GRAVITATION AT SHORT DISTANCES : THEORY

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Lowering the string scale in the TeV region provides a theoretical framework for solving the mass hierarchy problem and unifying all interactions. The apparent weakness of gravity can then be accounted by the existence of large internal dimensions, in the submillimeter region, and transverse to a braneworld where our universe must be confined. I review the main properties of this scenario, as well as the warped case, and its implications for observations at non-accelerator gravity experiments.

1 Strings and extra dimensions

In all physical theories, the number of dimensions is a free parameter fixed to three by observation, with one exception: string theory, which predicts the existence of six new spatial dimensions (seven in the case of M-theory). For a long time, string physicists thought that strings were extremely thin, having the smallest possible size of physics, associated to the Planck length $\sim 10^{-35}$ meters. However, the situation changed drastically over the recent years. It has been realized that the "hidden" dimensions of string theory may be much larger than what we thought in the past and they become within experimental reach in the near future, together with the strings themselves [1,2,3]. These ideas lead in particular to experimental tests of string theory that can be performed in particle colliders, such as LHC.

The main motivation came from considerations of the so-called mass hierarchy problem: why the gravitational force remains much weaker than the other fundamental forces (electromagnetic, nuclear strong and weak), at least up to present energies? In a quantum theory, the masses of elementary particles receive important quantum corrections which are of the order of the higher energy scale present in the theory. Thus, in the presence of gravity, the Planck mass $M_P \sim 10^{19}$ GeV attracts all Standard Model particles to become 10^{16} times heavier than what they are. To avoid this catastrophy, one has to adjust the parameters of the theory up to 32 decimal places, resulting in a very ugly fine tuning.

A possible solution is provided by the introduction of supersymmetry, which may be a new fundamental symmetry of matter. One of its main predictions is that every known elementary particle has a partner, called superparticle. Since none of these superparticles have ever been produced in accelerators, they should be heavier than the observed particles. Supersymmetry should therefore be broken. However, protection of the mass hierarchy requires that its breaking scale, i.e. the mass splitting between the masses of ordinary particles and their partners, cannot be larger than a few TeV. They can therefore be produced at LHC, which will test the idea of supersymmetry [4].

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On the other hand, a new idea was proposed that solves the problem if the fundamental string length is fixed to $10^{-18} - 10^{-19}$ meters[3]. In this case, quantum corrections are controlled by the string scale, which is in the TeV region, and do not destabilize the masses of elementary particles. Moreover, it offers the remarkable possibility that string physics may be testable soon in particle colliders.

2 The string scale at the TeV

An attractive and calculable framework allowing the dissociation of the string and Planck scales without contradicting observations is provided by the so-called type I string theory. In this theory, gravity is described by closed strings which propagate in all nine dimensions of space, while matter and all other Standard Model interactions are described by open strings ending on the so-called D-branes (where D stands for Dirichlet)[5]. This leads to a braneworld description of our universe, localized on a hypersurface, i.e. a membrane extended in p spatial dimensions, called p-brane (see Figure 1). Closed strings propagate in all nine dimensions of string theory: in those extended along the p-brane, called parallel, as well as in the transverse ones. On the contrary, open strings are attached on the p-brane. Obviously, our p-brane world must have

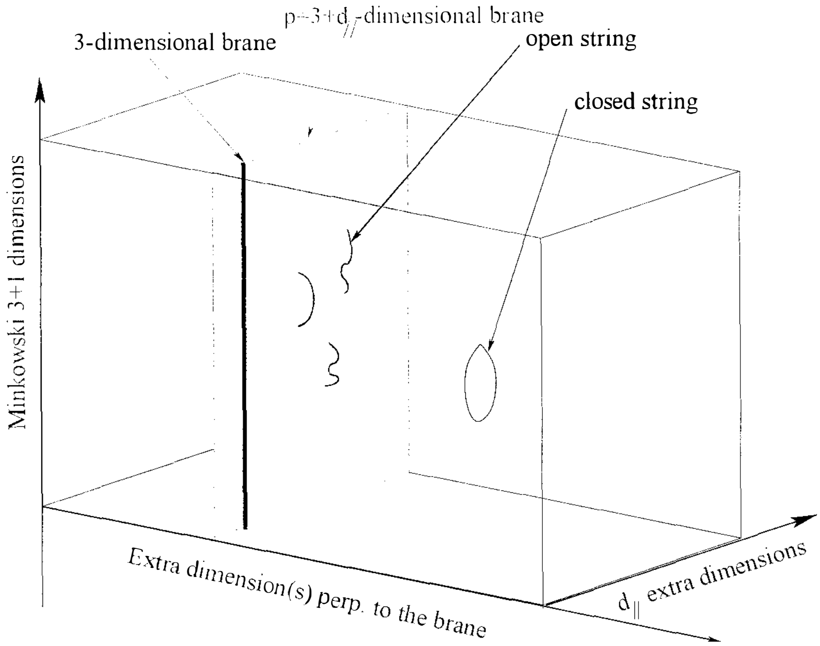

Figure 1: *In the type I string framework, our Universe contains, besides the three known spatial dimensions (denoted by a single blue line), some extra dimensions ($d_{\parallel} = p - 3$) parallel to our world p-brane (green plane) where endpoints of open strings are confined, as well as some transverse dimensions (yellow space) where only gravity described by closed strings can propagate.*

at least the three known dimensions of space. But it may contain more: the extra $d_{\parallel} = p - 3$ parallel dimensions must have a finite size, in order to be unobservable at present energies, and can be as large as $\text{TeV}^{-1} \sim 10^{-18}$ m[1]. On the other hand, transverse dimensions interact with us only gravitationally and experimental bounds are much weaker: their size could reach 0.1

mm[6].

In the framework of type I string theory, the string scale M_s can be lowered in the TeV region at the expense of introducing large transverse dimensions of size much bigger than the string length. Actually, the string scale fixes the energy at which gravity becomes strongly coupled with a strength comparable to the other three interactions, realizing the unification of all fundamental forces at energies lower by a factor 10^{16} from what we thought in past. On the other hand, gravity appears to us very weak at macroscopic distances because its intensity is spread in the large extra dimensions [2]. The basic relation between the fundamental (string) scale and the observed gravitational strength is:

$$\text{total force} = \text{observed force} \times \text{transverse volume}\,,$$

expressing the Gauss law for higher-dimensional gravity. In order to increase the gravitational force at the desired magnitude without contradicting present observations, one has to introduce at least two extra dimensions of size that can be as large as a fraction of a millimeter. At distances smaller than the size of extra dimensions, gravity should start deviate from Newton's law, which may be possible to explore in laboratory tabletop experiments[6,7,8] (see Figure 2).

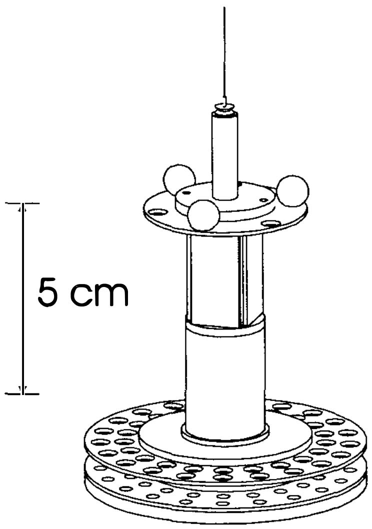

Figure 2: *Torsion pendulum that tested the validity of Newton's law at 55 μm.*

Type I string theory provides a realization of this idea in a coherent theoretical framework. Calculability of the theory implies that parallel dimensions should not be much bigger than the string length, while the size of transverse dimensions is fixed from the observed value of Newton's constant; it should thus vary from the fermi scale (10^{-14} meters) to a fraction of a millimeter, depending on their number (varying from six to two, respectively). It is remarkable that this possibility is consistent with present observations and presents a viable and theoretically well motivated alternative to low energy supersymmetry, offering simultaneously a plethora of spectacular new phenomena that can be tested in laboratory experiments and be a surprise in LHC and other particle accelerators. The main experimental signal is gravitational radiation in the bulk from any physical process on the world-brane, that gives rise to missing-energy. Explicit computation of these effects leads to the collider bounds given in Table 1.

Table 1: Collider bounds on the size of gravitational extra dimensions $R_\perp$ in mm.

Experiment	$n=2$	$n=4$	$n=6$
LEP 2	5×10^{-1}	2×10^{-8}	7×10^{-11}
Tevatron	5×10^{-1}	10^{-8}	4×10^{-11}
LHC	4×10^{-3}	6×10^{-10}	3×10^{-12}

3 Short range forces

There are three categories of predictions in "table-top" experiments that measure gravity at short distances:

(i) Deviations from the Newton's law $1/r^2$ behavior to $1/r^{2+n}$, which can be observable for $n=2$ large transverse dimensions of sub-millimeter size. This case is particularly attractive on theoretical grounds because of the logarithmic sensitivity of Standard Model couplings on the size of transverse space [9], that allows to determine the hierarchy [10].

(ii) New scalar forces in the sub-millimeter range, related to the mechanism of supersymmetry breaking, and mediated by light scalar fields φ with masses:

$$m_\varphi \simeq \frac{m_{susy}^2}{M_P} \simeq 10^{-4} - 10^{-6}\ \mathrm{eV}\,, \tag{1}$$

for a supersymmetry breaking scale $m_{susy} \simeq 1 - 10$ TeV. They correspond to Compton wavelengths of 1 mm to 10 μm. m_{susy} can be either the compactification scale of parallel dimensions $1/R_\parallel$ if supersymmetry is broken by compactification [11], or the string scale if it is broken "maximally" on our world-brane [2,3]. A universal attractive scalar force is mediated by the radion modulus $\varphi \equiv M_P \ln R$, with R the radius of the longitudinal ($R_\parallel$) or transverse ($R_\perp$) dimension(s). In the former case, the result (1) follows from the behavior of the vacuum energy density $\Lambda \sim 1/R_\parallel^4$ for large $R_\parallel$ (up to logarithmic corrections). In the latter, supersymmetry is broken primarily on the brane, and thus its transmission to the bulk is gravitationally suppressed, leading to (1). For $n=2$, there may be an enhancement factor of the radion mass by $\ln R_\perp M_s \simeq 30$ decreasing its wavelength by an order of magnitude [10].

The coupling of the radius modulus to matter relative to gravity can be easily computed and is given by:

$$\sqrt{\alpha_\varphi} = \frac{1}{M}\frac{\partial M}{\partial\varphi}\ ; \quad \alpha_\varphi = \begin{cases} \frac{\partial \ln \Lambda_{QCD}}{\partial \ln R} \simeq \frac{1}{3} & \text{for } R_\parallel \\ \frac{2n}{n+2} = 1 - 1.5 & \text{for } R_\perp \end{cases} \tag{2}$$

where M denotes a generic physical mass. In the longitudinal case, the coupling arises dominantly through the radius dependence of the QCD gauge coupling [11], while in the case of transverse dimension, it can be deduced from the rescaling of the metric which changes the string to the Einstein frame and depends slightly on the bulk dimensionality ($\alpha = 1 - 1.5$ for $n = 2 - 6$) [10]. Such a force can be tested in microgravity experiments and should be contrasted with the change of Newton's law due the presence of extra dimensions that is observable only for $n=2$[6,7]. The resulting bounds for the higher dimensional gravity scale M_*, from an analysis of the radion effects, are [12]:

$$M_* \gtrsim 6\,\mathrm{TeV} \quad (\text{for } R_\perp)\,. \tag{3}$$

In principle there can be other light moduli which couple with even larger strengths. For example the dilaton, whose vacuum expectation value determines the string coupling, if it does not acquire large mass from some dynamical mechanism, can lead to a force of strength 2000 times bigger than gravity [13].

(iii) Non universal repulsive forces much stronger than gravity, mediated by possible abelian gauge fields in the bulk [2,14]. Such fields acquire tiny masses of order M_s^2/M_P, as in (1), due to brane localized anomalies [14]. Although their gauge coupling is infinitesimally small, $g_A \sim M_s/M_P \simeq 10^{-16}$, it is still bigger that the gravitational coupling E/M_P for typical energies $E \sim 1$ GeV, and the strength of the new force would be $10^6 - 10^8$ stronger than gravity.

In Figure 3 we depict the actual information from previous, present and upcoming experiments [6,7,8]. The solid lines indicate the present limits from the experiments indicated. The

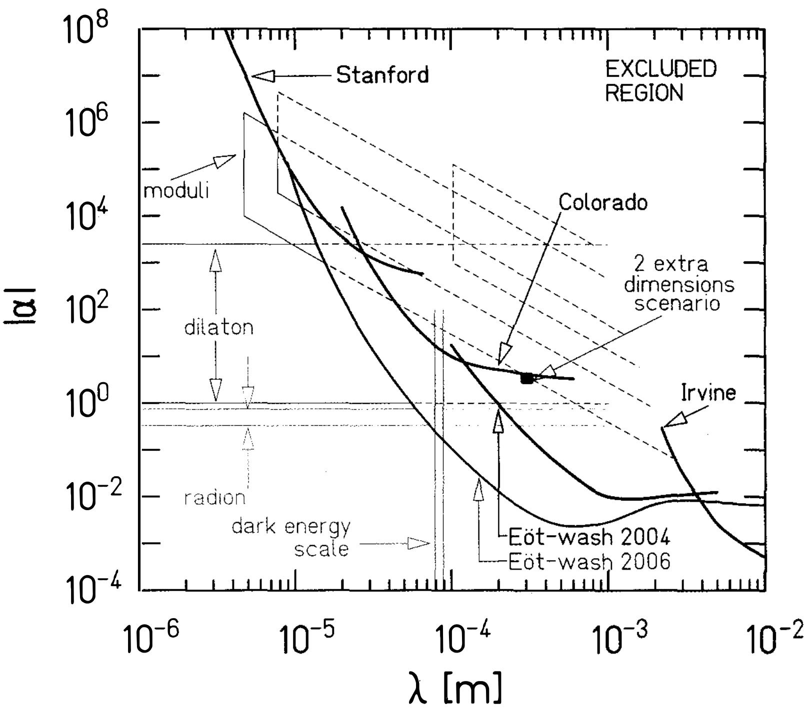

Figure 3: Present limits on new short-range forces (yellow regions), as a function of their range λ and their strength relative to gravity α. The limits are compared to new forces mediated by the graviton in the case of two large extra dimensions, and by the radion.

excluded regions lie above these solid lines. Measuring gravitational strength forces at short distances is challenging. The horizontal lines correspond to theoretical predictions, in particular for the graviton in the case $n = 2$ and for the radion in the transverse case. Finally, in Figures. 4, 5 and 6, recent improved bounds for new forces at very short distances are displayed by focusing on the left hand side of Figure 3, near the origin [7,8].

4 Warped spaces

Braneworld models in curved space (warped metric) with non-compact extra dimensions may lead also to gravity modification at short distances. In particular in RS2, space-time is a slice of anti de Sitter space (AdS) in $d = 5$ dimensions while our universe forms a four-dimensional

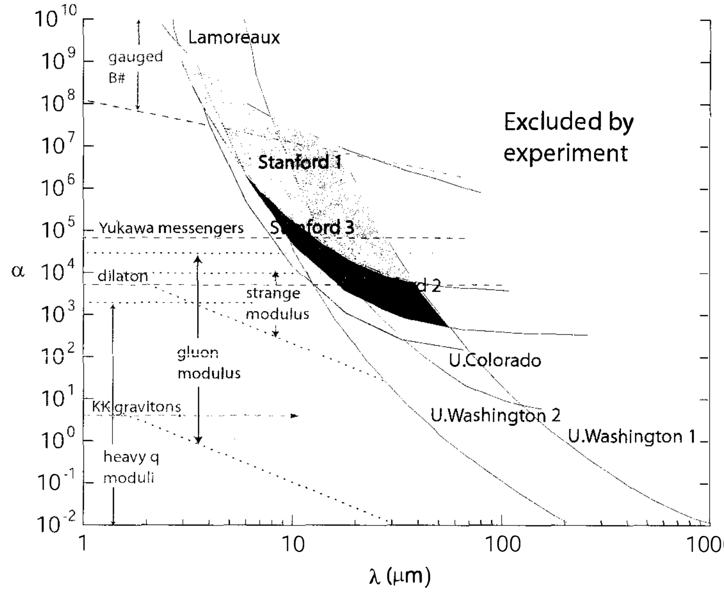

Figure 4: Bounds on non-Newtonian forces in the range 6-20 μm (see S. J. Smullin et al.[7]).

(4d) flat boundary[15]. The 4d Planck mass is given by: $M_P^2 = M_*^3/k$, with $k^2 = -\Lambda/24M_*^3$ in terms of the 5d cosmological constant Λ. Note that M_P is finite, despite the non-compact extra dimension in the 5d AdS space, because of the finite internal volume. As a result, gravity is kept localized on the brane, while the Newtonian potential gets corrections, $1/r + 1/k^2r^3$, which are identical with those arising in the compact case of two flat extra dimensions. Using the experimental limit $k^{-1} \lesssim 0.1$ mm, one finds a bound for the 5d gravity scale $M_* \gtrsim 10^8$ GeV, corresponding to a brane tension $T \gtrsim 1$ TeV. Notice that this bound is not valid in the compact case of six extra dimensions, because their size is in the fermi range and thus the $1/r^3$ deviations of Newton's law are cutoff at shorter distances.

In the presence of the string dilaton, the RS setup has a different solution, which is a linear dilaton background with flat metric in the string frame[16]. An exponential hierarchy is then obtained via the string coupling $g_s^2 = e^{-\alpha r_c}$ with α a mass parameter and r_c the distance of the Planck from the Standard Model brane in the 5th dimension. The 4d Planck mass is now given by: $M_P^2 \sim \frac{M_*^3}{\alpha} e^{\alpha r_c}$. This case extrapolates between flat extra dimension and RS warping with a graviton Kaluza-Klein spectrum $m_n^2 = (n\pi/r_c)^2 + \alpha^2/4$. Because of the mass gap given by α, one extra dimension is possible, for $\alpha^{-1} \lesssim 0.1$ mm with possible deviations of Newton's law in microgravity experiments.

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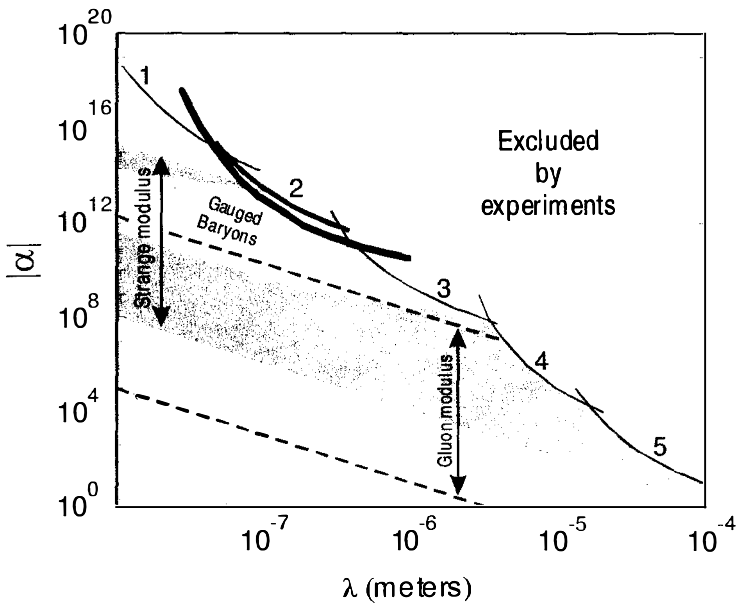

Figure 5: Bounds on non-Newtonian forces in the range of 10-200 nm (see R. S. Decca et al. in Ref. [7]). Curves 4 and 5 correspond to Stanford and Colorado experiments, respectively, of Figure 4 (see also J C. Long and J. C. Price of Ref. [7]).

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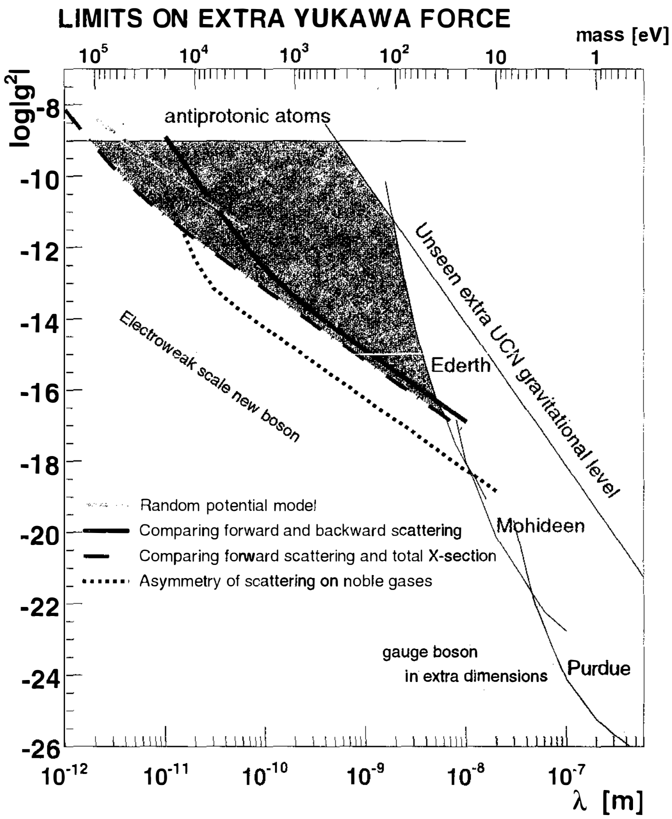

Figure 6: Bounds on non-Newtonian forces in the range of 1 pm-1 nm[8].

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