

# VERTICAL DISPERSION AND BETATRON COUPLING CORRECTION FOR FCC-EE

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## Abstract

The FCC-ee project foresees to build a 100 km e+/e- circular collider for precision studies and rare decay observations in the range of 90 to 350 GeV center of mass energy with luminosities in the order of  $10^{35} \text{ cm}^{-2} \text{ s}^{-1}$ . To reach such performances, an extreme focusing of the beam is required in the interaction regions with a low vertical beta function of 2 mm at the IPs. Moreover, the FCC-ee physics program requires very low emittances never achieved in a collider with 1 nm for  $\epsilon_x$  and 2 pm for  $\epsilon_y$ , bringing down the coupling ratio to 2/1000. Thus, coupling and vertical dispersion sources have to be controlled carefully. This paper describes the tolerance of the machine to magnet alignment errors as well as the optics correction methods that were implemented, such as the Orbit Dispersion Free Steering, in order to bring the vertical dispersion to reasonable values. The correction of the betatron coupling, being also a very important source of emittance growth, has been integrated to a challenging correction scheme to keep the vertical emittance as low as possible.

## INTRODUCTION

The Z, W, Higgs and top resonance precision measurements in FCC-ee require luminosities in the order of  $10^{35} \text{ cm}^{-2} \text{ s}^{-1}$ , with horizontal and vertical beta functions at the interaction point (IP) of about 1 m and 2 mm - see the optics figures Fig. 1. The FCC-ee lattice is based on 90/90 degree phase advance in the arcs and two interaction points. Since the top energy is the most critical case for emittance tuning, the beam parameters at 175 GeV are presented in Table 1. Chromaticities are known to be a hard limit in electron machines for low emittance tuning and FCC-ee is no exception since the focusing provided by the final focus quadrupoles (FF) generates a very large chromaticity of nearly -1000 units in the vertical plane. Therefore a complex sextupole scheme has been established [1] in order to achieve the required momentum acceptance of 2%, with sextupole strengths of two orders of magnitude larger than LEP or LHC. However any established alignment errors at the sextupoles and quadrupoles generate optics distortions, dispersion and linear coupling, which are particularly problematic as they strongly participate to the emittance growth in electron machines [2],

$$\epsilon_{x,y} = 1.47 \times 10^{-6} \frac{E^2}{R} \frac{1}{l} \int_0^l H(s) ds \quad (1)$$

$\epsilon_{x,y}$  is the equilibrium emittance of the beam, with  $E$  being the beam energy in GeV,  $R$  the bending radius,  $l$  is the length of the magnet and  $\mathcal{H}(s)$  is a function described in Eq. 2. This

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emittance of the beam is averaged of all possible photon energies and emission probabilities from the area of the phase space ellipse of a single particle, given by,

$$\mathcal{E}_y = \left( \frac{dp}{p} \right)^2 (\gamma D_y^2 + 2\alpha D_y D'_y + \beta D_y'^2) = \left( \frac{dp}{p} \right)^2 H(s) \quad (2)$$

where  $D_y$  is the vertical dispersion,  $D'_y$  the dispersion derivative with  $s$ ,  $\frac{dp}{p}$  the momentum spread and  $\gamma, \beta, \alpha$  are the Twiss parameters. Referring to the vertical plane, Eq. 2 shows a quadratic component of the vertical emittance with  $D_y$  and thus any spurious vertical dispersion has to be corrected carefully. In the first part of this paper, the correction methods used to establish an efficient correction scheme are presented in detail. Then the different contributions at 175 GeV to the emittance, error by error, are analyzed and the results of the emittances after correction are presented with their associated correction schemes. Unless explicitly indicated, the quadrupoles of the final focus are left perfectly aligned, as they might need a special treatment.

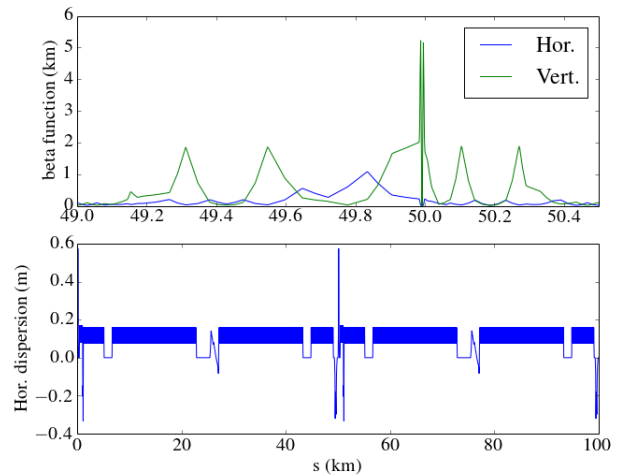


Figure 1: Beta function in the IR and horizontal dispersion function along the machine in the FCC-ee lattice. On this figure, one of the IP stands at 50 km.

## CORRECTION METHODS

As explained in the previous section, coupling and vertical dispersion have to be controlled with a particular emphasis on vertical dispersion, since it increases quadratically the vertical emittance. For a linear lattice, i.e. without sextupole field, the Dispersion Free Steering is used (DFS) to correct

Table 1: Baseline Beam Parameters in FCC-ee

Beam Energy (GeV)	175
Beam current (mA)	6.6
Bunch/beam	81
Bunch population ( $10^{11}$ )	1.7
Horizontal $\epsilon$ (nm)	1.3
Vertical $\epsilon$ (nm)	0.0025
Momentum compaction ( $10^{-5}$ )	0.7
Hor. $\beta^*$ at the IP (mm)	1000
Vert. $\beta^*$ at the IP (mm)	2
Energy loss/turn (GeV)	7.55
Total RF Voltage (GV)	10

vertical orbit and dispersion coming from dipolar kicks, generated by offsets at the quadrupoles. This method was used in LEP to minimize the vertical emittance [3] and is based on response matrices that relates orbit and dispersion to the corrector kick,

$$\begin{pmatrix} (1-\alpha)\vec{y} \\ \alpha\vec{D}_y \end{pmatrix} + \begin{pmatrix} (1-\alpha)\mathbf{A} \\ \alpha\mathbf{B} \end{pmatrix} \vec{\theta} = 0 \quad (3)$$

where  $\mathbf{A}$ ,  $\mathbf{B}$  are the response matrices of the orbit and the dispersion due to a corrector kick,  $\theta$ ,  $\alpha$  is a weight factor. A singular value decomposition (SVD) is then applied

$$T = UWV^t \quad (4)$$

where  $W$  is a diagonal matrix, decomposed in singular values  $w_i$ . A cut-off is applied to optimize the efficiency of the correction. The DFS efficiently reduces the vertical dispersion and the orbit, even in cases where the maximum weight ( $\alpha = 1$ ) is applied on the dispersion. This is convenient for machines very sensitive to BPM reading errors.

The next important step for an electron machine is the coupling correction. Skew components were installed at the sextupoles - extra windings can be mounted on the sextupoles - to get a skew quadrupole for coupling correction purposes. In this study, the method based on the correction of the linear coupling resonant driving terms (RDT)  $f_{1001}$  and  $f_{1010}$  has been used. A matrix based on the response of the RDT and of the vertical dispersion to a skew component  $J_w$  can be computed from MADX, since [4] [5],

$$\Delta D_y = -(J_w)D_x \frac{\sqrt{\beta_y\beta_{y0}}}{2 \sin(\pi Q)} \cos(\pi Q - |\phi_{y0} - \phi_y|) \quad (5)$$

with the RDT

$$f_{1010}^{1001} = \frac{\sum_w^W J_w \sqrt{\beta_x^w \beta_y^w} e^{i(\Delta\phi_{w,x} + / - \Delta\phi_{w,y})}}{4(1 - e^{2\pi i(Q_u + / - Q_v)})} \quad (6)$$

where  $J_w$  is the  $w$ -th integrated skew quadrupole strength. It is possible to combine the response of the RDT with the vertical dispersion to control these quantities during the correction, weighting them as necessary. Finally, horizontal offsets at sextupoles cause beta beating that compromise

the  $\beta^*$  and the luminosity. Quadrupolar components were installed at the sextupoles and we studied the response of the phase advance, horizontal dispersion and tunes to the gradient  $k_1$  of these quadrupoles, such as,

$$(\Delta\phi_{x,y}, \Delta D_x, \Delta Q_x, \Delta Q_y) = R\Delta k_1 \quad (7)$$

## TOLERANCES - RESULTS

### Roll Angles in Arc Quadrupoles

The roll angles of the arc quadrupoles have a fairly moderate influence. Tilts of  $100 \mu\text{rad}$  in all quadrupoles, gaussian distributed along the machine and truncated at 2.5 sigma, give the design vertical emittance of 2 pm, even before correction. The linear coupling is iteratively corrected, i.e. ( $Re(f_{1001}), Im(f_{1001}), Re(f_{1010}), Im(f_{1010})$ ) are minimized at the BPMs. As a second step, the vertical dispersion is included in the response matrix with equal weight. The results are presented in the Table 2. At  $100 \mu\text{rad}$ , the vertical dispersion is decreased by a factor 1000 (200 seeds were tested). The emittances are computed with MADX [6], and the machine is supposed to be fully tapered, i.e. every magnets, correctors included, have their strength adapted to the energy losses due synchrotron radiation in order to avoid a large so-called "sawtooth effect".

Table 2: Roll Angles Only in the Quadrupoles at 175 GeV

Emittance	100 $\mu\text{rad}$	200 $\mu\text{rad}$
$\epsilon_y$ pm	0.002+/-0.00017	0.008+/-0.0006
$\epsilon_x$ nm	1.29 +/-5.0e - 6	1.29+/-5.0e - 6
$\epsilon_y/\epsilon_x$	1.65e - 6	6.6e - 6

### Transverse Displacements in Arc Quadrupoles

While the errors are introduced, the sextupoles are switched off in order to avoid problems of numerical non-convergence. The vertical dispersion and the orbits are corrected, then the sextupoles are increased in steps of 10% of their final strength, and for each step, coupling and vertical dispersion are corrected. Table 3 presents the results of vertical displacements in quadrupoles and Table 4 for horizontal and vertical transverse displacements, for 100 and 200  $\mu\text{m}$  gaussian distributed truncated at 2.5 sigma. Fig. 2 shows the correction of the vertical dispersion for transverse displacements of the arc quadrupoles with orbit correction and DFS, sextupoles off. The RMS  $D_y$  goes from 100 m to the cm level, and with DFS to 0.1 mm RMS. Fig. 3 shows the vertical dispersion correction with coupling correction while the sextupoles are on: these final steps allow to reduce again the vertical dispersion by a factor 10.

Including 100  $\mu\text{m}$  BPM reading errors in the simulations from the Table 3 for the vertical plane only results in, as a first estimation, a vertical emittance increase from 0.008 pm to 0.2 pm. BPM reading errors are not negligible and more studies are needed to determine their exact impact on the emittances.

Table 3: Vertical Displacements in the Quadrupoles at 175 GeV

Emittance	100 $\mu\text{m}$	200 $\mu\text{m}$
$\epsilon_y$ pm	0.008+/-0.001	0.032+/-0.004
$\epsilon_x$ nm	1.29 +/-0.005	1.29+/-0.0004
$\epsilon_y/\epsilon_x$	$6.47e-6$	$2.6e-5$

Table 4: H and V Displacements in the Quadrupoles at 175 GeV

Emittance	100 $\mu\text{m}$	200 $\mu\text{m}$
$\epsilon_y$ pm	0.047	0.049
$\epsilon_x$ nm	1.29 +/-0.01	1.29+/-0.03
$\epsilon_y/\epsilon_x$	$3.8e-5$	$2.6e-5$

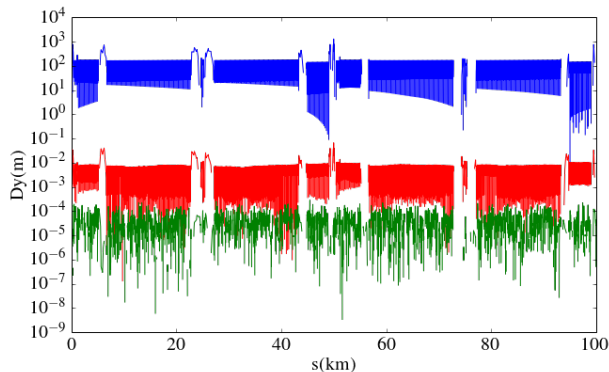


Figure 2: Vertical dispersion for vertical displacement of the arc quadrupoles of  $100\mu\text{m}$ . In blue, the initial vertical dispersion along the machine, 100m RMS, in red is the vertical dispersion after closed orbit correction (centimeter level) and in green, the final vertical dispersion after DFS (fraction of a millimeter).

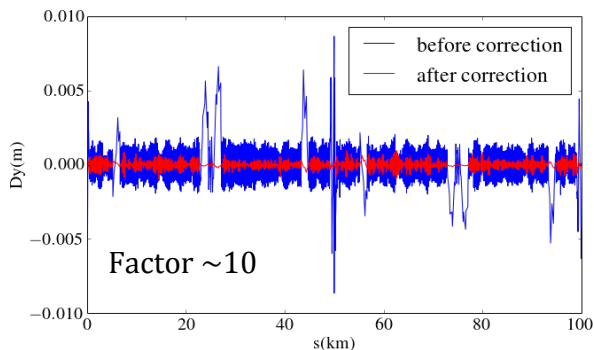


Figure 3: Vertical dispersion for vertical displacement of the arc quadrupoles of  $100\mu\text{m}$ . In blue, the vertical dispersion after DFS and with sextupoles switched on. In red, the vertical dispersion after coupling correction and vertical dispersion correction. A factor 10 is gained.

To obtain an overall picture, the roll angles and the transverse displacements are put together with  $100\mu\text{m}$  in displacement and  $100\mu\text{rad}$  in tilt in the arc quadrupoles for 700 samples. The results for the average emittances are:

$$\epsilon_x = 1.29 \text{ nm}, \epsilon_y = 1 \text{ pm}, \epsilon_y/\epsilon_x = 0.0008 \quad (8)$$

However, the spread for the vertical emittance for these simulations is large with emittances up to  $\epsilon_y = 20 \text{ pm}$  due to seeds with damping partition numbers of  $j_x = 1.39, j_y = 1.03, j_s = 1.58$ .

## SEXTUPOLE TRANSVERSE DISPLACEMENTS

A set of simulations was performed with sextupole displacements of  $\Delta x = \Delta y = 100\mu\text{m}$  RMS gaussian distributed truncated at 2.5 sigma. For this study, the quadrupoles are left perfectly aligned. FCC-ee carries a local chromaticity correction scheme since the IPs provide the largest part of the chromaticities. These strong sextupoles (SY) are located in high beta functions areas. The results of the average emittances are so far promising:

$$\epsilon_x = 1.29 \text{ nm}, \epsilon_y = 0.2 \text{ pm}, \epsilon_y/\epsilon_x = 0.0002 \quad (9)$$

## CONCLUSIONS - OUTLOOK

Alignment errors in the quadrupoles have a large impact on the vertical dispersion, and with the efficient correction methods inspired by light sources and colliders such LHC, the tolerance of the arc quadrupoles and of the sextupoles are kept to reasonable values. So far, the transverse displacements of the sextupoles have the strongest impact on the emittance tuning. While these simulations do not include yet the quadrupoles of the interaction region which might have a large impact. Furthermore the first simulations of BPM reading errors showed an increase by a factor 10 of the vertical emittance for quadrupole misalignment only. More studies are foreseen to determine the tolerance of the BPMs. Finally, the first results with  $100\mu\text{m}$  in the quadrupoles of the Final Focus multiply the vertical emittance by a factor 10, however this number has been taken very carefully because the sextupoles can also introduce a feed down effect in vertical dispersion. Therefore sextupole and quadrupole misalignments has to be merged in the same simulations with the quadrupoles of the IP. Then, the tapering of the machine has to be relaxed, in particular at the lowest energies where the energy loss by synchrotron radiation is less important, a possible solution would be to have a partial tapering as proposed in Ref. [7]. The correction tools developed and applied in these studies present efficient techniques and the resulting emittances are well within the given determined window.

## ACKNOWLEDGEMENT

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