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$U_A(1)$ Goldberger-Treiman Relation and the Proton Spin Problem

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ABSTRACT

We discuss the $U_A(1)$ Goldberger-Treiman (GT) relation and suggest that in contrast to the non-singlet channel the contribution from higher mass states to the singlet axial vector coupling of the nucleon, g_A^0, may be comparable to the contribution from the lowest-lying state η'. Experimental results from both J/ψ radiative decays and $p\bar{p}$ annihilation at rest on the $\eta(1440)$ particle are analyzed to support our suggestion. The expected cancelation between the contributions of these states is therefore possible.

The spin content of the proton measured by the EMC collaboration [1] is a very important result to reveal the structure of the nucleon. The result, together with the data from neutron and hyperon β decays gives an unexpected value of the flavour singlet axial coupling 0.12 ± 0.24 which is two standard deviation away from the expected value of 0.6 from a relativistic quark model [2]. The smallness of the value might indicate that the quark helicity component may make no net contribution to the spin of the proton .

Several proposals have been made to explain the EMC result [3-5],. especially the relation between the proton spin and the QCD $U_A(1)$ problem has been discussed in a series of papers [6-15] in which the GT relation in the singlet channel is studied to indicate the unexpected small flavour singlet axial vector coupling of the nucleon. The GT relation in the isotriplet channel,

$$g_A^0 = \frac{F_\pi}{M_N} g_{\pi NN} \tag{1}$$

which holds to within a few percent is very succesful. However, there are some controversial issues in discussing the $U_A(1)$ GT relation. To have a clear insight in it, we start with recalling the proof of the GT relation in the isotriplet channel.

The matrix element of the axial vector current is described by two form factors, G_1 and G_2.

$$< N(p)|J_\mu|N(p') >= \bar{u}(p)(G_1(q^2)\gamma_\mu\gamma_5 + G_2(q^2)q_\mu\gamma_5)u(p'),$$

$$q = p - p' \tag{2}$$

In the chiral limit, since pions are massless Goldstone bosons, the form factor G_2 acquires a pole at $q^2 = 0$, and then due to the current conservation we obtain the GT relation eq.(1). The more useful lesson we learn is from the derivation of eq.(1) in the real world. Because the pion acquires a mass in this case, the form factor $G_2(q^2)$ does not contain a Goldstone pole. The divergence of eq.(2) in the forward direction reads,

$$< p|\partial_\mu J^\mu|p >= 2M_N G_1(0)\bar{u}i\gamma_5 u. \tag{3}$$

To obtain the desired relation, we insert a complete set of physical states $|X >$ with quantum number 0^- into the left-hand side of eq.(3) and obtain,

$$2M_N G_1(0)\bar{u}i\gamma_5 u = \sum_X \sqrt{2} f_X g_{XNN}\bar{u}i\gamma_5 u \tag{4}$$

where g_{XNN} is the X-nucleon coupling and f_X is the decay constant of the X state defined as

$$< 0|J^\mu|X >= f_X q^\mu. \tag{5}$$

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Now assuming the higher mass single particle states and the continuum states give a vanishingly small contribution to the right-hand side of eq.(4) (pion pole dominance). we obtain the GT relation eq.(1) (with $f_\pi = \sqrt{2}F_\pi$). Equivalently the same result can be achieved by using the PCAC hypothesis,

$$\partial_\mu J^\mu = f_\pi m_\pi^2 \pi. \tag{6}$$

The PCAC condition eq.(6) is equivalent to the pion pole dominance. In the isotriplet channel, the suppression of the contribution from continuum states is because the multiparticle phase space vanishes at $q^2 \to 0$ in the chiral limit independent of the current quark mass $m \to 0$ [16]. The smallness of the contribution from higher mass single particle (for example, $\pi(1300)$) states is because the coupling strength of these states to J_μ vanish in the chiral limit and in reality they are very small (proportional to u,d quark masses). This can be clearly seen in the proof of the Goldstone theorem [17]. In the chiral limit the axial vector current is conserved exactly. The spontaneous symmetry breaking of chiral symmetry and the noninvariance of the physical vacuum under chiral rotation leads to the existence of a massless Goldstone boson. The current conservation indicates that any massive state must have zero coupling strength (decay constant) to J_μ.

Things are quite different in the singlet channel. Because the QCD $U_A(1)$ symmetry is explicitly broken by the axial anomaly [18], there is no current conservation and massless Goldstone boson in the $U_A(1)$ channel [19]. Even when we assume that multiparticle state contributions are small[20], there is no general guidance to constrain the contribution from higher single particle states above the η' particle. This means, if we assume a single particle states dominance hypothesis, we will have another PCAC-like relation in the singlet channel,

$$\partial_\mu J_\mu = f_{\eta'} m_{\eta'}^2 \eta' + \sum_X f_X M_X^2 X, \tag{7}$$

where f_X is the decay constant of the higher mass state X which couples to J_μ in the singlet channel. Based on a similar derivation to eq.(4), the GT relation in the singlet channel is,

$$G_1(0) = \frac{\sqrt{3}}{2M_N}(f_{\eta'} g_{\eta' NN} + \sum_X f_X g_{XNN}). \tag{8}$$

Therefore, in principle, the observed smallness of $G_1(0)$ will not necessarily require the decoupling of the physical η' from the nucleon, but probably indicate the cancellation between contributions from the η' and the higher mass states. Note that the higher mass states X in eq.(8) are not only the $q\bar{q}$ states but also include non-$q\bar{q}$ states. The state in the singlet channel above η' would be the radial excitation state of η' or a

0^{-+} glueball state. Physically, the $\eta(1440)$ (denoted by ι in the following) state comes next to η'. The possible glueball contributions to the singlet axial coupling have been pointed out first by Veneziano [8] and Shore and Veneziano [9] to account for the cancelation to the axial coupling . They have shown that $G_1(0)$ can be expressed in terms of a suitably defined coupling of the nucleon to the NG boson η_0 of the OZI limit of QCD. The smallness of $G_1(0)$ may imply the decoupling of the nucleon from the η_0 rather than from the η'[3,4]. They also rewrite it as η' and a glueball couplings. The possible cancellation between η' and the glueball has also been suggested by Ji [12].

In spite of different theoretical arguments on the smallness of the singlet axial coupling, it is crucial to make an independant analysis, on the basis of eq.(8), to see whether the contribution of ι is indeed comparable to that of η' to make the cancellation possible. In the following we will demonstrate by analyzing the related experimental data that the contribution of ι, $f_\iota g_{\iota NN}$ is of the same order of magnitude as the contribution of η', $f_{\eta'} g_{\eta' NN}$. The decay constant f_ι can be evaluated from the J/ψ radiative decay,

$$J/\psi \to \gamma\iota \to \gamma K\bar{K}\pi.$$

In the chiral limit, the divergence of the singlet axial current is

$$\partial_\mu J^\mu = \frac{3\alpha_s}{4\pi} G\tilde{G}.$$

It is a pure gluonic operator and appears in the matrix elements of J/ψ radiative decays into light (non $c\bar{c}$) pseudoscalar meson states P,

$$A(J/\psi \to \gamma P) \sim < 0|\frac{\alpha_s}{4\pi} G\tilde{G}|P> = \frac{1}{3} f_P M_P^2. \tag{9}$$

It is known that in these processes the higher power of the gluon field are suppressed because the charm quark is heavy, and the calculated decay rates for both $J/\psi \to \gamma\eta$ and $J/\psi \to \gamma\eta'$ are satisfactory [21,22]. For η and η' the gluonic anomaly matrix elements may be evaluated on the basis of chiral and large N_c approach[22].For ι,however,we will invoke the experimental data. Present experiments give $Br(J/\psi \to \gamma\eta') \sim 4.2 \times 10^{-3}$ and $Br(J/\psi \to \gamma\iota) \times Br(\iota \to K\bar{K}\pi) \sim 4.8 \times 10^{-3}$[23]. We have

$$\frac{Br(J/\psi \to \gamma\iota)}{Br(J/\psi \to \gamma\eta')} = [\frac{< 0|\frac{\alpha_s}{\pi} G\tilde{G}|\iota >}{< 0|\frac{\alpha_s}{\pi} G\tilde{G}|\eta' >}]^2 \frac{PS(J/\psi \to \gamma\iota)}{PS(J/\psi \to \gamma\eta')}, \tag{10}$$

where PS denote the phase space. We know that the decay constant is renormalization scale dependent, so we should have to obtain the decay constant at low energy scale rather than the one from eq.(9) and (10) which scaled at the mass of J/ψ. However, since we are only interested in estimating the ratio of the two decay constants, f_ι and $f_{\eta'}$ which is renormalization scale independent, so we need not worry about the scale we are discussing [24]. The decay constants can then be extracted from eq.(10),

$$f_\iota = 0.58 f_{\eta'} [Br(\iota \to K\bar{K}\pi)]^{-1/2}. \tag{11}$$

The branching ratio of ι decay into $K\bar{K}\pi$ is probably dominant but not known explicitly at present from experiment since ι is an isosinglet state it may also decay into $\eta\pi\pi$ or other states . We simply use $Br(\iota \to K\bar{K}\pi) \geq 0.5$ and obtain from eq.(11)

$$f_\iota / f_{\eta'} \sim 0.58 - 0.83. \tag{12}$$

It is very difficult to extract information of $g_{\iota NN}$ from experiments of t-channel hadron-hadron scattering. However, there exist experimental data of ι prduction from proton-antiproton annihilation at rest (see [25] for a review). If low energy strong interactions are described by an effective meson-baryon lagrangian, the annihilation cross section is proportional to the $\iota - N$ coupling strength $g^2_{\iota NN}$. The given branching ratio of s-wave $p\bar{p}$ annihilation into $\eta'\pi^+\pi^-$ (after a subtraction of $p\bar{p} \to \eta'\rho, \rho \to \pi^+\pi^-$) is about 1.65×10^{-3} [26] whereas the branching ratio of s-wave $p\bar{p}$ annihilation into $\iota\pi^+\pi^-(\iota \to K^\pm K^0 \pi^\mp)$ is about 7.1×10^{-4} [27]. Suppose a $SU(3)$ symmetric decay mode in the $K\bar{K}\pi$ channel, we have in s-wave $\mathrm{Br}(\,p\bar{p} \to \iota\pi^+\pi^-) \times Br(\iota \to KK\pi) = \frac{3}{2} Br(p\bar{p} \to \iota\pi^+\pi^-) \times Br(\iota \to K^\pm K^0 \pi^\mp) \sim 1.1 \times 10^{-3}$. Considering the fact that the phase space suppression to ι production is much greater than η' production, we conclude that the coupling constant $g_{\iota NN}$ should be at least as large as, if not larger than $g_{\eta' NN}$,

$$g_\iota / g_{\eta'} \geq 1. \tag{13}$$

The above relation is obtained for on shell coupling constants. To obtain the GT relation one needs an extrapolation for the coupling constants from mass shell to $q^2 = 0$. As we discussed before, unlike the case for $g_{\pi NN}$, the smoothness hypothesis is questionable here. However, we assume that instead of talking about each coupling constant separately the ratio of the coupling constants varies slowly for the off-shell extrapolation. Then together with our previous discussion on the decay constant f_ι we can conclude that the contribution of ι to the axial coupling is of the same order as that of η'.

It is difficult to our knowledge to discuss the relative sign of $f_{\eta'} g_{\eta'}$ to $f_\iota g_\iota$, if the ι particle is a gluball. If ι is the first radial excitation of η' then the node structure of its wave function may lead to the negative sign of $g_{\iota NN}$ to $g_{\eta' NN}$, since the meson-nucleon coupling is essentially determined by the overlap of the meson-nucleon wave functions, and the sign of the overlap integral for the first radially excited state can be

different from the ground state meson. No matter what the situation is the smallness of g^0_A means that the contributions from the two particles should cancel each other if their magnitudes are separately large.

There is no reliable extraction of $g_{\eta' NN}$ from experiments [28]. The theoretical estimation of $g_{\eta' NN}$ is either about 7 as predicted by SU(3) quark model or about zero as predicted by the Skryme model because the η' particle decouples from the SU(3) sector of the skrymion lagrangian and hence from the nucleon, a soliton made of octet mesons [4]. However, the recent experiment on $p\bar{p}$ annihilation at rest may also shed light on the issue. There are two observed s-wave branching ratios [26], $Br(p\bar{p} \to \eta\pi^+\pi^-) = (13.7 \pm 1.46) \times 10^{-3}$ and $Br(p\bar{p} \to \eta'\pi^+\pi^-) = (3.46 \pm 0.67) \times 10^{-3}$ (without the subtraction of $\rho \to 2\pi$). Considering that the phase space is much in favour of $\eta\pi\pi$, we might presumably have

$$g_{\eta'} / g_\eta \sim O(1). \tag{14}$$

If we use

$$g_{\eta'} \simeq g_\eta$$

and believe g_η is about 7 as indicated by some experiment results[28], then we would find the contribution of η' to $G_1(0)$ to be,

$$\Delta\Sigma = \frac{\sqrt{3}}{2M_N} f_{\eta'} g_{\eta' NN} \simeq 1. \tag{15}$$

Very recently a partial wave analysis of the decay $J/\psi \to \gamma K\bar{K}\pi$ in the $K\bar{K}\pi$ invariant mass range 1.35-1.6 GeV has been presented by the Mark III collaboration [29]. The results show that ι is not a single 0^{-+} resonance, but a mixture of three overlapping states: two 0^{-+} states respectively at about 1420 MeV (an $a_0(980)\pi$ resonance) and at about 1490 MeV (a K^*K resonance), and one 1^{++} state at about 1440 MeV (a K^*K resonance). The 0^{-+} state at about 1420 MeV is likely to be identified with $\eta(1400)$ which was observed in both $\pi N \to \eta\pi\pi N$ and $J/\psi \to \gamma\eta\pi\pi$, also in $\pi N \to K\bar{K}\pi N$ [29]. This may indicate that the 0^{-+} state at about 1420 MeV could be a radially excited state rather than a gluonium state and the 0^{-+} state at 1490 MeV could still be a non $q\bar{q}$ state, probably a glueball. This new experimental analysis is consistent with our previous analysis in the sense that as conventional $q\bar{q}$ states in the singlet channel, the radially excited state of η' is allowed to have appreciable coupling to $\frac{\alpha_s}{\pi} G\tilde{G}$ and hence to the $U_A(1)$ current, as now possibly suggested by the Mark III data for J/ψ radiative decays. A partial cancellation in $G_1(0)$ due to η' and its first radial excitation is also possible. However, if in the mass range 1.35-1.6 GeV the 0^{-+} channel is dominated by a glueball state, which could be at $\sim$ 1490 MeV or somewhere else, we will still expect that the cancellation in $G_1(0)$ is mainly due to η' and the glueball. As far as the experimental status is concerned, it is certainly helpful to further clarify the structures in the $\eta(1440)$ region in J/ψ radiative decays by other groups e.g., the DM2 collaboration [30] or by future experiments with higher statistics e.g., at BEPC(Beijing Electron-Positron Collider). It is also very useful if there are more detailed data in the ι region in $p\bar{p}$ annihilation at rest not only for $\iota \rightarrow K\bar{K}\pi$ but also for $\iota \rightarrow \eta\pi\pi$ to clarify the structure in this energy region and their coupling strengths to the nucleon.It will be very interesting to know how many 0^{-+} states there are in this energy range,and which state couples more stronly to both the $U_A(1)$ current and the nucleon,and whether it is a radially excited $q\bar{q}$ state or a glueball. In any case we believe that the qualitative feature of our argument will remain valid that the smallness of g_A^0 is po ssibly due to the cancelation betreen η' and the higher mass states, in spite of possible uncertainties in the interpretation of these 0^{-+} states which couple to both the $U_A(1)$ current and the nucleon.

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