Higgs Amplitudes from $\mathcal{N}=4$ Supersymmetric Yang-Mills Theory

Andreas Brandhuber,[1,2,*] Martyna Kostacińska,[1,†] Brenda Penante,[3,‡] and Gabriele Travaglini[1,2,4,§]
[1]*Centre for Research in String Theory, School of Physics and Astronomy, Queen Mary University of London, Mile End Road, London E1 4NS, United Kingdom*
[2]*Kavli Institute for Theoretical Physics, University of California, Santa Barbara, California 93106, USA*
[3]*CERN Theory Division, 1211 Geneva 23, Switzerland*
[4]*Institut für Physik und IRIS Adlershof, Humboldt-Universität zu Berlin, Zum Großen Windkanal 6, 12489 Berlin, Germany*
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Higgs plus multigluon amplitudes in QCD can be computed in an effective Lagrangian description. In the infinite top-mass limit, an amplitude with a Higgs boson and n gluons is computed by the form factor of the operator $\mathrm{Tr}F^2$. Up to two loops and for three gluons, its maximally transcendental part is captured entirely by the form factor of the protected stress tensor multiplet operator $\mathcal{T}_2$ in $\mathcal{N}=4$ supersymmetric Yang-Mills theory. The next order correction involves the calculation of the form factor of the higher-dimensional, trilinear operator $\mathrm{Tr}F^3$. We present explicit results at two loops for three gluons, including the subleading transcendental terms derived from a particular descendant of the Konishi operator that contains $\mathrm{Tr}F^3$. These are expressed in terms of a few universal building blocks already identified in earlier calculations. We show that the maximally transcendental part of this quantity, computed in non-supersymmetric Yang-Mills theory, is identical to the form factor of another protected operator, $\mathcal{T}_3$, in the maximally supersymmetric theory. Our results suggest that the maximally transcendental part of Higgs amplitudes in QCD can be entirely computed through $\mathcal{N}=4$ super Yang-Mills theory.

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Introduction.—Whether or not it will be discovered in present or future searches at the Large Hadron Collider (LHC), supersymmetry is a powerful organizational principle of perturbative calculations in quantum field theory. One example of such success is the one-loop supersymmetric decomposition [1], whereby the calculation of a one-loop scattering amplitude in pure Yang-Mills theory—a crucial ingredient for constructing QCD amplitudes—is traded for three simpler calculations: that of the same amplitude in the $\mathcal{N}=4$ (or maximally) supersymmetric Yang-Mills (SYM) theory, plus the contributions of an $\mathcal{N}=1$ chiral multiplet and a scalar running in the loop. The technical difficulty in dealing with gluons in the loop is thus replaced by three simpler calculations, two of which are performed in supersymmetric theories.

Supersymmetry makes a remarkable appearance in the principle of maximal transcendentality [2,3], allowing anomalous dimensions of twist-two operators in $\mathcal{N}=4$ SYM theory to be obtained from those computed in QCD [4,5] by simply deleting all terms with degree of transcendentality less than maximal ($2L$ at L loops). Conversely, one can say that $\mathcal{N}=4$ SYM captures the "most complicated," or maximally transcendental part of this QCD result (of course, technically such terms are much easier to obtain than, for instance, rational terms that may require the use of D-dimensional unitarity). Alas, scattering amplitudes in general do not satisfy the principle of maximal transcendentality. For instance, an n-point maximally helicity violating (MHV) amplitude computed in pure Yang-Mills theory for generic n receives additional contributions that have maximal transcendental degree already at one loop [1,6,7].

Multigluon Higgs amplitudes seem to provide a fortunate exception where the principle of maximal transcendentality may in fact apply [8]. To discuss this, we recall that gluon fusion through a top-quark loop is the dominant mechanism for Higgs production at the LHC; in an approximation where the mass of the top, m_t, is much larger than the mass of the Higgs boson, m_H, an effective Lagrangian description can be used to compute such amplitudes. The leading-order term is a dimension-five operator $\mathcal{L}^{(0)} \sim H\mathrm{Tr}F^2$, where H represents the Higgs field and F is the gluon field strength [9–11]. Hence, Higgs plus multigluon amplitudes at leading order are form factors of $\mathrm{Tr}F^2$. The surprising result of [8] is that the form factor of an "appropriate translation" of the operator $\mathrm{Tr}F^2$ to $\mathcal{N}=4$ SYM, computed in the maximally supersymmetric theory, is identical to the maximally transcendental part of the Higgs plus three-gluon amplitude in QCD of [13], and is independent of the gluon helicities. The same maximally transcendental part appears in a nonminimal two-loop form factor of the Konishi operator in $\mathcal{N}=4$ SYM [12]. This appropriate translation turns out to be the simplest composite operator in the theory, namely, the stress tensor multiplet operator $\mathcal{T}_2$. It is free from quantum corrections, or Bogomol'nyi-Prasad-Sommerfield (BPS) protected, and as such it does not mix with any other operator. Furthermore, its form factors have only infrared divergences. Two components of $\mathcal{T}_2$ are particularly relevant here: the chiral on-shell Lagrangian (the precise expression of $\mathcal{L}_{\text{on-shell}}$ can be found in [14,15]), $\mathcal{L}_{\text{on-shell}} \ni \text{Tr}F^2_{\text{ASD}}$, and $\text{Tr}X^2$, where F_{ASD} is the anti-self-dual part of the gluon field strength and X is one of the three complex scalar fields of $\mathcal{N}=4$ SYM. Note that it is $\mathcal{L}_{\text{on-shell}}$ that does not mix under renormalization, and not $\text{Tr}F^2_{\text{ASD}}$.

In parallel, one can study subleading corrections in m_H/m_t, which in the effective Lagrangian setup are captured by higher-dimensional operators. The first corrections arise at dimension seven and include the interactions $\mathcal{L}^{(1)} \sim H\text{Tr}F^3$ and $\mathcal{L}^{(2)} \sim H\text{Tr}(D_\mu F_{\nu\rho} D^\mu F^{\nu\rho})$ [16–20]. In this paper we compute the two-loop form factor of an appropriate translation to the $\mathcal{N}=4$ theory of the operator $\text{Tr}F^3$ in the case of three gluons. Our key finding is that its maximally transcendental part is identical to that of the contribution arising from the operator $\mathcal{L}^{(1)} \sim H\text{Tr}F^3$ to the Higgs plus multigluon amplitude in QCD. As we show, this maximally transcendental contribution turns out to be identical to the form factor of another special operator, namely, the trilinear half-BPS operators $\mathcal{T}_3$. This is an appropriate supersymmetrization of $\text{Tr}X^3$, whose minimal form factor has been computed in [21] at two loops. Hence the simplest operators in the maximally supersymmetric theory in four dimensions, the half-BPS operators, compute the maximally transcendental part of the nonsupersymmetric Higgs plus multigluon amplitudes.

To identify this appropriate translation, we observe that $\text{Tr}F^3_{\text{ASD}}$ is a trilinear, nonprotected operator that at one loop has the same anomalous dimension as the Konishi operator. A natural choice is to take the descendant obtained by acting with eight $\bar{Q}$ supersymmetries on the Konishi operator $\epsilon^{ABCD}\text{Tr}(\phi_{AB}\phi_{CD})$, landing on $\text{Tr}F^3_{\text{ASD}}$ dressed with appropriate additional terms as required by supersymmetry. Here ϕ_{AB} denote the scalar fields of the theory, with $A,\ldots,D = 1,\ldots,4$ being fundamental $SU(4)$ indices. Note that since this descendant is obtained by acting with tree-level supersymmetry generators, any potential mixing is deferred to one loop. We also pick an external state containing three positive-helicity gluons—a state that is produced by $\text{Tr}F^3_{\text{ASD}}$ acting on the vacuum. Tree-level form factors of the full Konishi multiplet have recently been studied in [22,23].

An earlier two-loop calculation is also relevant here: in [24] we considered the form factors of a particular trilinear descendant of the Konishi operator made mostly of scalar fields, rather than field strengths, namely, $\mathcal{O}_\mathcal{K} = \mathcal{O}_B - gN/(8\pi^2)\mathcal{O}_F$, where $\mathcal{O}_B \coloneqq \text{Tr}(X[Y,Z])$ and $\mathcal{O}_F \coloneqq (1/2)\text{Tr}(\psi\psi)$. The three scalar fields $X \coloneqq \phi_{12}$, $Y \coloneqq \phi_{23}$, $Z \coloneqq \phi_{31}$ and the fermion $\psi_\alpha \coloneqq \psi_{123,\alpha}$ are the letters of the $SU(2|3)$ closed subsector of the $\mathcal{N}=4$ theory [25] (the second term in $\mathcal{O}_\mathcal{K}$ is induced by mixing, and does not contribute to the maximally transcendental part of the result). The maximally transcendental part of the form factor of $\mathcal{O}_\mathcal{K}$ is identical to that of $\mathcal{T}_3$. It is accompanied by additional terms that are subleading in transcendentality, which feature in our discussion below.

Our results can be summarized as follows. First, the infrared-finite two-loop remainder of the form factor of the Konishi descendant containing $\text{Tr}F^3_{\text{ASD}}$, with a state of three positive-helicity gluons, has maximal degree of transcendentality equal to 4. Its maximally transcendental part is identical to the remainder of the half-BPS operator $\mathcal{T}_3$ of [21], and to that of $\mathcal{O}_B$. Remarkably, the universality of this contribution was also found in [24,26,27] for the three closed sectors $SU(2)$, $SU(2|3)$, and $SL(2)$ in the $\mathcal{N}=4$ theory, respectively. Second, our form factor remainder also contains terms of transcendentality ranging from 3 to 0, similarly to the form factor of $\mathcal{O}_B$ [24]. Unlike the case of $\mathcal{O}_B$, our present result is accompanied by polylogarithms multiplied by ratios of kinematic invariants. We find that only a few universal building blocks are needed to describe all such contributions and, interestingly, they are the same as those that appeared in [24] as well as in related computations of the spin chain Hamiltonian performed in [26,27], suggesting the universality of these quantities. Finally, we observe that the computation of the four-dimensional cut-constructible part of the form factor of $\text{Tr}F^3$ in QCD differs from our calculation in $\mathcal{N}=4$ SYM only by certain single-scale integrals of submaximal transcendentality, in the three-gluon case considered in detail here. Hence $\mathcal{N}=4$ SYM captures the maximally transcendental part not only of the leading-order Higgs plus three-gluon amplitudes, as found in [8], but also of the subleading corrections arising from $\text{Tr}F^3$. That the maximally supersymmetric theory may be relevant for computing phenomenologically interesting amplitudes is a happily surprising result.

The rest of the paper is organized as follows. In Sec. I we introduce the building blocks of our two-loop calculation, including the tree-level and one-loop form factors of the relevant operators, and discuss the methodology used to derive the result (details of the calculation will appear in [28]). In Sec. II we present our two-loop result. We conclude in Sec. III by discussing the modifications needed in a calculation performed in nonsupersymmetric Yang-Mills theory and why these do not alter the maximally transcendental part of the $\mathcal{N}=4$ result.

Outline of the computation.—In this Letter we consider form factors of the dimension-six operator $\mathcal{O}_1 \sim \text{Tr}F^3_{\text{ASD}} + \mathcal{O}(g)$ with three positive-helicity gluons up to two loops in $\mathcal{N}=4$ SYM. Note that $\text{Tr}F^3_{\text{ASD}}$ appears in the decomposition of $\text{Tr}(F^3) = \text{Tr}(F^3_{\text{SD}}) + \text{Tr}(F^3_{\text{ASD}})$ and the extra terms denoted by $\mathcal{O}(g)$ have length 4 or higher and are produced by acting with eight tree-level supercharges $\bar{Q}^A_{\dot\alpha}$ on the Konishi operator. In other words $\mathcal{O}_1$ is the (tree-level) descendant of

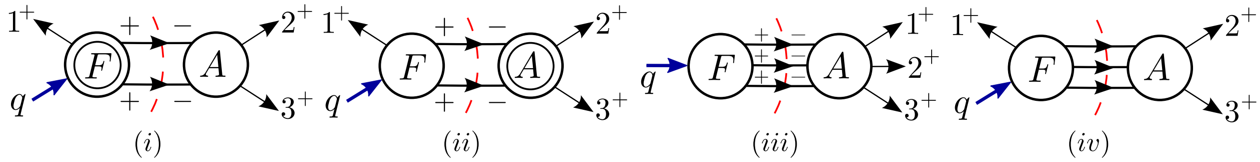

FIG. 1. Four distinct types of cuts considered in the calculation of the two-loop form factor. We also need to include cyclic permutations of the external lines. (i) Two-particle cut in the s_{23} channel with a one-loop form factor and a tree-level amplitude. (ii) Two-particle cut in the s_{23} channel with a tree-level form factor and a one-loop amplitude. (iii) Three-particle cut in the q^2 channel. (iv) Three-particle cut in the s_{23} channel. In this cut we sum over all possible helicity assignments of the internal particles.

the Konishi operator and any corrections due to mixing must appear at one-loop order or higher. The overall normalization of $\mathcal{O}_1$ is fixed so that the minimal tree-level form factor is

$$F^{(0)}_{\mathcal{O}_1}(1^+,2^+,3^+;q) = [12][23][31], \tag{1}$$

where $q \coloneqq p_1 + p_2 + p_3$ is the momentum carried by the operator. For future reference we also introduce the dimensionless ratios of Mandelstam variables $u \coloneqq s_{12}/q^2$, $v \coloneqq s_{23}/q^2$, and $w \coloneqq s_{31}/q^2$, which obey $u + v + w = 1$.

The other operators that can mix with $\mathcal{O}_1$ at this order and with the particular on-shell state we have picked are $\mathrm{Tr}(D^\mu F^{\nu\rho} D_\mu F_{\nu\rho})$, two further operators with different Lorentz contractions, and $q^2\mathrm{Tr}(F^2_{\mathrm{ASD}})$. See [20] for a discussion of suitable operator bases. In practice we need to choose a linear combination of these operators, which we call $\mathcal{O}_2 \sim \mathrm{Tr}(DFDF)$, and which produces the only other possible Lorentz structure with the correct dimension and spinor weights in addition to that of (1). Explicit forms of the operators are not necessary since we use unitarity in our calculation and only tree-level form factors and amplitudes are needed as input. The relevant form factor of the appropriately normalized operator $\mathcal{O}_2$ is

$$F^{(0)}_{\mathcal{O}_2}(1^+,2^+,3^+;q) = \frac{q^6}{\langle 12\rangle\langle 23\rangle\langle 31\rangle} = -\frac{F^{(0)}_{\mathcal{O}_1}(1^+,2^+,3^+;q)}{uvw}, \tag{2}$$

which is the only other possible ultraviolet counterterm form factor.

The minimal one-loop form factor of the operator $\mathcal{O}_1$ is obtained from two-particle cuts involving (1) and four-point tree-level gluon amplitudes. It is given by (see also [18])

$$F^{(1)}_{\mathcal{O}_1}(1^+,2^+,3^+;q) = iF^{(0)}_{\mathcal{O}_1}(1^+,2^+,3^+;q)[2\mathrm{Bub}(s_{12}) + s_{12}\mathrm{Tri}(s_{12}) + \mathrm{cyclic}(1,2,3)], \tag{3}$$

with $\mathrm{Bub}(s) = \{ic_\Gamma/[\epsilon(1-2\epsilon)]\}(-s/\mu^2)^{-\epsilon}$, $\mathrm{Tri}(s) = [(ic_\Gamma)/\epsilon^2](-s/\mu^2)^{-\epsilon}/s$, and $c_\Gamma = \{[\Gamma(1+\epsilon)\Gamma^2(1-\epsilon)]/[(4\pi)^{2-\epsilon}\Gamma(1-2\epsilon)]\}$. From (3) we can infer the one-loop anomalous dimension $\gamma^{(1)}_{\mathcal{O}_1} = 12a$, where $a = g^2N/(4\pi)^2$ is the 't Hooft coupling.

We now proceed to the minimal two-loop form factor of the operator $\mathcal{O}_1$. For a detailed discussion of the computation we refer the reader to the forthcoming paper [28]. In order to completely fix the two-loop integrand we use four types of cuts.

First, we consider the two-particle cut in the kinematic s_{23} channel shown in Fig. 1(i) where as building blocks we use the one-loop form factor (3) and a tree-level MHV amplitude. Note that Fig. 1(ii) presents the two-particle cut in this channel with the tree-level form factor of (1) and a one-loop amplitude, but this term gives no extra constraint on the integrand.

Second, we turn to the three-particle cut in the q^2 channel, as presented in Fig. 1(iii). Importantly, the internal loop legs involve gluons with fixed helicity, rendering this cut completely universal and theory independent.

Finally, we consider the three-particle cut in the s_{23} channel, shown in Fig. 1(iv). In this case, the form factor entering the cut is nonminimal and we have several possible helicity configurations for the momenta entering the loops, including fermions and scalars. The relevant nonminimal form factors can be calculated with MHV diagrams [29] applied to form factors [15,20,30] or more recent methods introduced in [22,23,31]. For convenience, we quote some of the nonminimal form factors entering the two-loop cut computations (see also [18,32,33] for related investigations of these quantities),

$$F^{(0)}_{\mathcal{O}_1}(1^+,2^+,3^+,4^-;q) = \frac{([12][23][31])^2}{[12][23][34][41]},$$
$$F^{(0)}_{\mathcal{O}_1}(1^+,2^+,3^+,4^+;q) = \frac{[12][23][34][41]}{s_{12}}\left(1+\frac{[31][4|q|3\rangle}{s_{23}[41]}\right) + \mathrm{cyclic}(1,2,3,4). \tag{4}$$

Extracting the integrand from the cut information is rather involved and we present details of this calculation in [28]. With the help of the *Mathematica* package LiteRed [34,35] the two-loop integrand can be reduced to a basis of master integrals, whose explicit expressions were computed in [36,37]. Finally, whenever possible we have simplified the answer by means of the symbol of transcendental functions [38].

In the next section we use the result of this calculation and present the two-loop remainder function obtained after subtracting infrared divergences.

Results.—The two-loop remainder function of the form factor of a general operator $\mathcal{O}$ was first written in [8] using the same infrared subtraction scheme as its amplitude counterpart [39,40]. It is given by

$$\mathcal{R}_{\mathcal{O}}^{(2)} \coloneqq \mathcal{F}_{\mathcal{O}}^{(2)}(\epsilon) - \frac{1}{2}[\mathcal{F}_{\mathcal{O}}^{(1)}(\epsilon)]^2 - f^{(2)}(\epsilon)\mathcal{F}_{\mathcal{O}}^{(1)}(2\epsilon) + \mathcal{O}(\epsilon), \quad (5)$$

where $\mathcal{F}_{\mathcal{O}}^{(L)} = F_{\mathcal{O}}^{(L)}/F_{\mathcal{O}}^{(0)}$, $f^{(2)}(\epsilon) = -2(\zeta_2 + \epsilon\zeta_3 + \epsilon^2\zeta_4)$ and we have taken out a factor of $a[4\pi e^{-\gamma_E}(-\mu^2/q^2)]^\epsilon$ per loop.

The remainder functions of scattering amplitudes or form factors of protected operators are finite quantities as they are free from ultraviolet (UV) divergences. However, in the case of nonprotected operators, the remainder does contain UV divergences. For the operator in question, we confirm that all infrared (IR) and mixed IR and UV divergences cancel and all $1/\epsilon^k$ terms of the remainder vanish for $k = 2, 3, 4$.

We find that the remainder contains a $1/\epsilon$ UV pole with coefficient $12 - \pi^2 + [1/(uvw)]$. The constant $-\pi^2$ arises from the subtraction scheme (5) and is not part of the anomalous dimension, as in [24]. The $1/(uvw)$ term is an indication of the mixing with the operator $\mathcal{O}_2$ introduced in Sec. II [see Eq. (2)]. Therefore, we define the one-loop corrected operator $\tilde{\mathcal{O}}_1 = \mathcal{O}_1 + Ca\mathcal{O}_2$ and demand that the two-loop UV divergence of the form factor of $\tilde{\mathcal{O}}_1$ is proportional to $F_{\mathcal{O}_1}^{(0)}(1^+, 2^+, 3^+; q)$; i.e., the $1/(uvw)$ term is canceled. This requirement fixes $C = 1/6$ and the coefficient of the two-loop UV divergence to be 12. From this we infer the expected two-loop anomalous dimension of $\tilde{\mathcal{O}}_1$ as $\gamma_{\tilde{\mathcal{O}}_1}^{(2)} = -48a^2$, in agreement with that of the Konishi multiplet at this loop order.

The finite part of the remainder of the form factor of $\mathcal{O}_1$ consists of functions of degree of transcendentality ranging from 4 to 0. We present it here in "slices" of uniform transcendentality m, starting from the maximal degree 4 and denoting each slice as $\mathcal{R}_{\mathcal{O}_1;m}^{(2)}$. The complete remainder is just the sum of all slices. The answer is remarkably simple—it contains only classical polylogarithms and, as we detail below, its building blocks are closely related to those of form factors of other nonprotected operators [24,26,27] hinting at a universal structure encompassing general classes of operators.

Degree 4: The key observation is that the maximally transcendental part of the two-loop remainder of $\mathcal{O}_1$ is identical to that of the BPS operator $\mathcal{O}_{\rm BPS} = {\rm Tr}X^3$, computed in [21] and already recognized as a universal building block in [24,26],

$$\begin{aligned}\mathcal{R}_{\mathcal{O}_1;4}^{(2)} = \mathcal{R}_{\rm BPS}^{(2)} = &-\frac{3}{2}{\rm Li}_4(u) + \frac{3}{4}{\rm Li}_4\left(-\frac{uv}{w}\right) - \frac{3}{2}\log(w){\rm Li}_3\left(-\frac{u}{v}\right) + \frac{1}{16}\log^2(u)\log^2(v) + \frac{\log^2(u)}{32}[\log^2(u) - 4\log(v)\log(w)] \\ &+ \frac{\zeta_2}{8}\log(u)[5\log(u) - 2\log(v)] + \frac{\zeta_3}{2}\log(u) + \frac{7}{16}\zeta_4 + {\rm perms}(u, v, w). \end{aligned} \quad (6)$$

Degree 3: At transcendentality three new interesting structures appear as we get two types of terms: those consisting of pure transcendental functions and those multiplied by rational prefactors taken from the list $\{u/v, v/u, v/w, w/v, u/w, w/u\}$. The terms without any rational prefactors take the form

$$\begin{aligned}\mathcal{R}_{\mathcal{O}_1;3}^{(2)}\Big|_{\rm pure} = &{\rm Li}_3(u) + {\rm Li}_3(1-u) - \frac{1}{4}\log^2(u)\log\left(\frac{vw}{(1-u)^2}\right) \\ &+ \frac{1}{3}\log(u)\log(v)\log(w) \\ &+ \zeta_2\log(u) - \frac{5}{3}\zeta_3 + {\rm perms}(u, v, w), \end{aligned} \quad (7)$$

which, remarkably, is almost identical to the transcendentality-three part $\mathcal{R}_{\text{non-BPS};3}^{(2)}$ of the two-loop remainder of the operator $\mathcal{O}_B = {\rm Tr}(X[Y, Z])$ found in Eq. (4.11) of [24]. Specifically, we have [up to a $\log(-q^2)$ term]

$$\mathcal{R}_{\mathcal{O}_1;3}^{(2)}\Big|_{\rm pure} = \frac{1}{2}(\mathcal{R}_{\text{non-BPS};3}^{(2)} + 4\zeta_2\log(uvw) - 24\zeta_3). \quad (8)$$

We now move on to the terms with rational prefactors, which we label by one of the possible ratios listed above. For concreteness we present the term with prefactor u/w,

$$\begin{aligned}\mathcal{R}_{\mathcal{O}_1;3}^{(2)}\Big|_{u/w} = &\left[-{\rm Li}_3\left(-\frac{u}{w}\right) + \log(u){\rm Li}_2\left(\frac{v}{1-u}\right) - \frac{1}{2}\log(1-u)\log(u)\log\left(\frac{w^2}{1-u}\right) + \frac{1}{2}{\rm Li}_3\left(-\frac{uv}{w}\right) + \frac{1}{2}\log(u)\log(v)\log(w) \right. \\ &\left. + \frac{1}{12}\log^3(w) + (u \leftrightarrow v)\right] + {\rm Li}_3(1-v) - {\rm Li}_3(u) + \frac{1}{2}\log^2(v)\log\left(\frac{1-v}{u}\right) - \zeta_2\log\left(\frac{uv}{w}\right). \end{aligned} \quad (9)$$

Another surprising observation can be made at this point. Comparing (9) with the remainder density $(R_i^{(2)})^{XYX}_{XXY}|_3$ of form factors in the $SU(2)$ sector introduced in Eq. (3.22) of [26], we observe that these are related [up to a $\log(-q^2)$ term],

$$\mathcal{R}^{(2)}_{\mathcal{O}_1;3}\Big|_{u/w} = -(R_i^{(2)})^{XYX}_{XXY}\Big|_3 - \zeta_2 \log(u). \qquad (10)$$

The remaining terms, multiplied by different rational prefactors, follow the same pattern and can be simply found by taking the appropriate permutations of u, v, and w.

Degree 2: At transcendentality 2, again we have two types of terms—those consisting of purely transcendental functions and those multiplied by rational coefficients. The pure part reads

$$\mathcal{R}^{(2)}_{\mathcal{O}_1;2}\Big|_{\text{pure}} = -\text{Li}_2(1-u) - \log^2(u) + \frac{1}{2}\log(u)\log(v) - \frac{13}{2}\zeta_2 + \text{perms}(u,v,w). \qquad (11)$$

The other part consists of terms multiplied by one of the following rational coefficients: $\{u^2/v^2, v^2/u^2, u^2/w^2, v^2/w^2, w^2/u^2, w^2/v^2\}$. The term multiplied by u^2/w^2 has the form

$$\mathcal{R}^{(2)}_{\mathcal{O}_1;2}\Big|_{u^2/w^2} = \text{Li}_2(1-u) + \text{Li}_2(1-v) + \log(u)\log(v) - \zeta_2, \qquad (12)$$

where again the remaining terms are obtained by appropriate permutations of u, v, and w.

Degree 1 and 0: The degree-1 terms can be presented in a very compact form as

$$\mathcal{R}^{(2)}_{\mathcal{O}_1;1} = \left(-4 + \frac{v}{w} + \frac{u^2}{2vw}\right)\log(u) + \text{perms}(u,v,w), \qquad (13)$$

while the degree-0 terms read

$$\mathcal{R}^{(2)}_{\mathcal{O}_1;0} = 7\left(12 + \frac{1}{uvw}\right). \qquad (14)$$

As a final comment we note that the constant part of (14) times $-4/7$ equals the value of the two-loop Konishi anomalous dimension—the same observation was made in [26] for operators in the $SU(2)$ sector.

Beyond $\mathcal{N}=4$ SYM.—In this final section, we argue that the universality of the maximally transcendental part of the remainder function of $F_{\mathcal{O}_1}(1^+, 2^+, 3^+; q)$ is not confined to $\mathcal{N}=4$ SYM, and in fact extends to theories with less supersymmetry, including pure Yang-Mills theory and QCD.

All deviations from $\mathcal{N}=4$ SYM are due to a different matter content (scalars and fermions), and we now analyze how these affect the cuts in Fig. 1. First, we note that the diagrams (i) and (iii) are purely gluonic and, therefore, theory independent. Second, the diagram in Fig. 1(ii) contains a four-point one-loop amplitude. If the matter content is changed compared to $\mathcal{N}=4$ SYM, this amplitude receives modifications through additional bubble integrals [1,6,7], which can only produce two-loop integrals of lower transcendentality. This leaves us with Fig. 1(iv), and we need to analyze the individual contributions from fermions and scalars propagating across the cut. Our computation shows that such contributions appear through the integral topology shown in Fig. 2, which, due to nontrivial cancellations, is absent for $\mathcal{N}=4$ SYM. Evaluating explicitly the integrals with appropriate numerators coming from fermions and scalars crossing the cut, we find again that they only contribute at submaximal transcendental weight. Hence we conclude that the transcendentality-4 slice of the remainder function is indeed universal for this particular form factor in Yang-Mills theories with any amount of supersymmetry and QCD (the presence of fermions in the fundamental representation does not alter this statement).

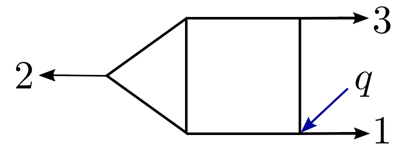

FIG. 2. The single-scale integral topology that incorporates the effect of having a different field content compared to that of $\mathcal{N}=4$ SYM.

We end by commenting on possible extensions of our work that are currently under investigation [28]. An obvious important step is to generalize our calculation to theories with less supersymmetry, including pure Yang-Mills theory and QCD. Here it is important to address potential rational terms that may be missed in less supersymmetric theories when four-dimensional cuts are employed (rather than D-dimensional ones). Note that issues encountered with dimensional regularization in the case of Konishi operator [26] did not arise in [12], where dimensional reduction was used, and in [24] and the present work, where the operator definition does not involve state sums. Other aspects to be discussed in future work are form factors of other dimension-six operators such as $\text{Tr}(DFDF)$ appearing in the effective theory for Higgs plus multiparton scattering, and studies of the operator mixing using subminimal or nonminimal form factors as in [24]. Finally, we are also investigating form factors with more general helicity configurations than the one considered in this Letter. We expect that in all these considerations supersymmetry will emerge as a powerful organizational principle and that results for form factors in QCD will reveal further remarkable similarities with $\mathcal{N}=4$ SYM.

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[*]a.brandhuber@qmul.ac.uk
[†]m.m.kostacinska@qmul.ac.uk
[‡]b.penante@cern.ch
[§]g.travaglini@qmul.ac.uk

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