Assuming Regge trajectories in holographic QCD: from OPE to Chiral Perturbation Theory

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The Soft Wall model in holographic QCD has Regge trajectories but wrong operator product expansion (OPE) for the two-point vectorial QCD Green function. We correct analytically this problem and describe the axial sector and chiral symmetry breaking. The low energy chiral parameters, F_π and L_{10}, are well described analytically by the model in terms of Regge spacing and QCD condensates. The model nicely supports and extends previous theoretical analyses advocating Digamma function to study QCD two-point functions in different momentum regions.

1 Introduction

QCD Green functions are well described as an interpolation from the IR region, chiral perturbation theory (CHPT) and perturbative QCD region, but it is phenomenologically and theoretically interesting to go beyond this simple matching, for instance by implementing Regge trajectories [1]. Since Veneziano's model several authors have proposed to describe two point functions with Digamma functions [2], which produce a Regge spectrum and an OPE in the euclidean region. Also, a phenomenological matching among the various regions has been proposed [3]. We believe that there are theoretical motivations to insist on having Regge trajectories, *i.e.* a progression for the spectrum like $M_V(n)^2 \propto n$, and arrange for QCD OPE of the vector and axial vector two-point function

$$\Pi_{V,A}(Q^2) \underset{Q^2\to\infty}{\sim} \frac{N_c}{24\pi^2}\ln\left(\frac{\Lambda_V^2}{Q^2}\right) + \frac{\alpha_s\left\langle G^2\right\rangle}{24\pi}\frac{1}{Q^4} - \frac{14\pi}{9}c_{V,A}\,\alpha_s\,\left\langle\bar\psi\psi\right\rangle^2\frac{1}{Q^6} \qquad (1)$$

with $c_V = 1$ and $c_A = -11/7$. We should then be able to take into account the difference appearing at order $1/Q^6$ in the OPE expansion of the vector and axial correlators. QCD literature [1,3] has addressed the issue to predict

$$F_\pi^2 = 2\,\mathrm{Res}\left[\Pi_{LR}\left(Q^2\right),0\right] \qquad \text{and} \qquad L_{10} = \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}Q^2}\left[Q^2\Pi_{LR}(Q^2)\right]\Big|_{Q^2=0}, \qquad (2)$$

by phenomenologically matching the OPE to the pion and lowest mesons (ρ and a_1) with Regge resonances. A good knowledge of the three regions of QCD also leads to address properly the issue of duality violations in tau and b-decays and pion form factor [4]. Holographic QCD has: *i)* Hard Wall models (HW) good IR, chiral perturbation theory, and UV parton log, but Kaluza Klein NOT Regge and *ii)* Soft Wall model (SW): Regge and UV parton log plus terms in $1/Q^2$

but no χPT predictions. A recent review of those models and their relations with light front holographic QCD is Ref. [5]. Our idea in Rcf. [6] is that a correct OPE is pertinent for the study of the low Q^2 limit of $\Pi_{LR}(Q^2) = 1/2(\Pi_V - \Pi_A)$ allowing us to extract the chiral constants in eq. (2).

Indeed, in the SW model the AdS metric, $1/z$, is deformed by the presence of a scalar, $\Phi(z)$, the dilaton, describing confinement scale and hence Regge spacing: the effective metric is $w_0(z) = e^{-\Phi(z)}/z$ with $\Phi(z) = \kappa^2 z$. The equation of motion for the vectorial field associated expressed in its Fourier transform $f_V(z)$ becomes, [7]

$$\partial_z^2 f_V + \partial_z \left[\ln w_0(z)\right] \partial_z f_V + q^2 f_V = 0 \,, \tag{3}$$

This turns in an *harmonic-oscillator-like* problem, up to the $1/z$ term, for the eigenvalue problem $q^2 = M_V(n)^2$

$$\partial_z^2 \phi_n - \left(\frac{1}{z} + 2\kappa^2 z\right) \partial_z \phi_n + M_V(n)^2 \phi_n = 0 \,, \tag{4}$$

and leading to the two-point function expressed in terms of a Digamma function ψ,

$$Q^2 \Pi_V^{(0)}(Q^2) = -\frac{2\kappa^2}{g_5^2}\left(\frac{Q^2}{4\kappa^2}\right)\left[\gamma_E + \psi\left(\frac{Q^2}{4\kappa^2}+1\right)\right] \,, \tag{5}$$

Practically, the same solution suggested in the past in 4D QCD models [2].

2 The vectors, the axials and chiral symmetry breaking

We propose to modify the dilaton perturbatively for z small adding a polynomial in z, $B(\sqrt{\theta} z)$. The damage for z large should be cured by the fact that Regge is dynamically obtained: θ is an auxiliary parameter ordering the expansion to match the OPE. Our proposal to have a correct OPE turns in a set of perturbative differential equations for f_V,

$$\partial_z^2 f_V - \partial_z \left[\frac{1}{z} + 2\kappa^2 z + b_2 \theta z^2 + b_4 \theta^2 z^4 + b_6 \theta^3 z^6\right] \partial_z f_V + q^2 f_V = 0 \,, \tag{6}$$

with a corresponding solution for the vector two-point function

$$\Pi_V = \Pi_V^{(0)} + \theta \Pi_V^{(1)} + \theta^2 \Pi_V^{(2)} + \theta^3 \Pi_V^{(3)} + \mathcal{O}(\theta^4) \,, \tag{7}$$

which takes the form (where $\mathcal{P}$ is a polynomial)

$$Q^2 \Pi_V^{(n)}(Q^2) = \sum_{k=0}^{n} \mathcal{P}_{k,n}\left(\frac{Q^2}{4\kappa^2}\right) \psi^{(k)}\left(\frac{Q^2}{4\kappa^2}\right) \,. \tag{8}$$

The coefficients b_n of the polynomial $B(\sqrt{\theta} z)$ are obtained by imposing that the large euclidean expansion of eq. (8) coincides with the QCD one eq. (1). The spectrum is Figure 1 while $\Pi_V^{(0)}$ is in eq. (5) and more specifically $\Pi_V^{(1)}$ and $F_V(n)^2$ are

$$\Pi_V^{(1)}(Q^2) = \sum_{n=0}^{\infty} \frac{F_V(n)^2}{Q^2 + M_V(n)^2} = \frac{b_2}{4\kappa^2 g_5^2}\left(\frac{4\kappa^2}{Q^2}\right)\left[1 + \left(\frac{Q^2}{4\kappa^2}\right) - \left(\frac{Q^2}{4\kappa^2}\right)^2 \psi'\left(\frac{Q^2}{4\kappa^2}\right)\right] \,, \tag{9}$$

with

$$\frac{F_V(n)^2}{4\kappa^2} = (1-6n)\left(\frac{3}{4}\frac{\langle \mathcal{O}_4\rangle}{\kappa^4} - \frac{5}{32}\frac{\langle \mathcal{O}_6\rangle_V}{\kappa^6}\right) - \frac{75}{32}\frac{\langle \mathcal{O}_6\rangle_V}{\kappa^6} n^2 \,. \tag{10}$$

We use the same dilaton $\Phi(z)$ and polynomial $B(\sqrt{\theta}z)$, and, consistently with the AdS prescription for chiral symmetry breaking, we add a scalar field with the vacuum profile

$$\left(\frac{v(\sqrt{\theta}z)}{z}\right)^2 = \beta_0 + \beta_2 z^2\theta + \beta_4 z^4\theta^2 + \beta^* z\delta(z) \ , \tag{11}$$

where in Ref. [6], compared to typical HW or SW metric in Ref.[9], we have added $\beta_0 = 4\kappa^2/g_5^2$ and also a contact term $\beta^* z\delta(z)$. Based on the phenomenological relation

$$M_{a_1}^2 = 2\ M_\rho^2 \ , \tag{12}$$

we reconcile the axial spectrum in Figure 2 with the phenomenological one as a shifted vectorial one leading to

$$\Pi_A(Q^2) = \Pi_V(Q^2 + 4\kappa^2) + \begin{pmatrix}\text{Corrections to obtain OPE} \\ \text{involving } \beta_{2k} \text{ coefficients}\end{pmatrix} \ . \tag{13}$$

Also the contact term β^* is needed in order to obtain the correct OPE.

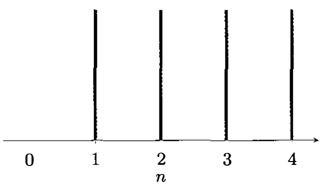

Figure 1 – Vectorial spectrum

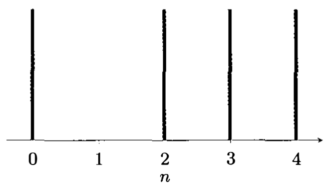

Figure 2 – Axial spectrum

3 The chiral constants

Now we believe that equations (2), ruling F_π and L_{10}, are theoretically more clean than those of $\Pi_{V,A}$:

$$F_\pi^2 = \frac{N_c\kappa^2\left(180\zeta(3) + 191 - 41\pi^2\right)}{72\pi^2} + \frac{5\left(\pi^2 - 10\right)}{2}\frac{\langle\mathcal{O}_4\rangle}{\kappa^2} - \frac{45\left(\pi^2 - 10\right)}{56}\frac{\langle\mathcal{O}_6\rangle_V}{\kappa^4} \ , \tag{14}$$

using $N_c = 3$, $\kappa = \sqrt{1.43/4}\,\text{GeV} \simeq 0.6\,\text{GeV}$[8] and the values of the condensates $\langle\mathcal{O}_4\rangle = (-0.635\pm 0.04)\cdot 10^{-3}\,\text{GeV}^4$ and $\langle\mathcal{O}_6\rangle_V = (14\pm 3)\cdot 10^{-4}\,\text{GeV}^6$ from [3], we obtain

$$F_\pi \simeq \sqrt{4099.9 + 579 + 1147.8}\ \text{MeV} \simeq 76\ (\pm 3)_{\text{ext.}}\ \text{MeV} \ , \tag{15}$$

the error in (15) are coming from the errors quoted for σ and the condensates [a]. While for L_{10}

$$\begin{aligned} L_{10} = & \frac{N_c(8010\zeta(3) + 495 - 585\pi^2 - 46\pi^4)}{8640\pi^2} \\ & + \frac{-72\zeta(3) - 12 + 11\pi^2}{64}\frac{\langle\mathcal{O}_4\rangle}{\kappa^4} + \frac{5[5216\zeta(3) + 67 - 33\pi^2]}{1792}\frac{\langle\mathcal{O}_6\rangle_V}{\kappa^6} \ , \end{aligned} \tag{16}$$

$$10^3 L_{10} \simeq -4.6 - 0.8 + 0.1 \simeq -5.3\,(\pm 1)_{\text{ext.}} \ \cdot \tag{17}$$

Both are in incredible good agreement with experiments. Particularly interesting is F_π where numerically all chiral symmetry breaking effects ($\beta_0, \beta_2, \beta_4$ and β^*) are relevant see Figures 3 and 4.

[a] Let us notice that if we had made the choice for $\sigma \simeq 0.9\,\text{GeV}^2$ as in [7] the values obtained would have been $F_\pi \simeq 80$ MeV and $10^3 L_{10} \simeq 80$ MeV and $10^3 L_{10} \simeq -6.2$ which remain quite acceptable too.

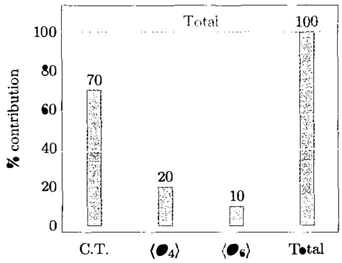

Figure 3 – Relative contributions to F_π^2

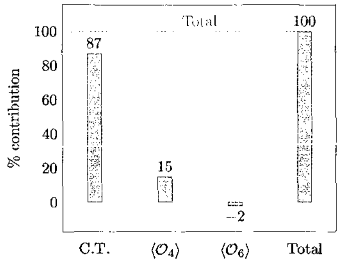

Figure 4 – Relative contributions to L_{10}

4 Conclusions

We think that it is important to realize chiral properties from a Regge theory analytically. Our results are phenomenologically very interesting (see Section 3) even if more work is needed. Somewhat maybe this work is on the spirit of Coleman Witten theorem that in large N_c established chiral symmetry breaking if there is confinement: we have followed their prescription but differently from their assumptions we show that several chiral breaking symmetry mechanisms are needed.

Acknowledgments

We thank the organizers for the invitation to *Moriond QCD* 2015 in particular Pr. Maria Krawczyk and the secretariat in particular Mrs. Vera de Sa-Varanda.

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