CERN-TH.6211/91

REGGE CALCULUS: A BIBLIOGRAPHY AND BRIEF REVIEW

Ruth M. Williams [1]

Theory Division, CERN
CH-1211 Geneva 23, Switzerland

and

Philip A. Tuckey [2]

School of Physics and Materials, Lancaster University, Lancaster LA1 4YB, U.K.

ABSTRACT

This paper consists of a brief review of the theory and applications of Regge calculus in classical and quantum gravity, followed by a comprehensive bibliography which we hope will be of use to workers in the subject. Any omissions are unintentional, and we will be pleased to hear of further references to be included in future revisions.

CERN-TH.6211/91
August 1991

[1] Permanent addresses: Girton College, Cambridge CB3 OJG, and D.A.M.T.P, Silver Street, Cambridge CB3 9EW, U.K. E-mail: rmw7 @ uk.ac.cambridge.phoenix

[2] E-mail: P.A. Tuckey @ uk.ac.lancaster

1 INTRODUCTION

The theory of what has come to be called Regge calculus was developed by Regge in 1961. This involved the application of a branch of mathematics, piecewise linear spaces, to general relativity. Rather than considering spaces with smooth curvature, Regge looked at spaces where the curvature is restricted to subspaces of codimension two, based on the division of the manifold into simplicial blocks. He evaluated the Einstein-Hilbert action for such spaces and hence constructed the variational equations which are the analogues of Einstein's equations in this case. These simplicial Einstein spaces are interesting in their own right and are also useful for providing approximation schemes in numerical classical relativity and in quantum gravity. For example, Hawking [1978], following a suggestion of Wheeler [1964], has discussed the way in which Regge calculus can be used to model "space-time foam", where space-time appears nearly smooth on large length scales, but is highly curved, with all possible topologies, on the scale of the Planck length. (Regge calculus also has other uses; for an application to the analysis of image perception, see Jasinchi and Yuille [1989].)

Regge's ideas were expounded in detail by Wheeler [1964] and Misner, Thorne and Wheeler [1973], but for almost twenty years there was only a handful of papers on Regge calculus. However, in the last ten years there has been a great deal of activity, with fruitful results in both classical and quantum areas. We shall give a very brief review of these results and attempt to provide a comprehensive bibliography (see also Williams [1991]). For a simple introduction to the concepts of Regge calculus, see Williams [1986b], and for a review of progress in the first part of the current activity, see Lewis's thesis [1982b]. A discussion of the use of Regge calculus in quantum gravity is included in Isham's review talk at GR11 (Isham [1987]).

For convenience, the review is divided into sections, but the division is sometimes rather artificial and there is often a great deal of overlap.

2 DEVELOPMENT OF THE CLASSICAL THEORY

2.1 The Geometry of a Simplicial Space-Time

There is a considerable amount of mathematical literature on piece-wise linear spaces (see, for example, Banchoff [1982] and references therein). No attempt will be made to survey this here, nor will we describe other discrete approaches to gravity, classical or quantum, which do not directly involve Regge calculus. Some of the earliest work on Regge calculus was done by Sorkin, and this is described in his thesis [1974]. He showed how to formulate the initial-value and time-evolution problems in simplectic terms [1975a]. This work has led to the recent realization that the Regge evolution equations are local and that vertices on a space-like surface can be evolved forward in time individually or even sometimes in parallel (Barrett, Miller, Sorkin, Tuckey and Williams, [1991]). Sorkin also extended Regge's approach to the electromagnetic case [1975b]. This involved the development of an affine tensor formalism and exterior calculus, which were discussed further by Warner [1982] and Brewin [1986]. Hartle and Sorkin [1981] extended Regge's action to include the extra term needed to describe a simplicial manifold with boundary. Bruno, Ellis and Shepley studied the singularity structure of a simplicial space-time with some null hinges [1987].

The consequences of Wheeler's suggestion that the principle that "the boundary of a boundary is zero" be applied to Regge calculus were investigated by Miller [1986a]. This paper also contains a discussion of the Bianchi identities in Regge calculus, which were given a very simple topological interpretation by Regge [1961]. They were discussed further by Roček and Williams [1982], Miller [1986b], Brewin [1988a] and Tuckey [1988]. In studying the relationship between Regge calculus and Einstein gravity in three dimensions, Roček and Williams [1985] showed that the Bianchi identities imply momentum conservation. This work was extended to four dimensions and its application to cosmic strings pointed out by Bezarra [1988]. Other work on cosmic strings, using a Regge calculus approach, includes that of Clarke, Ellis and Vickers [1990] on the limits on bending, and of Hellaby [1989] on collisions.

The construction of conformal transformations for a Regge calculus space-time was discussed by Roček and Williams [1982] and used by Piran and Williams [1986a] to solve the initial value problem according to York's prescription.

A method for tracing geodesics of particles and light rays through Regge calculus space-times was developed by Williams and Ellis [1981] and applied to a number of problems in the Schwarzschild geometry [1984]. Brewin [1988e] gave a local construction for a marginally trapped surface in a simplicial space-time, which should be useful in investigating the existence of black holes. Brewin [1981a] also showed how to calculate the Arnowitt-Deser-Misner energy and three-momentum for a finite simplicial space.

2.2 The Continuum Limit

The relationship between an approximation scheme and the corresponding continuum theory is of crucial importance and there have been extensive studies on this aspect of Regge calculus in the last ten years. To complement Regge's derivation of the simplicial action, Cheeger, Müller and Schrader [1982,1984], while considering the more general class of Lipschitz-Killing curvatures, showed rigorously that the Regge action converges to the continuum action, in the sense of measures, provided that certain conditions on the fatness of the simplices are satisfied. (See also Budach [1989].)

As part of a more general programme on discrete physics, T.D. Lee and his collaborators looked at the relationship between Regge calculus and Einstein gravity. Approaching in the opposite direction from Cheeger, Müller and Schrader, Friedberg and Lee [1984] derived the Regge action from the continuum action. Feinberg, Friedberg, Lee and Ren [1984] then showed that for any lattice with typical link length l, the deviation from its continuum limit can be expressed as a power series in l^2. They argued for the possibility that lattice theories might be more fundamental than continuum ones.

Barrett, in a series of papers, also explored the continuum limit of Regge calculus. Rather than considering the action, he looked at the relationship between the Regge variational equations and Einstein's equations, and showed [1986, 1987a] the equivalence between the Regge variational equations and the vanishing of the energy momentum flow across a certain hypersurface. He went on to formulate the fundamental theorem of linearized Regge calculus [1987b], that the space of small deviations of the edge lengths away from flat space, modulo the subspace of deviations due to translations of the vertices leaving the space flat, is equivalent to the space of linearized deficit angles which satisfy the Bianchi identities. This theorem was proved [1988a] using results on exact sequences in simplicial cohomology. Barrett then proposed a convergence criterion for sequences in linearized Regge calculus [1988b]; this definition ensures that solutions of the linearized Regge equations converge to analytic solutions of the linearized Einstein equations. Barrett and Williams [1988] constructed examples of such sequences for a hypercubic lattice divided into simplices. A start has been made on extending this work on convergence to the general (non-linearized) case (Barrett and Parker, [1990]).

Using a different approach from Barrett, Brewin [1989b] proved that the Regge and Einstein actions have equal values on almost-flat simplicial space-times, and that the Regge and Einstein equations are equivalent there.

2.3 Three-Plus-One Formulations of Regge Calculus

Three-plus-one versions of Regge calculus are attempts to construct a simplicial analogue of continuum 3+1 general relativity, which involves a preferred set of space-like hypersurfaces. The main motivations are firstly for ease of interpretation in classical time-evolution calculations, and secondly as a basis for discrete canonical quantum gravity. The Regge calculus versions are of two types, discrete time and continuous time formulations.

In the discrete time versions, the hypersurfaces are separated by finite time intervals, and the shapes of the blocks distinguish between space and time. The general method developed by Porter [1982] and the null-strut calculus developed by Miller [1986c] are of this type, as are the numerical calculations of Brewin [1983] and Dubal [1987] (see section 3 for more details). Further work on this approach has been done by Tuckey [1988], who considered the problem of finding independent variables to parametrize the metric in his method using truncated simplices [1989], and studied the structure of the time-evolution equations [1990]. (See also Tuckey [1991].)

There are two main approaches to the continuous time formulation of 3+1 Regge calculus. The first starts with a discrete time formulation and takes the limit where the hypersurfaces become infinitesimally close. This was the approach of Collins and Williams [1973, 1974], and more recently of Brewin, who derived formulae for the Riemann and extrinsic curvature tensors [1988b] and used the Gauss-Codacci equations to relate the deficit angles in the three-dimensional space of a 3+1 discrete space-time to the four-dimensional deficits [1988c]. The validity of the scheme was tested successfully for a simple cosmological model (Brewin [1988d]).

The other approach to continuous time 3+1 Regge calculus involves a direct discretization of the quantities in the continuum theory. (This has the advantage that one needs to visualize only three-dimensional discrete spaces, not four-dimensional discrete space-times.) The earliest work was by Lund and Regge [1974], who wrote down the form of the action for homogeneous and isotropic spaces, and applied the formalism to some simple model universes. In the homogeneous and isotropic case, the momentum constraints are automatically satisfied, so there is no need to introduce a shift vector, and the Hamiltonian constraint is preserved in time by the evolution equations. In extending the formalism to inhomogeneous spaces, Piran and Williams [1986a, 1986b], chose to omit the shift vector, arguing that it did not have a natural representation in the simplicial structure of the hypersurfaces. As a consequence, their theory contained no momentum constraints, and the Hamiltonian constraints were not conserved in time. Progress on this problem was made by Friedman and Jack [1986], who set up a theory which included a shift vector and momentum constraints. In contrast to the continuum theory, the algebra of the Hamiltonian and momentum constraints failed to close, leading to problems with the preservation of constraints. Friedman and Jack applied the Dirac procedure for constrained dynamical systems and were able to satisfy the constraint equations by solving them for the lapse and shift functions. Miller and Tuckey [1990] used the Friedman and Jack formalism to study a simple model universe and showed that the method is more complicated than expected. Tuckey and Williams [1990] applied the Dirac procedure to the Piran-Williams formalism and obtained a consistent set of constraints for an inhomogeneous anisotropic universe with cosmological constant.

The fact that for discrete theories the constraint algebra does not close as it does in the continuum theory, has led to various alternative approaches to 3+1 Regge calculus. Waelbroeck [1989, 1990] reviewed the problem and provided a solution for the simpler case of 2+1 gravity. In his formulation, Brewin [1988a] claimed that the constraints were preserved by the evolution as a result of the Bianchi identities. Bander attempted to prove closure of the constraints by two methods, firstly [1987] by transcribing to a simplicial lattice the work of Teitelboim on the relation between constraints and deformations, and secondly [1988] by using a discrete moving frame formalism to express Hamiltonian gravity in lattice terms. However, neither method provides a complete solution to the problem. Khatsymovsky [1989] extended Bander's work on the tetrad formulation, derived its continuous time limit [1991a] and wrote down Ashtekar-like variables for Regge calculus [1991b].

3 CLASSICAL APPLICATIONS

3.1 Simple Simplicial Einstein Spaces

Some simple solutions of Regge's variational equations have been found in cases of high symmetry. Hartle [1986a] found extrema of the Regge action for triangulations of S^4 and CP^2 which approximate solutions of the continuum theory. This is part of his "simplicial minisuperspace" work (see Section 5.2) and an earlier paper in this series (Hartle [1985a]) gives many useful formulae for practical Regge calculus. Piran and Strominger [1986] considered various triangulations of S^3 and S^4, finding solutions in each case corresponding to continuum solutions, and one solution on S^3 with no continuum analogue. Hartle, Sorkin and Williams [1991] studied the problem of finding solutions on product manifolds, illustrating it with a solution on $S^2 \times S^2$.

3.2 The Initial-Value Problem and the Time-Development of Simple Model Universes

Some of the earliest applications of Regge calculus were to the initial-value problem. Wong [1971] constructed initial hypersurfaces for the Schwarzschild geometry using two different types of block. Collins and Williams [1972] extended his ideas to a geometry with a non-spherical throat and one with two throats, and also solved the initial-value problem for a space with topology T^3. Williams [1985] solved the initial-value problem for the Taub universe using a tessellation of S^3 with 600 blocks.

The time-development of a number of simple model universes has been calculated using Regge calculus. Collins and Williams, following a suggestion of Wheeler [1964], investigated the Friedmann universe [1973] and the Tolman universe [1974] containing dust and radiation respectively. The Friedmann universe was also studied by Connors [1978]. Using his formulation of the action integral for pure dust in a Regge space-time [1984], Brewin re-examined and corrected the Friedmann universe calculation, and introduced new models by subdivision [1987a]. In his Ph.D. thesis [1983], he set up a comprehensive formalism for studying Friedmann cosmologies and discussed how to generalize the usual Regge calculus ways of imposing homogeneity and isotropy. Lewis [1982a] studied the time-development of the spatially flat Friedmann-Robertson-Walker universe and of the Kasner universe with topology T^3, and showed that the equations agreed with the analytic ones in the continuum limit. This was the first work in which this feature was demonstrated explicitly.

Some recent calculations have involved the use of 3+1 formulations of Regge calculus with continuous time; these include studies of inhomogeneous anisotropic cosmologies with a massive scalar field, by Piran and Williams [1985], and the Taub universe by Tuckey and Williams [1988], in addition to those already mentioned in Section 2.3. Other time-development calculations are closer to genuine numerical relativity and will be described in the next subsection.

3.3 Numerical Schemes in Classical Relativity

One of the principal uses envisioned for Regge calculus was as a tool in numerical relativity, somewhat related to the finite element method. Early work in this direction was done by Sorkin [1974, 1975a], but this did not lead to any numerical applications. The first extensive work was by Porter [1982] who considered the time-evolution of certain inhomogeneous spaces. He set up a 3+1 formalism with discrete time [1987a] and applied it to spherically symmetric space-times containing a black hole [1987b]. His results agreed very well with the analytic solution. He also solved the initial-value problem for polytropic stars [1987c].

Continuing from where Porter left off, Dubal [1985, 1986] developed a technique for including matter flow between blocks using velocity potentials. He set up [1989b] and solved [1990] the time-evolution equations for the spherical collapse of polytropic stars. The Regge calculus results agree well with the continuum ones, except when a bounce occurs in the collapse process. Dubal also solved the initial-value problem for axisymmetric non-rotating vacuum space-times as a prelude to studying axisymmetric collapse [1989a]. For a summary of this work, see Dubal [1990].

An alternative approach to the usual numerical schemes has been developed in Texas by Wheeler, Miller and their collaborators. The essential idea of "null-strut calculus" is to build simplicial space-times with the maximal number of null edges. This reduces the number of variables in the equations to be solved and results in great simplification of one type of equation, which becomes a linear relation between deficit angles. Null simplices were first discussed by Miller and Wheeler [1985] and the development of null-strut calculus reviewed by Miller [1986b] and described in detail in his Ph.D. thesis [1986c]. In a series of papers, Kheyfets, LaFave and Miller developed the theory of null-strut geometrodynamics further, with both classical and quantum applications. The topics discussed include parallel transport, extrinsic curvature [1989a, 1989b] and spinor variables of both Penrose and Ashtekar types [1988]. Kheyfets, Miller and Wheeler [1988] described the first application of null-strut calculus, to the Kasner universe, which agrees with the analytic solution to very high accuracy. This was followed by an application to the Friedman universe (Kheyfets, LaFave and Miller [1990a, 1990b]).

4 FOUNDATIONS OF SIMPLICIAL QUANTUM GRAVITY

4.1 A Model Theory; Diffeomorphisms

The difficulties, both technical and conceptual, of formulating a continuum theory of quantum gravity, have led to various approaches using simplicial space-times. Most work so far has been based on path-integral methods, rather than on canonical quantization. As an example, we shall now briefly describe one such theory.

Rather than just starting with the Regge calculus formulation and attempting to quantize it in some way, Lehto, Nielsen and Ninomiya, in a series of papers, went back to the basic notion of a simplicial space-time and asked some very fundamental questions about the quantum theory based on it. This work, described in detail in Lehto's thesis [1988], considered an abstract simplicial complex, quantized using Euclidean path integral methods. The abstract simplicial complex was realized as a Regge lattice on a four-dimensional manifold, with the simplest topologies dominating, and the resulting theory was non-local. However, this non-locality was shown to be only of finite range [1986a], using an ingeneous generalization of a correlation decay theory in statistical mechanics [1984] and the theory is equivalent to Einstein's in the long wave-length limit. This result was illustrated by an explicit calculation in the case of one-dimensional simplicial quantum gravity [1987] but the corresponding calcuation in higher dimensions would be much more difficult.

In addition to the locality condition, another requirement of a theory of quantum gravity is invariance under diffeomorphisms: Lehto, Nielsen and Ninomiya [1986b] established this in the case where space is flat or almost flat, and gave suggestions as to how to extend the arguments to more highly curved spaces. The recovery of the diffeomorphism group in the continuum limit was also discussed by Hartle [1985a]. Roček and Williams [1984] showed how to write down "gauge transformations" which leave the action invariant, for almost flat spaces. Römer and Zähringer [1986] proposed a way of avoiding double counting in the functional integral (and effectively dividing out by the volume of the diffeomorphism group) by summing over just one simplicial representative of each geometry, the simplicial complex with edges which are most nearly equilateral. They showed that this gauge fixing provides a good approximation in the strong coupling régime.

4.2 The Measure; Matter Fields; Relation to Gauge Theories

There are many technical questions which must be answered in order to set up a quantum field theory for gravity. In an unpublished paper, Fröhlich [1982] discussed many of these questions, including the definition of the measure, the establishement of unitarity using reflection positivity, the use of renormalization group techniques and the introduction of matter fields. The measure was also discussed by Bander [1986] and Hartle [1985a]. Martellini and Marzuoli [1986] outlined a possible application of the renormalization group using cone subdivision. Jourjine [1985a, 1985b, 1986a, 1986b] discussed the relation between gauge theories and Regge calculus [1987] and described how to introduce matter fields. This was also studied by Ren [1988] and by Drummond [1986] in his first-order formulation of Regge calculus (see also Martellini [1979]). Casselle, D'Adda and Magnea [1989] reformulated Regge calculus on the dual lattice in a way which can be interpreted as either a first- or second-order formalism. Their compactified form of the Regge action (involving sine of the deficit angle) was also used by Kawamoto and Nielsen [1990] in their version of lattice gauge gravity with fermions. Early work by Weingarten [1977, 1982] set up a general formulation for lattice quantum gravity in the style of a gauge theory.

5 QUANTUM APPLICATIONS

5.1 Some Analytic Calculations

Two Dimensions

The relationship between a Regge calculus version of two-dimensional quantum gravity and the Polyakov string was first discussed by Jevicki and Ninomiya [1985]. Using invariants, they derived [1986] a non-local form of the measure appropriate to integrating over Regge manifolds. Further work on two-dimensional quantum Regge calculus and string theory was done by Förster [1987a, 1987b].

Three Dimensions

The first application of Regge calculus in quantum gravity was by Ponzano and Regge [1968], who pointed out the relationship between a sum involving the asymptotic values of $6j$-symbols associated with a triangulated three-manifold, and the path integral for three-dimensional simplicial quantum gravity with the Regge action. This work was developed further by Hasslacher and Perry [1981]and Lewis [1983]. Monssouris [1983] gave a new proof of the Ponzano-Regge result and showed how to generalize the $SU(2)$ recoupling theory to other groups.

Four Dimensions

The earliest work on quantum Regge calculus in four dimensions involved the study of small perturbations about a flat space background. Roček and Williams [1981, 1984] derived the propagator in the Euclidean case and showed that it agreed with the continuum propagator in the long wavelength limit. Williams [1986a] performed a similar calculation in the Lorentzian case and derived gravitational wave solutions. An expression for the graviton propagator was also derived by Feinberg, Friedberg, Lee and Ren [1984] and for the scalar and fermion propagators by Ren [1988].

5.2 Simplicial Minisuperspace

In a series of papers dealing with applications of Regge calculus in both classical and quantum gravity, Hartle developed the subject of "simplicial minisuperspace" (see also Section 3.1). In quantum cosmology, it has been proposed that the wave function of the universe can be calculated by summing the exponential of minus the Euclidean gravitational action with cosmological constant; the sum is over all compact four-geometries which have the required three-geometry of the universe as boundary. Clearly, to sum over all possible such geometries is extremely hard, and, to investigate the proposal, the sum must be approximated. One method of doing this is to sum over simplicial geometries which are described by only a finite number of parameters. Hartle [1985a] described in detail how this might be done. The unboundedness of the Einstein action leads to convergence problems for the functional integral, and in the first application of simplicial minisuperspace using a five-simplex model [1989] having one integration parameter, he showed how to rotate the contour to make the integral converge. In a three-dimensional calculation using a model with two integration variables, Louko and Tuckey [1991] showed that the conformal degree of freedom could be separated and its integration contour deformed to give a convergent integral, at least in the case of vanishing cosmological constant.

In principle, the sum over histories in the path integral approach should involve not only a sun over metrics, but also over manifolds with different topologies. The unsolvable problem of classifying manifolds in four and higher dimensions led Hartle to suggest a sum over a more general class of objects than manifolds, unruly topologies [1985b, 1985c, 1986b]. He showed that for two-dimensional quantum gravity, the extension of the sum to the class of pseudo-manifolds satisfies the usual requirements, but the problem is still unsolved in higher dimensions.

5.3 Numerical Calculations in Simplicial Quantum Gravity

Recent progress in the understanding of functional integral methods for simplicial quantum gravity, and the need for a non-perturbative approach, have led to some large-scale numerical studies, using ideas developed in lattice gauge theories. For a review of the use of Regge calculus in numerical quantum gravity, see Hartle [1986c]. Jacobs [1989] and Ambjørn [1991] discussed Regge calculus in the context of current numerical work on discretized two-dimensional random surfaces. Recent numerical work on three-dimensional simplicial quantum gravity has also used the Regge form of the action (see, for example, Ambjørn, Durhuus and Jonsson [1991] and Godfrey and Gross [1991]).

In a series of papers, Hamber and Williams described the use of Monte Carlo procedures to study the quantum fluctuations of a Regge calculus space about an equilibrium configuration to which it has evolved. A higher derivative term, quadratic in the curvature, was introduced [1984] to ensure that the action remained positive and so to avoid problems of convergence of the functional integral. The expectation values of certain operators could then be calculated. The basic theory was reviewed by Hamber [1986] and the results summarized by Hamber and Williams [1985]. In two dimensions [1986a], the lattice propagator was found to agree with the continuum one in the weak field limit, and the numerical results on T^2 exhibited a number of features which are instructive for work in higher dimensions. Similar results were found by Koibuchi and Yamada [1989a, 1989b]. In four dimensions [1986b], the weak-field expansion of the action for the regular triangulation α_5 of S^4 was calculated. In numerical work on T^4, it was found that at strong coupling the system developed an average negative curvature, and evidence was found for a phase transition between $\lambda = 1.0$ and 1.5.

In more recent work, Gross and Hamber [1991] performed a two-dimensional simulation keeping the total area constant, in order to compare the results with those of Knizhnik, Polyakov and Zamolodchikov using conformal field theory. There was good agreement for the torus, and also subsequently for the sphere when an appropriate triangulation was used. The Haussdorf dimension for the model was found to be infinite. Continuing with earlier work in four dimensions, Hamber carried out [1990b, 1991d] further investigation of the phase diagram and critical exponents for pure gravity, using lattices of size up to 16^4. The computer simulations in both two and four dimensions are described in various review talks (Hamber [1990a, 1991a, 1991b]). For a more elementary description of the programme, see Hamber [1991d].

Eliezer [1989] considered the continuum limit of the Hamber-Williams formulation of higher derivative gravity in Regge calculus, and showed in two dimensions that geodesic lengths should be used to ensure convergence.

The other main work on numerical quantum gravity using Regge calculus is that of Berg, who did Monte Carlo simulations using a hypercubic lattice and keeping the total volume constant

[1985a, 1985b]. His results indicated that an exponentially decreasing entropy factor from the measure might cure the problem of the unboundedness of the gravitational action [1986]. These results were summarized in a review [1988], where the unresolved problem of the choice of measure was also discussed.

6 CONCLUSIONS

As is clear from this review, there has been a great deal of progress in the last ten years in both classical and quantum applications of Regge calculus. Many unresolved problems remain. For example, at the classical level, there are questions about degrees of freedom and the rôle of the Bianchi identities. At the quantum level, further work is needed on the continuum limit, unitarity, summing over topologies, operator ordering in canonical simplicial quantum gravity and many other issues. On the numerical side, there is as yet no general "three-dimensional" (i.e., with four space-time dimensions) Regge calculus code for classical evolution equations. It will be very useful to compare computer simulations of quantum gravity in three dimensions based on Regge calculus (Hamber [1991e]) with results obtained with equilateral triangulations. In four dimensions, Regge calculus has been the basis for pioneering work on numerical quantum gravity and it will be extremely interesting to see whether similar results are obtained by other methods.

REFERENCES

Ambjørn J 1991 "2D Gravity, Random Surfaces and All That" *Nucl. Phys.* B (Proc. Suppl.) **20** 701–710

Ambjørn J, Durhuus B and Jonsson T 1991 "Three-Dimensional Simplicial Quantum Gravity and Generalized Matrix Models" *Mod. Phys. Lett.* A **6** 1133–1146

Banchoff T F 1982 "Critical Points and Curvature for Embedded Polyhedra" in *Differential Geometry; Proceedings of the Special Year 1981–1982, University of Maryland* Progress in Mathematics Series **32** (Boston: Birkhäuser) pp 34–55

Bander M 1986 "Functional Measure for Lattice Gravity" *Phys. Rev. Lett.* **57** 1825–1827

Bander M 1987 "Hamiltonian Lattice Gravity: I. Deformations of Discrete Manifolds" *Phys. Rev.* D **36** 2297–2300

Bander M 1988 "Hamiltonian Lattice Gravity: II. Discrete Moving frame Formulation" *Phys. Rev.* D **38** 1056–1062

Barrett J W 1986 "The Einstein Tensor in Regge's Discrete Gravity Theory" *Class. Quantum Grav.* **3** 203–206

Barrett J W 1987a "The Geometry of Classical Regge Calculus" *Class. Quantum Grav.* **4** 1565–1576

Barrett J W 1987b "The Fundamental Theorem of Linearised Regge Calculus" *Phys. Lett.* **190B** 135–136

Barrett J W 1988a "Linearised Regge Calculus" University of Newcastle preprint

Barrett J W 1988b "A Convergence Result for Linearised Regge Calculus" *Class. Quantum Grav.* **5** 1187–1192

Barrett J W, Miller W A, Sorkin R, Tuckey P A and Williams R M 1991 "An Explicit Evolution Scheme for Regge Calculus" in preparation

Barrett J W and Parker P E 1990 "Smooth Limits of Piecewise-Linear Approximations" Cambridge University preprint DAMTP/R-90/11 and Newcastle University preprint NCL-90/TP6

Barrett J W and Williams R M 1988 "The Convergence of Lattice Solutions of Linearised Regge Calculus" *Class. Quantum Grav.* **5** 1543–1556

Berg B 1985a "Exploratory Numerical Study of Discrete Quantum Gravity" *Phys. Rev. Lett.* **55** 904–907

Berg B 1985b "Simulation of Discrete Euclidean Quantum Gravity" Florida State University (Tallahassee) preprint FSU-SCRI-85-3

Berg B 1986 "Entropy Versus Energy on a Fluctuating Four-Dimensional Regge Skeleton" *Phys. Lett.* B **176** 39–44

Berg B 1988 "Quantum Gravity Motivated Computer Simulations" Florida State University (Tallahassee) preprint FSU-SCRI-88-35 (Invited lectures given at XXVII Internationale Universitätwochen für Kernphysik, Schladming, 1988)

Bezarra V B 1988 "Einstein Gravity in the Loop Variable Approach" *Class. Quantum Grav.* **5** 1065–1072

Brewin L 1983 "The Regge Calculus in Numerical Relativity" Ph.D. Thesis, Monash University

Brewin L 1984 "The Energy Momentum Action in Regge Calculus" Vancouver preprint

Brewin L 1986 "Exterior Differentiation in the Regge Calculus" *J. Math. Phys.* **27** 296–301

Brewin L 1987a "Friedman Cosmologies via the Regge Calculus" *Class. Quantum Grav.* **4** 899–928

Brewin L 1988a "A Continuous Time Formulation of the Regge Calculus" *Class. Quantum Grav.* **5** 839–847

Brewin L 1988b "The Riemann and Extrinsic Curvature Tensors in the Regge Calculus" *Class. Quantum Grav.* **5** 1193–1203

Brewin L 1988c "The Gauss-Codacci Equation on a Regge Spacetime" *Class. Quantum Grav.* **5** 1205–1213

Brewin L 1988d "An Application of a First-Order Formulation of the Regge Calculus" *Gen. Rel. Grav.* **20** 383–393

Brewin L 1988e "Marginally Trapped Surfaces in a Simplicial Space" *Phys. Rev.* D **38** 3020–3022

Brewin L 1989a "Arnowitt-Deser-Misner Energy and Three-Momentum for a Simplicial Space" *Phys. Rev.* D **39** 2258–2262

Brewin L 1989b "Equivalence of the Regge and Einstein Equations for Almost Flat Simplicial Space-Times" *Gen. Rel. Grav.* **21** 565–583

Bruno R U, Ellis G F R and Shepley L C 1987 "Quasi-regular Singularities Based on Null Planes" *Gen. Rel. Grav.* **19** 973–982

Budach L 1989 "On the Combinatorial Foundations of Regge Calculus" *Ann. Physik Leipzig* **46** 1–14

Caselle M, D'Adda A and Magnea L 1989 "Regge Calculus as a Local Theory of the Poincaré Group" *Phys. Lett.* B **232** 457–461

Cheeger J, Müller W and Schrader R 1982 "Lattice Gravity or Riemannian Structure on Piecewise Linear Spaces" in *Unified Theories of Elementary Particles (Heisenberg Symposium 1981) Lecture Notes in Physics* ed P Breitenlohner and H P Dürr (Berlin, Heidelberg, New York: Springer)

Cheeger J, Müller W and Schrader R 1984 "On the Curvature of Piecewise Flat Spaces" *Comm. Math. Phys.* **92** 405–454

Clarke C J S, Ellis G F R and Vickers J A 1990 "The Large Scale Bending of Cosmic Strings" *Class. Quantum Grav.* **7** 1–14

Collins P A and Williams R M 1972 "Application of Regge Calculus to the Axially-Symmetric Initial-Value Problem in General Relativity" *Phys. Rev.* D **5** 1908–1912

Collins P A and Williams R M 1973 "Dynamics of the Friedmann Universe Using Regge Calculus" *Phys. Rev.* D **7** 965–971

Collins P A and Williams R M 1974 "Regge Calculus Model for the Tolman Universe" *Phys. Rev.* D **10** 3537–3538

Connors P A 1978 "Computations in Relativistic Astrophysics" D.Phil. Thesis, University of Oxford

Drummond I T 1986 "Regge-Palatini Calculus" *Nucl. Phys.* B **273** 125–136

Dubal M R 1985 "Numerical Methods in General Relativity" M.Phil. Thesis, SISSA, Trieste

Dubal M R 1986 "Regge Calculus in Numerical Relativity" in *Proceedings of the 14th Yamada Conference on Gravitational Collapse and Relativity* ed H Sato and T Nakamura (Singapore: World Scientific) pp 339–349

Dubal M R 1987 "Numerical Computations in General Relativity" Ph.D. Thesis, SISSA, Trieste

Dubal M R 1989a "Initial Data for Time-Symmetric Gravitational Radiation Using Regge Calculus" *Class. Quantum Grav.* **6** 141–155

Dubal M R 1989b "Relativistic Collapse Using Regge Calculus: I. Spherical Collapse Equations" *Class. Quantum Grav.* **6** 1925–1941

Dubal M R 1990 "Relativistic Collapse Using Regge Calculus: II. Spherical Collapse Results" *Class. Quantum Grav.* **7** 371–384

Dubal M R 1991 "Hydrodynamic Computations Using Regge Calculus" to appear in the proceedings of the "Workshop on Numerical Applications of Regge Calculus and Related Topics", Amalfi, Italy (18–21 September 1990) ed I Pinto

Eliezer D 1989 "On the Continuum Limit of Curvature-Squared Actions in the Regge Calculus" *Nucl. Phys.* B **319** 667–686

Feinberg G, Friedberg R, Lee T D and Ren H C 1984 "Lattice Gravity Near the Continuum Limit" *Nucl. Phys.* B **245** 343–368

Förster D 1987a "Non Perturbative $d = 2$ Euclidean Gravity Without a Lattice" *Nucl. Phys.* B **283** 669–680

Förster D 1987b "On $d = 2$ Regge Calculus Without Triangulation: The Supersymmetric Case" *Nucl. Phys.* B **291** 813–828

Friedman J L and Jack I 1986 "3 + 1 Regge Calculus with Conserved Momentum and Hamiltonian Constraints" *J. Math. Phys.* **27** 2973–2986

Friedberg R and Lee T D 1984 "Derivation of Regge's Action from Einstein's Theory of General Relativity" *Nucl. Phys.* B **242** 145–166

Fröhlich J 1982 "Regge Calculus and Discretized Gravitational Functional Integrals" IHES preprint (unpublished)

Godfrey N and Gross M 1991 "Simplicial Quantum Gravity in More Than Two Dimensions" *Phys. Rev.* D **43** R1749–R1753

Gross M and Hamber H W 1991 "Critical Properties of Two-Dimensional Simplicial Quantum Gravity" *Nucl. Phys.* B **364** 703-733

Hamber H W 1986 "Simplicial Quantum Gravity" in *Critical Phenomena, Random Systems, Gauge Theories* Proceedings of the Les Houches Summer School 1984 ed K Osterwalder and R Stora (Amsterdam: North Holland)

Hamber H W 1990a "Simplicial Quantum Gravity from Two to Four Dimensions" in *Probabilistic Methods in Field Theory and Quantum Gravity* (Cargèse NATO Workshop August 1989) ed P H Damgaard, H Hüffel and A Rosenblum, NATO Advanced Study Institute Series B, vol 224 (New York: Plenum Press) pp 243–257

Hamber H W 1990b "Non-Trivial Fixed Point for Quantum Gravity in Four Dimensions" UC Irvine preprint UCI-Th-90-60

Hamber H W 1991a "A Review of Simplicial Quantum Gravity" invited talk presented at the Elba International Conference on Monte Carlo Methods (27 June to 6 July 1990), to appear in the proceedings published by Editrice Compositori

Hamber H W 1991b "Simplicial Gravity in Two Dimensions" in the proceedings of the international conference "Lattice 90", Tallahassee, Florida (7–11 October 1990) ed A Kennedy et al *Nucl. Phys.* B (Proc. Suppl.) **20** 728–732

Hamber H W 1991c "Simulations of Discrete Quantized Gravity" *Int. J. Supercomputer Applications* (in press)

Hamber H W 1991d "Phases of 4-d Simplicial Quantum Gravity" UC Irvine preprint UCI-Th-91-11 to appear in *Phys. Rev.* D

Hamber H W 1991e "Phases of Simplicial Quantum Gravity" to appear in the proceedings of the 1991 Barcelona conference on "Two-Dimensional Gravity and Random Surfaces" ed P Damgaard et al *Nucl. Phys.* B (Proc. Suppl.)

Hamber H W and Williams R M 1984 "Higher Derivative Quantum Gravity on a Simplicial Lattice" *Nucl. Phys.* B **248** 392–414

Hamber H W and Williams R M 1985 "Non-Perturbative Simplicial Quantum Gravity" *Phys. Lett.* **157B** 368–374

Hamber H W and Williams R M 1986a "Two Dimensional Simplicial Quantum Gravity" *Nucl. Phys.* B **267** 482–496

Hamber H W and Williams R M 1986b "Simplicial Quantum Gravity with Higher Derivative Terms: Formalism and Numerical Results in Four Dimensions" *Nucl. Phys.* B **269** 712–743

Hartle J B 1985a "Simplicial Minisuperspace: I. General Discussion" *J. Math. Phys.* **26** 804–814

Hartle J B 1985b "Unruly Topologies in Two-Dimensional Quantum Gravity" *Class. Quantum Grav.* **2** 707–720

Hartle J B 1985c "Simplicial Quantum Gravity" in *Proceedings of the Third Seminar on Quantum Gravity* ed M A Markov, V A Berezin and V P Frolov (Singapore: World Scientific) pp 123–140

Hartle J B 1986a "Simplicial Minisuperspace: II. Some Classical Solutions on Simple Triangulations" *J. Math. Phys.* **27** 287–295

Hartle J B 1986b "Simplicial Quantum Gravity and Unruly Topology" in *Proceedings of the 4th Marcel Grossmann Meeting on General Relativity, Rome 1985*, Part A ed R Ruffini (Amsterdam and New York: North Holland) pp 85–99

Hartle J B 1986c "Numerical Quantum Gravity" in *Proceedings of the 14th Yamada Conference on Gravitational Collapse and Relativity* ed H Sato and T Nakamura (Singapore: World Scientific) pp 329–339

Hartle J B 1989 "Simplicial Minisuperspace: III. Integration Contours in a Five-Simplex Model" *J. Math. Phys.* **30** 452–460

Hartle J B and Sorkin R 1981 "Boundary Terms in the Action for the Regge Calculus" *Gen. Rel. Grav.* **13** 541–549

Hartle J B, Sorkin R and Williams R M 1991 "Simplicial Minisuperspace: IV. Product Solutions on Product Manifolds" in preparation

Hasslacher B and Perry M J 1981 "Spin Networks are Simplicial Quantum Gravity" *Phys. Lett.* **103B** 21–24

Hawking S W 1978 "Space-time Foam" *Nucl. Phys.* B **144** 349–362

Hellaby C 1989 "When Strings Collide" University of Cape Town preprint 89/6

Isham C J 1987 "Quantum Gravity" in *Proceedings of the 11th International Conference on General Relativity and Gravitation, Stockholm 1986* ed M A H MacCallum (Cambridge University Press) pp 99–129

Jacobs L 1989 "Recent Progress in Simplicial Quantum Gravity" MIT preprint CTP#1811

Jasinchi R and Yuille A 1989 "Nonrigid Motion and Regge Calculus" *J. Opt. Soc. Am.* **A6** 1088–1095

Jevicki A and Ninomiya M 1985 "Lattice Gravity and Strings" *Phys. Lett.* B **150** 115–118

Jevicki A and Ninomiya M 1986 "Functional Formulation of Regge Gravity" *Phys. Rev.* D **33** 1634–1637

Jourjine A N 1985a "Dimensional Phase Transitions: Coupling of Matter to the Cell Complex" *Phys. Rev.* D **31** 1443–1452"

Jourjine A N 1985b "Gauging of Dirac-Kähler Fermions on the Cubic Lattice" *Phys. Rev.* D **32** 459–461

Jourjine A N 1986a "Reply to Comment on the Gauging of Dirac-Kähler Fermions on Lattices" *Phys. Rev.* D **34** 1234

Jourjine A N 1986b "Dimensional Phase Transitions: Spinors, Gauge Fields and Background Gravity on a Cell Complex" *Phys. Rev.* D **34** 3058–3068

Jourjine A N 1987 "Discrete Gravity Without Coordinates" *Phys. Rev.* D **35** 2983–2986

Kawamoto N and Nielsen H B 1990 "Lattice Gauge Gravity with Fermions" preprint KUNS 1011/HE(TH)90/03

Khatsymovsky V 1989 "Tetrad and Self-Dual Formulations of Regge Calculus" *Class. Quantum Grav.* **6** L249–L255

Khatsymovsky V 1991a "Continuous Time Regge Gravity in the Tetrad Connection Variables" *Class. Quantum Grav.* **8** 1205–1216

Khatsymovsky V 1991b "On the Two-Dimensional Model of Quantum Regge Gravity" Novosibirsk preprint

Kheyfets A, LaFave N J and Miller W A 1988 "Quantum Insights from Null-strut Geometrodynamics" Loyola Conference on Mathematics and Interpretational Problems in Relativistic Quantum Theory, New Orleans, 1987 *Int. J. Theor. Phys.* **27** 133–158

Kheyfets A, LaFave N J and Miller W A 1989 "Pseudo-Riemannian Geometry on a Simplicial Lattice and the Extrinsic Curvature Tensor" *Phys. Rev.* D **39** 1097–1108

Kheyfets A, LaFave N J and Miller W A 1990a "Null-Strut Calculus: I. Kinematics" *Phys. Rev.* D **41** 3628–3636

Kheyfets A, LaFave N J and Miller W A 1990b "Null-Strut Calculus: II. Dynamics" *Phys. Rev.* D **41** 3637–3651

Kheyfets A, Miller W A and Wheeler J A 1988 "Null-strut Calculus: The First Test" *Phys. Rev. Lett.* **61** 2042–2045

Koibuchi H and Yamada M 1989a "A Monte Carlo Study of 2-Dimensional Higher Derivative Quantum Gravity on a Triangulated Random Surface" *Mod. Phys. Lett.* A **4** 1249–1255

Koibuchi H and Yamada M 1989b "Statistical Mechanics of a Rigid Bubble: Higher Derivative Quantum Gravity on a Spherical Random Surface Embedded in $E|3$" *Mod. Phys. Lett.* A **4** 2417–2428

Lehto M 1988 "Simplicial Quantum Gravity" Ph.D. Thesis, University of Jyväskylä

Lehto M, Nielsen H B and Ninomiya M 1984 "A Correlation Decay Theorem at High Temperature" *Comm. Math. Phys.* **93** 483–493

Lehto M, Nielsen H B and Ninomiya M 1986a "Pregeometric Quantum Lattice: A General Discussion" *Nucl. Phys.* B **272** 213–227

Lehto M, Nielsen H B and Ninomiya M 1986b "Diffeomorphism Symmetry in Simplicial Quantum Gravity" *Nucl. Phys.* B **272** 228–252

Lehto M, Nielsen H B and Ninomiya M 1987 "Semilocality of One-dimensional Simplicial Quantum Gravity" *Nucl. Phys.* B **289** 684–700

Lewis S M 1982a "Two Cosmological Solutions of Regge Calculus" *Phys. Rev.* D **25** 306–312

Lewis S M 1982b "Regge Calculus: Applications to Classical and Quantum Gravity" Ph.D. Thesis, University of Maryland

Lewis S M 1983 "Three-dimensional Regge Quantum Gravity and $6j$ Symbols" *Phys. Lett.* **122B** 265–267

Louko J and Tuckey P A 1991 "Regge Calculus in Anisotropic Quantum Cosmology" to appear in *Class. Quantum Grav.*

Lund F and Regge T 1974 "Simplicial Approximation to some Homogeneous Cosmologies" unpublished

Martellini M 1979 "Quantum Gravity on Skeleton Space: A Quantum Formulation of the Regge Calculus" DAMTP preprint

Martellini M and Marzuoli A 1986 "Cone Subdivision Procedure for Regge Lattices: A Renormalization Group Approach to Quantum Gravity" in *Proceedings of the Fourth Marcel Grossman Meeting on General Relativity, Rome 1985*, Part B ed R Ruffini (Amsterdam and New York: North Holland) pp 1173–1181

Miller S J and Tuckey P A 1990 "The Constraint Structure of Friedman-Jack-Regge Calculus" in preparation

Miller W A 1986a "The Geometrodynamic Content of the Regge Equations as Illuminated by the Boundary of a Boundary Principle" *Found. Phys.* **16** 143–169

Miller W A 1986b " Geometric Computation: Null-strut Geometrodynamics and the Inchworm Algorithm" in *Dynamical Spacetimes and Numerical Relativity* ed J Centrella (Cambridge and New York: Cambridge University Press) pp 256–303

Miller W A 1986c "Foundations of Null-strut Calculus" Ph.D. dissertation, University of Texas

Miller W A and Wheeler J A 1985 "4-geodesy" *Nuovo Cimento* **8** E 418–434

Misner C W, Thorne K S and Wheeler J A 1973 in *Gravitation* (San Francisco: Freeman) chapter 42

Moussouris J P 1983 "Quantum Theory of Space-Time Based on Recoupling Theory" D.Phil. Thesis, University of Oxford

Piran T and Strominger A 1986 "Solutions of the Regge Equations" *Class. Quantum Grav.* **3** 97–102

Piran T and Williams R M 1985 "Inflation in Universes with a Massive Scalar Field" *Phys. Lett.* **163B** 331–335

Piran T and Williams R M 1986a "Three-Plus-One Formulation of Regge Calculus" *Phys. Rev.* D **33** 1622–1633

Piran T and Williams R M 1986b "Three-Plus-One Regge Calculus: Formulation and Application to Inflationary Universes" in *Proceedings of the Fourth Marcel Grossmann Meeting on*

General Relativity, Rome 1985, Part B ed R Ruffini (Amsterdam and New York: North Holland) pp 1373–1384

Ponzano G and Regge T 1968 "Semiclassical Limit of Racah Coefficients" in *Spectroscopic and Group Theoretical Methods in Physics* ed F Block, S G Cohen, A De-Shalit, S Sambursky and I Talmi (Amsterdam: North Holland) pp 1–58

Porter J D 1982 "Numerical Study of Non-homogeneous Space-times using Regge Calculus" D.Phil. Thesis, University of Oxford

Porter J D 1987a "A New Approach to the Regge Calculus: I. Formalism" *Class. Quantum Grav.* **4** 375–389

Porter J D 1987b "A New Approach to the Regge Calculus: II. Application to Spherically Symmetric Vacuum Space-times" *Class. Quantum Grav.* **4** 391–410

Porter J D 1987c "Calculation of Relativistic Model Stars Using Regge Calculus" *Class. Quantum Grav.* **4** 651–661

Regge T 1961 "General Relativity Without Coordinates" *Nuovo Cimento* **19** 558–571

Ren H C 1988 "Matter Fields in Lattice Gravity" *Nucl. Phys.* B **301** 661–684

Roček M and Williams R M 1981 "Quantum Regge Calculus" *Phys. Lett.* **104B** 31–37

Roček M and Williams R M 1982 "Introduction to Quantum Regge Calculus" in *Quantum Structure of Space and Time* ed M J Duff and C J Isham (Cambridge University Press) pp 105–116

Roček M and Williams R M 1984 "The Quantization of Regge Calculus" *Z. Phys.* C **21** 371–381

Roček M and Williams R M 1985 "Three Dimensional Einstein Gravity and Regge Calculus" *Class. Quantum Grav.* **2** 701–706

Roček M and Williams R M 1991 "On the Euler Characteristic for Piecewise Linear Manifolds" Institute for Advanced Study preprint IASSNS-HEP-91/41, to appear in *Phys. Lett. B.*

Römer H and Zähringer M 1986 "Functional Integration and the Diffeomorphism Group in Euclidean Lattice Quantum Gravity" *Class. Quantum Grav.* **3** 897–910

Sorkin R 1974 "Development of Simplectic Methods of the Metrical and Electromagnetic Fields" Ph.D. Thesis, California Institute of Technology (available from University Microfilms, Ann Arbor, Michigan)

Sorkin R 1975a "The Time-Evolution Problem in Regge Calculus" *Phys. Rev.* D **12** 385–396

Sorkin R 1975b "The Electromagnetic Field on a Simplicial Net" *J. Math. Phys.* **16** 2432–2440

Tuckey P A 1988 "Approaches to $3+1$ Regge Calculus" Ph.D. Thesis, University of Cambridge

Tuckey P A 1989 "Independent Variables in $3+1$ Regge Calculus" *Class. Quantum Grav.* **6** 1–21

Tuckey P A 1990 "The Time-Evolution Problem in $3+1$ Regge Calculus" Durham University preprint DTP 90/40

Tuckey P A 1991 "Issues in Discrete-Time 3+1 Regge Calculus" to appear in the proceedings of the "Workshop on Numerical Applications of Regge Calculus and Related Topics", Amalfi, Italy (18–21 September 1990) ed I Pinto

Tuckey P A and Williams R M 1988 "A $3+1$ Regge Calculus Model of the Taub Universe" *Class. Quantum Grav.* **5** 155–166

Tuckey P A and Williams R M 1990 "Conserved Constraints in Three-Plus-One Regge Calculus" *Class. Quantum Grav.* **7** 2055–2071

Waelbroeck H 1989 "On Constraint Equations in Continuous Time Formulations of Lattice Gravity" progress report prepared for GR12, Boulder, Colorado

Waelbroeck H 1990 "$2+1$ Lattice Gravity" *Class. Quantum Grav.* **7** 751–769

Warner N P 1982 "The Application of Regge Calculus to Quantum Gravity and Quantum Field Theory in a Curved Background" *Proc. Roy. Soc. London* A **383** 359–377

Weingarten D 1977 "Geometric Formulation of Electrodynamics and General Relativity in Discrete Space-time" *J. Math. Phys.* **18** 165–170

Weingarten D 1982 "Euclidean Quantum Gravity on a Lattice" *Nucl. Phys.* B **210** 229–245

Wheeler J A 1964 "Regge Calculus and Schwarzschild Geometry" in *Relativity, Groups and Topology* ed B DeWitt and C DeWitt (New York: Gordon and Breach) pp 463–501

Williams R M 1985 "The Time-symmetric Initial Value Problem for a Homogeneouos Anisotropic Empty Closed Universe, Using Regge Calculus" *Gen. Rel. Grav.* **17** 559–571

Williams R M 1986a "Quantum Regge Calculus in the Lorentzian Domain and its Hamiltonian Formulation" *Class. Quantum Grav.* **3** 853–869

Williams R M 1986b "Building Blocks for Space and Time" *New Scientist* **110** No 1512, 48–51

Williams R M 1991 "A Brief Survey of Applications of Regge Calculus in Classical and Quantum Gravity" to appear in the proceedings of the "Workshop on Numerical Applications of Regge Calculus and Related Topics", Amalfi, Italy (18–21 September 1990) ed I Pinto

Williams R M and Ellis G F R 1981 "Regge Calculus and Observations: I. Formalism and Applications to Radial Motion and Circular Orbits" *Gen. Rel. Grav.* **13** 361–395

Williams R M and Ellis G F R 1984 "Regge Calculus and Observations: II. Further Applications" *Gen. Rel. Grav.* **16** 1003–1021

Wong C-Y 1971 "Application of Regge Calculus to the Schwarzschild and Reissner-Nordstrøm Geometries at the Moment of Time Symmetry" *J. Math. Phys.* **12** 70–78