$U_A(1)$ Goldberger-Treiman relation and the proton spin problem

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We discuss the $U_A(1)$ Goldberger-Treiman relation and suggest that in contrast with the nonsinglet channel the contribution from higher mass states to the singlet axial-vector coupling of the nucleon, g_A^0, may be comparable to the contribution from the lowest-lying state η'. Experimental results from both J/ψ radiative decays and $p\bar{p}$ annihilation at rest on the $\eta(1440)$ particle are analyzed to support our suggestion. The expected cancellation between the contributions of these states is therefore possible.

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The spin content of the proton measured by European Muon Collaboration (EMC) [1] is a very important result to reveal the structure of the nucleon. The result, together with the data from neutron and hyperon β decays, give an unexpected value of the flavor-singlet axial-vector coupling 0.12 ± 0.24 which is two standard deviations away from the expected value of 0.6 from a relativistic quark model [2]. The smallness of the value has induced many interesting discussions [3–15].

It has been suggested that this might indicate that the quark helicity component may make no net contribution to the spin of the proton. It is further argued that this small value may be due to the cancellation between contributions of the quark component and the gluon component [4]. However, this is still very controversial because there is only one gauge-invariant, dimension-three local operator with correct quantum numbers for each quark flavor and it is impossible to distinguish between the quark component and the gluon component [2,5]. Moreover, the relation between the proton spin and the QCD $U_A(1)$ problem has been discussed in a series of papers [6–15] in which Goldberger-Treiman (GT) relation in the singlet channel is studied to indicate the unexpected small flavor-singlet axial-vector coupling of the nucleon. The GT relation in the isotriplet channel

$$g_A=\frac{F_\pi}{M_N}g_{\pi NN} \tag{1}$$

which holds to within a few percent is very successful. However, there are some controversial issues in discussing the $U_A(1)$ GT relation. It has been argued that the smallness of the axial-vector coupling means the decoupling of the η' meson from the nucleon [3]. The contributions of π^0 and η are further included through the mixing due to isospin and SU(3) breaking (see, e.g., Ref. [11]) and the conclusion is still the same that the EMC data suggest the decoupling of η' from the nucleon.

In this article we would like to point out that theoretically because of the $U_A(1)$ anomaly, the $U_A(1)$ GT relation in the singlet channel differs essentially from that in the triplet channel and the higher mass physical states must be considered, and that experimentally there is a strong indication that a higher mass state in the 1400–1500 MeV region does couple to both the nucleon and the singlet axial-vector current, and the couplings are comparable to the corresponding couplings of the η'. Therefore the smallness of the flavor singlet axial-vector coupling for the nucleon will not necessarily mean the decoupling of the η' from the nucleon but indicate a possible cancellation between the contributions of η' and this higher mass state. Furthermore, this result may also shed some light on the issue of the separation of quark component from the gluon component.

To have clear insight into it, we start by recalling the proof of the GT relation in the isotriplet channel. The matrix element of the axial-vector current is described by two form factors G_1 and G_2:

$$\langle N(p)|J_\mu|N(p')\rangle =\bar{u}(p)[G_1(q^2)\gamma_\mu\gamma_5+G_2(q^2)q_\mu\gamma_5]u(p'), \tag{2}$$
$$q=p-p' .$$

In the chiral limit, since pions are massless Goldstone bosons, the form factor G_2 acquires a pole at $q^2=0$, and

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then due to the current conservation we obtain the GT relation Eq. (1). The more useful lesson we learn is from the derivation of Eq. (1) in the real world. Because the pion acquires a mass in this case, the form factor $G_2(q^2)$ does not contain a Goldstone pole. The divergence of Eq. (2) in the forward direction reads

$$\langle p|\partial_\mu J^\mu|p\rangle = 2M_N G_1(0)\bar{u}i\gamma_5 u \ . \qquad (3)$$

To obtain the desired relation, we insert a complete set of physical states $|X\rangle$ with quantum number 0^- into the left-hand side of Eq. (3) and obtain

$$2M_N G_1(0)\bar{u}i\gamma_5 u = \sum_X \sqrt{2} f_X g_{XNN}\bar{u}i\gamma_5 u \ , \qquad (4)$$

where g_{XNN} is the X-nucleon coupling and f_X is the decay constant of the X state defined as

$$\langle 0|J^\mu|X\rangle = f_X q^\mu \ . \qquad (5)$$

Now assuming the higher mass single particle states and the continuum states give a vanishingly small contribution to the right-hand side of Eq. (4) (pion pole dominance), we obtain the GT relation Eq. (1) (with $f_\pi = \sqrt{2}F_\pi$). Equivalently the same result can be achieved by using the PCAC (partial conservation of axial-vector current) hypothesis

$$\partial_\mu J^\mu = f_\pi m_\pi^2 \pi \ . \qquad (6)$$

The PCAC condition Eq. (6) is equivalent to the pion pole dominance. In the isotriplet channel, the suppression of the contribution from continuum states is because the multiparticle phase space vanishes at $q^2 \to 0$ in the chiral limit independent of the current quark mass $m \to 0$ [16]. The smallness of the contribution from higher mass single particle [for example, $\pi(1300)$] states is because the coupling strength of these states to J_μ vanish in the chiral limit and in reality they are very small (proportional to u,d quark masses). This can be clearly seen in the proof of the Goldstone theorem [17]. In the chiral limit the axial-vector current is conserved exactly. The spontaneous symmetry breaking of chiral symmetry and the noninvariance of the physical vacuum under chiral rotation leads to the existence of a massless Goldstone boson. The current conservation indicates that any massive state must have zero coupling strength (decay constant) to J_μ.

Things are quite different in the singlet channel. Because the QCD $U_A(1)$ symmetry is explicitly broken by the axial anomaly [18], there is no current conservation and massless Goldstone boson in the $U_A(1)$ channel [19]. Even when we assume that multiparticle state contributions are small [20], there is no general guidance to constrain the contribution from higher single particle states above the η' particle. This means, if we assume a single particle state dominance hypothesis, we will have another PCAC-like relation in the singlet channel:

$$\partial_\mu J_\mu = f_{\eta'} m_{\eta'}^2 \eta' + \sum_X f_X M_X^2 X \ , \qquad (7)$$

where f_X is the decay constant of the higher mass state X which couples to J_μ in the singlet channel. Based on a similar derivation to Eq. (4), the GT relation in the singlet channel is

$$G_1(0) = \frac{\sqrt{3}}{2M_N}\left[f_{\eta'} g_{\eta' NN} + \sum_X f_X g_{XNN} \right] . \qquad (8)$$

Therefore, in principle, the observed smallness of $G_1(0)$ will not necessarily require the decoupling of the physical η' from the nucleon, but probably indicate the cancellation between contributions from the η' and the higher mass states. Note that the higher mass states X in Eq. (8) are not only the $q\bar{q}$ states but also include non-$q\bar{q}$ states. The state in the singlet channel above η' would be the radial excitation state of η' or a 0^{-+} glueball state. Physically, then $\eta(1440)$ (denoted by ι in the following) state comes next to η'. Therefore it will be interesting to know the contribution of the $\eta(1440)$.

The possible gluonic contribution to the singlet axial-vector coupling has been pointed out by Veneziano [8] and Shore and Veneziano [9] to account for the cancellation to the axial-vector coupling. They have shown that $G_1(0)$ can be expressed in terms of a suitably defined coupling of the nucleon to the Nambu-Goldstone boson η_0 of the Okobo-Zweig-Iizuka (OZI) limit of QCD. The smallness of $G_1(0)$ may imply the decoupling of the nucleon from the η_0 rather than from the η'[3]. They also rewrite it as η' and a gluon component coupling but without notifying whether the gluon component corresponds to any physical states. The possible cancellation between η' and the glueball has also been suggested by Ji [12].

In spite of different theoretical arguments on the smallness of the singlet axial-vector coupling, it is crucial to make an independent analysis in terms of observable physical couplings, on the basis of Eq. (8), to see whether the contribution of higher mass states, e.g., the ι, is indeed comparable to that of η' to make the cancellation possible. In the following we will demonstrate by analyzing the related experimental data that the contribution of ι, $f_\iota g_{\iota NN}$ is of the same order of magnitude as the contribution of η', $f_{\eta'} g_{\eta' NN}$. The decay constant f_ι can be evaluated from the J/ψ radiative decay:

$$J/\psi \to \gamma\iota \to \gamma K\bar{K}\pi \ .$$

In the chiral limit, the divergence of the singlet axial-vector current is

$$\partial_\mu J^\mu = \frac{3\alpha_s}{4\pi} G\tilde{G} \ .$$

It is a pure gluonic operator and appears in the matrix elements of J/ψ radiative decays into light (non $c\bar{c}$) pseudoscalar hadron states P:

$$A(J/\psi \to \gamma P) \propto \left\langle 0 \left| \frac{\alpha_s}{4\pi} G\tilde{G} \right| P \right\rangle = \frac{1}{3} f_P M_P^2 \ . \qquad (9)$$

Equation (9) is based on the following observation. These OZI-forbidden radiative decays can proceed via two processes. The first is that the vector $c\bar{c}$ mixes via three virtual gluons with a light vector hadron which then turns into a light pseudoscalar by emitting the photon. The second, on the other hand, is that the vector $c\bar{c}$ turns by emitting the photon into a pseudoscalar $c\bar{c}$ which then mixes via two virtual gluons with a light pseudoscalar. The latter process is expected to be dominant since only two virtual gluons are involved. The matrix element for the gluonic operator in Eq. (9) describes the transition from two virtual gluons into a light pseudoscalar. It is known that in these processes the higher power of the gluon field is suppressed because the charm quark is heavy, and the calculated decay rates for both $J/\psi\to\gamma\eta$ and $J/\psi\to\gamma\eta'$ are satisfactory [21,22]. For η and η' the gluonic anomaly matrix elements may be evaluated on the basis of chiral and large-N_c approaches [22]. For ι, however, we will invoke the experimental data. Present experiments give $B(J/\psi\to\gamma\eta')\sim 4.2\times10^{-3}$ and $B(J/\psi\to\gamma\iota)B(\iota\to K\bar{K}\pi)\sim 4.8\times10^{-3}$ [23]. We have

$$\frac{B(J/\psi\to\gamma\iota)}{B(J/\psi\to\gamma\eta')}=\left[\frac{\left\langle 0\left|\frac{\alpha_s}{\pi}G\tilde{G}\right|\iota\right\rangle}{\left\langle 0\left|\frac{\alpha_s}{\pi}G\tilde{G}\right|\eta'\right\rangle}\right]^2\frac{S_P(J/\psi\to\gamma\iota)}{S_P(J/\psi\to\gamma\eta')}\ , \tag{10}$$

where S_P denotes the phase space. We know that the decay constant is renormalization scale dependent, so we should have to obtain the decay constant at a low-energy scale rather than the one from Eqs. (9) and (10) which scaled at the mass of J/ψ. However, since we are only interested in estimating the ratio of the two decay constants f_ι and $f_{\eta'}$ which is renormalization scale independent, we need not worry about the scale we are discussing [24]. The decay constants can then be extracted from Eq. (10):

$$f_\iota=0.58f_{\eta'}[B(\iota\to K\bar{K}\pi)]^{-1/2}\ . \tag{11}$$

The branching ratio of ι decay into $K\bar{K}\pi$ is probably dominant but not known explicitly at present from experiment since ι is an isosinglet state and may also decay into $\eta\pi\pi$ or other states. We expect $B(\iota\to K\bar{K}\pi)\geq 0.5$ and then obtain, from Eq. (11),

$$f_\iota/f_{\eta'}\sim 0.58-0.83\ . \tag{12}$$

It is very difficult to extract information about $g_{\iota NN}$ from experiments of t-channel hadron-hadron scattering. However, there exist experimental data of ι production from proton-antiproton annihilation at rest (see [25] for a review). If low-energy strong interactions are described by an effective meson-baryon Lagrangian, the annihilation cross section is proportional to the ι-N coupling strength $g^2_{\iota NN}$. The given branching ratio of s-wave $p\bar{p}$ annihilation into $\eta'\pi^+\pi^-$ (after a subtraction of $p\bar{p}\to\eta'\rho$, $\rho\to\pi^+\pi^-$) is about 1.65×10^{-3} [26] whereas the branching ratio of s-wave $p\bar{p}$ annihilation into $\iota\pi^+\pi^-(\iota\to K^\pm K^0\pi^\mp)$ is about 7.1×10^{-4} [27]. Suppose a SU(3)-symmetric decay mode in the $K\bar{K}\pi$ channel; then we have in the s wave

$$B(p\bar{p}\to\iota\pi^+\pi^-)\times B(\iota\to KK\pi)$$
$$=\tfrac{3}{2}B(p\bar{p}\to\iota\pi^+\pi^-)\times B(\iota\to K^\pm K^0\pi^\mp)\sim 1.1\times10^{-3}\ .$$

Considering the fact that the phase space suppression to ι production is much greater than η' production, we conclude that the coupling constant $g_{\iota NN}$ should be at least as large as, if not larger than, $g_{\eta' NN}$:

$$g_\iota/g_{\eta'}\geq 1\ . \tag{13}$$

The above relation is obtained for on-shell coupling constants. To obtain the GT relation one needs an extrapolation for the coupling constants from mass shell to $q^2=0$. As discussed before, unlike the case for $g_{\pi NN}$, the smoothness hypothesis is questionable here. However, we assume that instead of talking about each coupling constant separately the ratio of the coupling constants varies slowly for the off-shell extrapolation. Then combining (13) together with our previous discussion on the decay constants (12), we can conclude that the contribution of ι to the axial-vector coupling is of the same order as that of η'.

Next we discuss the cancellation between contributions of ι and η'. We will distinguish between two cases. In the first case the ι is identified with a glueball. Although it is difficult to know the relative sign of $f_{\eta'}g_{\eta'}$ to $f_\iota g_\iota$ from other theoretical sources, we can still use the argument [8,9] that the Skyrme nucleon, which lives in the SU(3) sector of U(3), may decouple from the η_0, which is the Nambu-Goldstone (NG) boson of the OZI limit, and this leads to the cancellation between η' and the gluon component. The experimental result is by no means trivial, however, because the cancellation can only occur when $f_\iota g_\iota$ is of the same size as $f_{\eta'}g_{\eta'}$, as has been analyzed by us. In the second case, the ι is identified with the first radial excitation of η'. In this case $f_\iota g_\iota$ is likely to have a different sign from $f_{\eta'}g_{\eta'}$. This can be due to the node structure of the radially excited wave function. Suppose both the ι and η' wave functions have a positive sign at the origin, so f_ι and $f_{\eta'}$ have the same sign. Then at large distances the wave function for ι changes its sign. The meson-nucleon coupling is essentially determined by the overlap integral of the meson-nucleon wave functions. Therefore g_ι is likely to have a different sign from $g_{\eta'}$. We emphasize that the large distance behavior of the meson wave function is important since in the meson-nucleon coupling the quark-antiquark pair creation is needed (in the S channel the meson turns into a nucleon-antinucleon pair via two quark-antiquark pair creation), and the quark-antiquark pair creation occurs mainly at large distances to make the initial quark-antiquark color flux tube in the meson state break into the final state hadrons. Therefore we believe in both cases the cancellation between $f_{\eta'}g_{\eta'}$ and $f_\iota g_\iota$ is possible. The smallness of g_A^0 therefore means that the contributions from the two particles should cancel each other if their magnitudes are separately large.

There is no reliable extraction of $g_{\eta' NN}$ from experiments [27]. The theoretical estimation of $g_{\eta' NN}$ is either about 7 as predicted by the SU(3) quark model or about zero as argued in the Skyrme model assuming the η' particle decouples from the SU(3) sector of the Skyrmion Lagrangian and hence from the nucleon, a soliton made of octet mesons [3]. However, the recent experiment on $p\bar{p}$

annihilation at rest may also shed light on the issue. There are two observed s-wave branching ratios [26]:

$$B(p\bar{p}\rightarrow\eta\pi^+\pi^-)=(13.7\pm1.46)\times10^{-3}$$

and $B(p\bar{p}\rightarrow\eta'\pi^+\pi^-)=(3.46\pm0.67)\times10^{-3}$ (without the subtraction of $\rho\rightarrow2\pi$). Considering that the phase space is much in favor of $\eta\pi\pi$, we might presumably have

$$g_{\eta'}/g_\eta\sim1\ . \tag{14}$$

If we use

$$g_{\eta'}\simeq g_\eta$$

and believe g_η is about 7 as indicated by some experiment results [28], then we find the contribution of η' to $G_1(0)$ to be

$$\Delta\Sigma=\frac{\sqrt{3}}{2M_N}f_{\eta'}g_{\eta'NN}\simeq1\ . \tag{15}$$

Very recently a partial wave analysis of the decay $J/\psi\rightarrow\gamma K\bar{K}\pi$ in the $K\bar{K}\pi$ invariant mass range 1.35–1.6 GeV has been presented by the Mark III Collaboration [29]. The results show that ι is not a single 0^{-+} resonance, but a mixture of three overlapping states: two 0^{-+} states, respectively, at about 1420 MeV [an $a_0(980)\pi$ resonance] and at about 1490 MeV (a K^*K resonance), and one 1^{++} state at about 1440 MeV (a K^*K resonance). The 0^{-+} state at about 1420 MeV is likely to be identified with $\eta(1400)$ which was observed in both $\pi N\rightarrow\eta\pi\pi N$ and $J/\psi\rightarrow\gamma\eta\pi\pi$, also in $\pi N\rightarrow K\bar{K}\pi N$ [29]. It is argued that the 0^{-+} state at about 1420 MeV could be a radially excited state rather than a gluonium state and the 0^{-+} state at 1490 MeV could still be a non $q\bar{q}$ state, probably a glueball [29]. Although the experimental situation has become more complicated than it was before, in our opinion, however, it does not change the qualitative feature of our analysis. The 0^{-+} components are still dominant in the ι region in J/ψ radiative decays. Furthermore, considering the fact that only a part of the decay modes of these 0^{-+} states are known experimentally, they (or one of them) may still strongly couple to the $U_A(1)$ axial-vector current and the nucleon, regardless of whether it is a glueball or a radially excited meson.

As far as the experimental status is concerned, it is certainly helpful to further clarify the structures in the $\eta(1440)$ region in J/ψ radiative decays by other groups, e.g., the DM2 Collaboration [30] or by future experiments with higher statistics, e.g., at BEPC (Beijing Electron-Positron Collider). It is also very useful if there are more detailed data in the ι region in $p\bar{p}$ annihilation at rest not only for $\iota\rightarrow K\bar{K}\pi$ but also for $\iota\rightarrow\eta\pi\pi$ to clarify the structure in this energy region and their coupling strengths to the nucleon. It will be very interesting to know how many 0^{-+} states there are in this energy range, and which state couples more strongly to both the $U_A(1)$ current and the nucleon, and whether it is a radially excited $q\bar{q}$ state or a glueball. In any case we believe that the qualitative feature of our argument, that the smallness of g_A^0 is possibly due to the cancellation between η' and the higher mass states, will remain valid in spite of possible uncertainties in the interpretation of these 0^{-+} states which couple to both the $U_A(1)$ current and the nucleon.

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