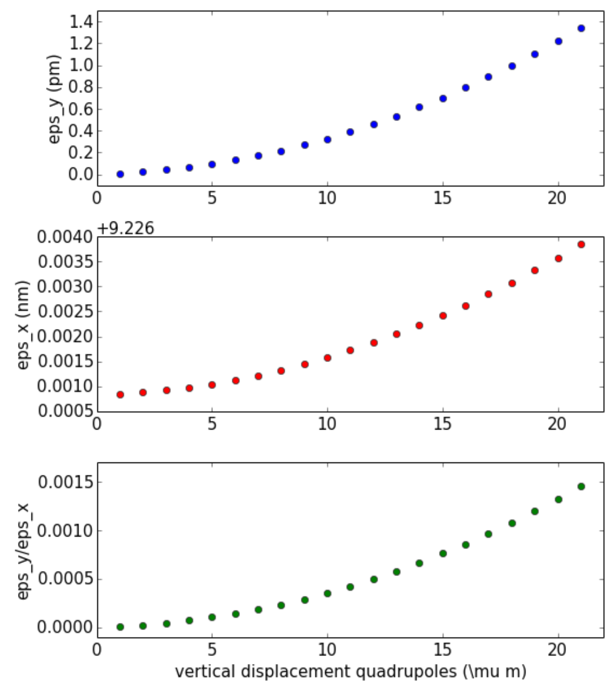

Figure 8: Vertical, horizontal emittance and coupling ratio as a function of the misalignements in the quadrupoles.

The same method can be applied to the BPMs errors for which the tolerance goes from 5 to 30 μm Fig. 10.

ROLLS IN QUADRUPOLES AND COUPLING CORRECTION

Current Skew Quadrupole Correctors Scheme for FCC-ee

In order to correct the betatron coupling, one skew quadrupole has been installed every 6 FODO cells, with a horizontal and vertical phase advance of $\Delta\phi_x = 540$ and

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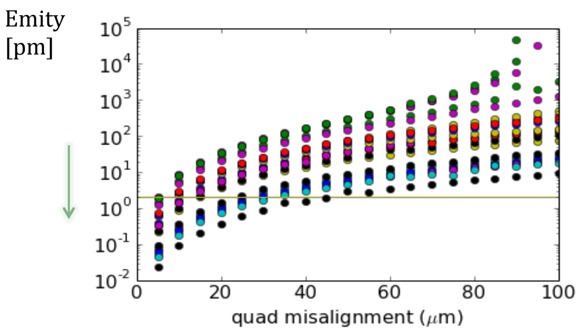

Figure 9: RMS vertical dispersion for several iterations of Dispersion Free Steering first without sextupole and then with sextupoles.

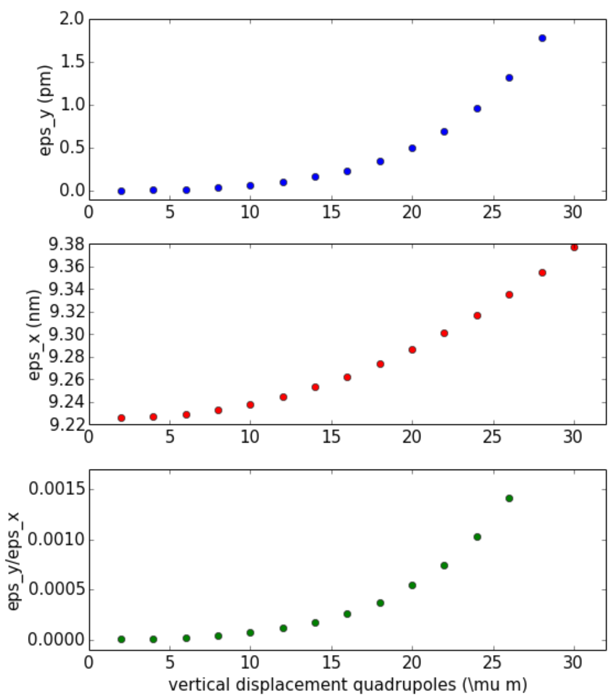

Figure 10: Vertical, horizontal emittance and coupling ratio as a function of the errors in the BPMs.

$\Delta\phi_y = 360$ degrees, since the lattice has a 90/60 degrees phase advance per cell. Therefore the total amount of skews in the machine is 272, installed in dispersive places. Currently, they are used to correct both betatron coupling and vertica dispersion. No local correction of the coupling at the IPs is performed, but is foreseen as next step in order to compensate the coupling generated by the roll angles of the final focus doublets. To correct the betatron coupling, the coupling resonance driving terms, so called f_{1001} for difference resonance and f_{1010} for the sum resonance, are mitigated, as successfully applied in LHC and at the ESRF [7] [8].

The closest tune approach is related to the complex coupling parameter, C^- - here the difference coupling parameter - which is directly a function of the coupling resonance driving terms (RDT) as [7] [8] [9]

$$\Delta Q_{min} = |C^-| = |\frac{4\Delta}{2\pi R} \oint ds f_{1001} e^{-i(\phi_x - \phi_y) + is\Delta/R}| \quad (7)$$

The resonant driving terms f_{1001} and f_{1010} can be computed from several ways, here the analytical formula

$$f_{1010}^{1001} = \frac{\sum_w J_w \sqrt{\beta_x^w \beta_y^w} e^{i(\Delta\phi_{w,x} -/+ \Delta\phi_{w,y})}}{4(1 - e^{2\pi i(Q_u -/+ Q_v)})} \quad (8)$$

where J are the skew strength, $\beta_x^w \beta_y^w$ are the horizontal and vertical beta function at the location of the skew strength, $\Delta\phi_{w,x}, \Delta\phi_{w,y}$ are the phase advance between the observation point and the skew componant.

Using the matrix formalism, a response matrix of the RDT using the quadrupole skews of the lattice can be computed,

$$(\vec{f_{1001}})_{meas} = -M\vec{J} \quad (9)$$

where J are the vector of the skew, $\vec{f_{1001}}$ are the complex coupling RDT at the BPMs, M is the response matrix of the RDT to skew quadrupole kicks. $\vec{f_{1010}}$ is neglected to the distance of the working point with respect to the sum coupling resonance.

Coupling Correction for a Lattice with Roll Angles in the Quadrupoles

The roll quadrupole tolerances are much less tigh compared to the transverse displacement with an amplification factor of 25 on the vertical dispersion. Let us consider the FCC-ee lattice at 175 GeV with 50 μrad roll angle gaussian distributed cut at 2 sigma. Since no other error is considered in this simulation, the coupling mainly comes the tilted quadrupoles.

The coupling RDT at the BPMs are computed and corrected with the corresping response matrix after a SVD, and skew quadrupoles strength are then applied. The resulting RDT are compared to the initial RDT. The successive corrections allows to correct by a factor 10 the RDT $\vec{f_{1001}}$.

This correction can be combined with a response matrix of the vertical dispersion to the skew quadrupoles:

$$(\vec{D_y}) = -M\vec{J} \quad (10)$$

where $\vec{D_y}$ is the vertical dispersion measured at the BPMs, M is the response matrix of the RDT to the skews, J are the skew strength. While introducing roll angles in the quadrupoles of the lattice, dispersion is transferred from the horizontal plane to the vertical one Eq. 4. The correction of the vertical dispersion with the skew quadrupoles allows to bring it down from 3.5 mm RMS to 0.5 mm.

Fig. 11 shows the real part of the coupling RDT before coupling correction in blue with 0.010 RMS, the RDT after coupling correction with 0.001 RMS en red, and finally in green, the RDT when the RF cavities are on with synchrotron damping in the simulation and with a sector wise tapering

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Fig.4. This latter brings extra source of coupling which was not present before switching on the RF cavities. A possible explanation is the coupling of the remaining sawtooth to the vertical plane, exciting a vertical orbit, which would then create additional coupling through the sextupoles.

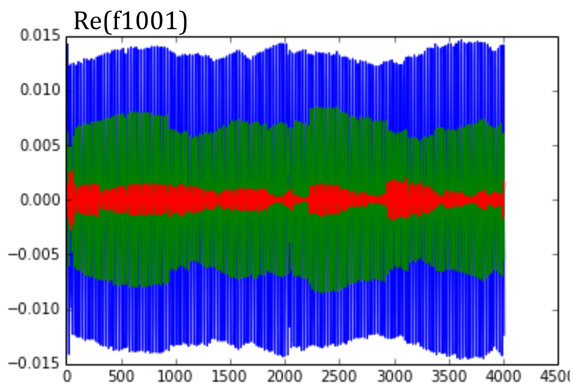

Figure 11: Real part of the coupling RDT at the BPMs before coupling correction in blue, after correction in red, with RF cavties and synchrotron damping in green.

Combining 20 μm misalignements in the quadrupoles with 50 μ m roll angles is acceptable for the tolerances for a sector wise tapared lattice.

SUMMARY-CONCLUSION

Lattice errors have a very large impact on the vertical dispersion and emittance due the high beta functions in the interaction regions. The challenges come from the the very low emittances of the order of pm in vertical plane with a very strong focus at the IPs resulting in a β_y^* of 2mm, which makes the machine very sensitive to alignement errors. In order to alleviate BPM errors and maximize the vertical dispersion correction, a Dispersion Free Steering algorithm was implemented without and with sextupoles combined with a local correction of the dispersion at the IPs. The tolerances could then be increased by a factor 4 and 20/30 μm misalignements with 50 μ rad roll angles is an acceptable tolerance for a sector wise tapering machine at 175 GeV. Futher studies will focus on the tolerance to errors of a lattice fully tapered which might relax the tolerances. An implementation of a local correction of the coupling at the IPs is also required, and futher investigations to improve the coupling correction needed.

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