QCD FACTORIZATION FOR $B \to \pi\pi$ DECAYS

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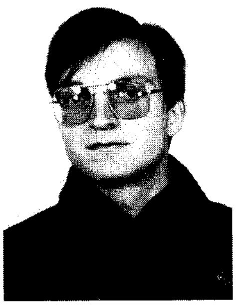

The strong-interaction dynamics of two-body hadronic B decays simplifies considerably in the heavy-quark limit, where, as has recently been proposed, a QCD factorization theorem can be written for the hadronic matrix elements. Here we discuss applications of this framework for $B \to \pi\pi$ decays.

1 QCD Factorization

The computation of hadronic matrix elements for exclusive nonleptonic B decays is among the most urgent theoretical problems in B physics. In some cases this problem can be circumvented (CP asymmetry in $B \to J/\Psi K_S$), or at least reduced using SU(2) or SU(3) flavour symmetries and an appropriate combination of various channels. Yet an improved understanding of the QCD dynamics in hadronic B decays would greatly enhance our capability to extract from these processes the underlying flavour physics.

A simple approach, which has been widely used in phenomenological studies [1,2], is the *naive factorization* of matrix elements, schematically

$$\langle\pi^+\pi^-|(\bar{u}b)_{V-A}(\bar{d}u)_{V-A}|\bar{B}\rangle \to \langle\pi^+|(\bar{u}b)_{V-A}|\bar{B}\rangle \cdot \langle\pi^-|(\bar{d}u)_{V-A}|0\rangle \qquad (1)$$

for the example of $B \to \pi\pi$. The justification for this procedure has been less clear. An obvious issue is the proper scheme and scale dependence of the matrix elements of four-quark operators, which is needed to cancel the corresponding dependence in the Wilson coefficients. This dependence is lost in naive factorization as the factorized currents are scheme independent objects. In many cases the factorization can be justified in the large-N_c limit of QCD [3], but this approximation is often too crude for a reliable phenomenology. In any case one would prefer not to rely exclusively on this argument. A different qualitative justification for factorization has been given by Bjorken [4]. It is based on the *colour transparency* of the hadronic environment

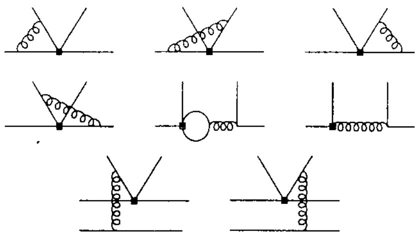

Figure 1: Order α_s corrections to the hard scattering kernels T_i^I (first two rows) and T_i^{II} (last row). In the case of T_i^I, the spectator quark does not participate in the hard interaction and is not drawn. The two lines directed upwards represent the two quarks forming the emitted pion.

for the highly energetic pion emitted in the decay of a B meson (the π^- in the above example, which is being created from the vacuum). Formally this is related to the decoupling of soft gluons from the small-size colour-singlet object that the emitted pion represents.

A new, systematic approach was recently formulated in [5]. There it was proposed that factorization can indeed be justified within QCD to leading order in the heavy quark limit for a large class of two-body hadronic B decays. The statement of *QCD factorization* in the case of $B \to \pi\pi$ can be schematically written

$$A(B \to \pi\pi) = \langle\pi|j_1|B\rangle \, \langle\pi|j_2|0\rangle \cdot \left[1 + \mathcal{O}(\alpha_s) + \mathcal{O}\left(\frac{\Lambda_{QCD}}{m_B}\right)\right]. \tag{2}$$

Up to corrections suppressed by Λ_{QCD}/m_B the amplitude is calculable in terms of simpler hadronic objects: It factorizes, to lowest order in α_s, into matrix elements of bilinear quark currents ($j_{1,2}$). To higher order in α_s, but still to leading order in Λ_{QCD}/m_B, there are 'nonfactorizable' corrections, which are however governed by hard gluon exchange. They are therefore again calculable in terms of few universal hadronic quantities. More explicitly, the matrix elements of 4-quark operators Q_i are expressed by the factorization formula

$$\begin{aligned}\langle\pi(p')\pi(q)|Q_i|\bar{B}(p)\rangle &= f^{B\to\pi}(q^2)\int_0^1 dx\, T_i^I(x)\Phi_\pi(x) \\ &+ \int_0^1 d\xi dx dy\, T_i^{II}(\xi,x,y)\Phi_B(\xi)\Phi_\pi(x)\Phi_\pi(y),\end{aligned} \tag{3}$$

which is valid up to corrections of relative order Λ_{QCD}/m_b. Here $f^{B\to\pi}(q^2)$ is a $B \to \pi$ form factor evaluated at $q^2 = m_\pi^2 \approx 0$, and Φ_π (Φ_B) are leading-twist light-cone distribution amplitudes of the pion (B meson). The $T_i^{I,II}$ denote hard-scattering kernels, which are calculable in perturbation theory. T_i^I starts at $\mathcal{O}(\alpha_s^0)$, T_i^{II} at $\mathcal{O}(\alpha_s^1)$ (see Fig. 1).

This treatment of hadronic B decays is based on the analysis of Feynman diagrams in the heavy quark limit, utilizing consistent power counting to identify the leading contributions. The framework is very similar in spirit to more conventional applications of perturbative QCD in exclusive hadronic processes with a large momentum transfer, as the pion electromagnetic form factor [6,7,8]. It may be viewed as a consistent formalization of Bjorken's colour transparency argument [4]. In addition the method includes, for $B \to \pi\pi$, the hard nonfactorizable spectator interactions, penguin contributions and rescattering effects (Fig. 1). As a corollary, one finds that strong rescattering phases are either of $\mathcal{O}(\alpha_s)$, and calculable, or power suppressed. In any case they vanish therefore in the heavy quark limit. QCD factorization is valid for cases where the emitted particle (the meson created from the vacuum in the weak process, as opposed to the one that absorbs the b-quark spectator) is a small size colour-singlet object, e.g. either a fast

Table 1: The QCD coefficients $a_i^p(\pi\pi)$ at NLO for three different renormalization scales μ. Leading order values are shown in parenthesis for comparison.

	$\mu = m_b/2$	$\mu = m_b$	$\mu = 2m_b$
$a_1^u(\pi\pi)$	$1.047 + 0.033i$	$1.038 + 0.018i$	$1.027 + 0.010i$
	(1.038)	(1.020)	(1.010)
$a_2^u(\pi\pi)$	$0.061 - 0.106i$	$0.082 - 0.080i$	$0.108 - 0.064i$
	(0.066)	(0.140)	(0.200)
$a_4^u(\pi\pi)$	$-0.030 - 0.019i$	$-0.029 - 0.015i$	$-0.026 - 0.013i$
	(−0.027)	(−0.020)	(−0.014)
$a_4^c(\pi\pi)$	$-0.038 - 0.009i$	$-0.034 - 0.008i$	$-0.031 - 0.007i$
	(−0.027)	(−0.020)	(−0.014)
$a_6^p(\pi\pi)r_\chi$	–	–	–
	(−0.036)	(−0.030)	(−0.024)

light meson (π, ϱ, K, K^*) or a J/Ψ. For the special case of the ratio $\Gamma(B \to D^*\pi)/\Gamma(B \to D\pi)$ the perturbative corrections to naive factorization have been evaluated in [9] using a formalism similar to the one described above.

Further details on QCD factorization in B decays and additional literature can be found in refs. [5,10]

2 Phenomenological Applications for $B \to \pi\pi$

Let us illustrate a few phenomenological applications of QCD factorization for $\bar{B} \to \pi^+\pi^-$ [5]. The decay amplitude can be written as

$$A(\bar{B}_d \to \pi^+\pi^-) = i\frac{G_F}{\sqrt{2}}m_B^2 f_+(0) f_\pi |\lambda_c| \cdot \cdot[R_b e^{-i\gamma}\left(a_1^u(\pi\pi) + a_4^u(\pi\pi) + a_6^u(\pi\pi)r_\chi\right) - \left(a_4^c(\pi\pi) + a_6^c(\pi\pi)r_\chi\right)]. \tag{4}$$

Here $R_b = (1 - \lambda^2/2)|V_{ub}/V_{cb}|/\lambda$, where $\lambda = 0.22$ is the sine of the Cabibbo angle, γ is the phase of V_{ub}^*, and we will use $|V_{cb}| = 0.039 \pm 0.002$, $|V_{ub}/V_{cb}| = 0.085 \pm 0.020$. We also take $f_\pi = 131\,\text{MeV}$, $f_B = (180 \pm 20)\,\text{MeV}$, $f_+(0) = 0.275 \pm 0.025$, and $\tau(B_d) = 1.56\text{ps}$; $\lambda_c \equiv V_{cd}^* V_{cb}$. The contribution of $a_6^p(\pi\pi)$ is multiplied by $r_\chi = 2m_\pi^2/(m_b(m_u + m_d)) \sim \Lambda_{QCD}/m_b$. It is thus formally power suppressed, but numerically relevant since $r_\chi \approx 1$. The coefficients a_i are evaluated in table 1. They have been calculated using the factorization formula (3) at next-to-leading order, that is including order α_s (except for the formally power-suppressed term with a_6, where (3) does not apply). We then find for the branching fraction

$$B(\bar{B}_d \to \pi^+\pi^-) = 6.5\,[6.1] \cdot 10^{-6} \left| e^{-i\gamma} + 0.09\,[0.18]\, e^{i\cdot 12.7\,[6.7]^\circ} \right|^2, \tag{5}$$

where the default values correspond to neglecting $a_6^p(\pi\pi)$ and the values in brackets use $a_6^p(\pi\pi)$ at leading order. The predictions for the $\pi^+\pi^-$ final state are relatively robust, with errors on the order of $\pm 30\%$ due to the input parameters. The direct CP asymmetry in the $\pi^+\pi^-$ mode is approximately $4\% \cdot \sin\gamma$.

As a further example we may use the factorization formula to compute the time-dependent, mixing-induced asymmetry in $B_d \to \pi^+\pi^-$ decay. For reasons discussed below the result should still be considered preliminary. However, it illustrates the predictivity of a systematic approach to calculate nonleptonic decay amplitudes. The time-dependent asymmetry can be expressed as

$$\mathcal{A}(t) = -S \cdot \sin(\Delta M_{B_d} t) + C \cdot \cos(\Delta M_{B_d} t). \tag{6}$$

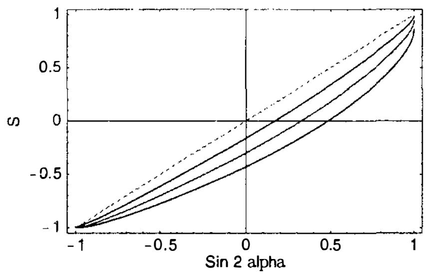

Figure 2: Coefficient of $-\sin(\Delta M_{B_d} t)$ vs. $\sin 2\alpha$. $\sin 2\beta = 0.7$ has been assumed. See text for explanation.

In the absence of a penguin contribution (defined as the contribution to the amplitude which does not carry the weak phase γ in standard phase conventions) $S = \sin 2\alpha$ (where α refers to one of the angles of the CKM unitarity triangle) and $C = 0$. Fig. 2 shows S as a function of $\sin 2\alpha$ with the amplitudes computed according to (3), (4). The central of the solid lines refers to the heavy quark limit including α_s corrections to naive factorization and including the power-suppressed term $a_6 r_\chi$ that is usually also kept in naive factorization. The other two solid lines correspond to dropping this term or multiplying it by a factor of 2. This exercise shows that formally power-suppressed terms can be non-negligible, but it also shows that a measurement of S can be converted into a range for $\sin 2\alpha$ which may already provide a very useful constraint on CP violation.

Much work remains to be done on the theoretical and phenomenological side. On the theoretical side, the proof of factorization for a final state of two light mesons has to be completed. Power corrections are an important issue, as m_b is not particularly large. There exist the 'chirally enhanced' corrections that involve the formally power suppressed, but numerically large parameter r_χ. All terms involving such chiral enhancements can be identified, but they involve nonfactorizable soft gluons. The size of these terms has to be estimated to arrive at a realistic phenomenology.

If this can be done, one may expect promising constraints and predictions for a large number of nonleptonic, two-body final states.

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