TESTS OF SCALE INVARIANCE IN THE "GARGAMELLE" NEUTRINO - EXPERIMENT

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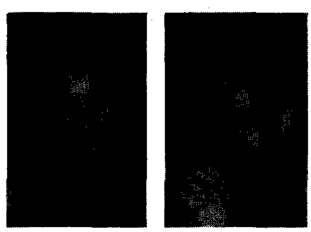
Abstract : The neutrino experiment performed with the CERN $\nu/\bar{\nu}$ beam in the large heavy liquid bubble chamber "Gargamelle" has yielded results in agreement with scale invariance predictions in the energy range 1-10 GeV. The signification of this "precocious" scaling is analyzed here, through the study of the distributions of the scaling variables $x' = q^2/(2M\nu + M^2)$, and of the variable $y = \nu/E$, in a sample of 2600 ν events and 1000 $\bar{\nu}$ events.

Résumé : L'expérience réalisée avec le faisceau $\nu/\bar{\nu}$ du CERN dans la grande chambre à bulles à liquide lourd "Gargamelle" a fourni des résultats en accord avec l'invariance d'échelle dans le domaine d'énergie de 1 à 10 GeV. Nous analysons ici la signification de cette invariance d'échelle "précoce" au moyen des distributions des variables $x' = q^2/(2M\nu + M^2)$ et $y = \nu/E$, obtenues à partir d'un lot de 2600 événements neutrinos et 1000 événements antineutrinos.



I. *INTRODUCTION*

Scale invariance is usually expected[(1)] in inclusive lepton-nucleon reactions :

$$1 + N \rightarrow 1' + \text{hadrons},$$

in the so-called "Bjorken limit", defined (using the notations of reference (5)) by :

$$\begin{cases} q^2 \rightarrow \infty \\ \nu \rightarrow \infty \end{cases},$$

the "scaling" variable : $x = \frac{q^2}{2M\nu}$ remaining at a fixed value.

In particular, scaling behaviour has been observed in the SLAC-MIT electroproduction experiment[(2)] for values of q^2 greater than 1 GeV^2, and for final hadron invariant masses W greater than 6 GeV.

As far as neutrino experiments are concerned, scale invariance predicts the following behaviour for the differential cross-sections :

$$(1) \quad \left.\begin{matrix} \dfrac{d^2\sigma^\nu}{dxdy} \\ \text{or} \\ \dfrac{d^2\sigma^{\bar\nu}}{dxdy} \end{matrix}\right\} = \frac{G^2ME}{\pi}\left[xy^2F_1^{\nu,\bar\nu}(x) + (1-y)\,F_2^{\nu,\bar\nu}(x) \mp xy(1-\tfrac{y}{2})\,F_3^{\nu,\bar\nu}(x)\right]$$

where G is the Fermi constant, M the nucleon mass, E the incident neutrino (anti-neutrino) energy, and y the ratio $\frac{\nu}{E}$. $F_1^{\nu,\bar\nu}$, $F_2^{\nu,\bar\nu}$ and $F_3^{\nu,\bar\nu}$ are respectively the limits in the Bjorken region of the more general structure functions :

$$\left.\begin{matrix} W_1^{\nu,\bar{\nu}}(q^2, \nu) \\ \frac{\nu}{M} W_2^{\nu,\bar{\nu}}(q^2, \nu) \\ \frac{\nu}{M} W_3^{\nu\ \bar{\nu}}(q^2, \nu) \end{matrix}\right\}$$

which in the general case can be considered as functions of x, and Ey = ν.

As far as one studies interactions on nucleons, with about as many protons as neutrons (this is the case in our experiment), the hypothesis of charge symmetry for "$\Delta S = 0$" weak hadron currents states that $F_i^{\nu} = F_i^{\bar{\nu}}$.

Using formula (1), and integrating over x and y, one predicts :

(a) a linear rise of the total $(\nu \to \mu^-)$ and $(\nu \to \mu^+)$ inclusive cross-sections with E ;

(b) a linear rise of the mean value of q^2 ($<q^2>$) with E, ($q^2 = 2ME\ x\ y$) since $<q^2> = 2ME <xy>$, due to the factorisation of E in formula (1).

The validity of formula (1) is expected in the following conditions : $q^2 = 2ME\ x\ y >> M^2$

$\nu = Ey >> M$

i.e. E >> M, since $0 \leqslant (x, y) \leqslant 1$. In this region the contributions of elastic and quasi-elastic channels should be small. This condition is fulfilled for the high energy part of the CERN neutrino spectrum in the "Gargamelle" experiment(5), as well as in both NAL counter experiments(3)(4). On the other hand, a large part of the "Gargamelle" data are out of the ordinary scaling region, and one should expect some deviation from scaling predictions for these low-energy events. It is then very surprising that both tests (a) and (b) are found to be well satisfied in the CERN energy range down to 1 GeV. Furthermore, the Gargamelle results on ν and $\bar{\nu}$ cross-sections and their ratio $R = \frac{\sigma^{\bar{\nu}}}{\sigma^{\nu}}$ (5)(6), interpreted in the framework of the validity of formula (1) and of charge symmetry for "$\Delta S = 0$" weak hadron currents, show a remarkable agreement with the predictions of the quark-parton model[7]. In order to understand these effects, one has to study the experimental distributions in the variables x and y in order to compare them to the predictions of formula (1), or any formula equivalent to (1) in the high energy limit. However, we have to note that x is not a good parameter in the low energy region in which the elastic contribution is important ; (elastic events are concentrated at $x = 1$, and thus cannot be considered on the same footing as other events). Following Bloom and Gilman[8], we shall use here the empirical variable :

$$x' = \frac{q^2}{2M\nu + M^2}$$

instead of x, which is equivalent to x in the ordinary scaling region and which allows a similar treatment of elastic and inelastic events.

In this paper, we first review the main features of the Gargamelle experiment and give new experimental results on the distributions in the variables x' and y.

II. EXPERIMENTAL ANALYSIS

The present analysis is based on 2600 ν events and 1000 $\bar{\nu}$ events. Detailed descriptions of the neutrino beam[6] and of the experimental procedures can be found elsewhere[5][6]. We recall here the main points of interest in the study of cross-sections, and of x' and y distributions.

A. Flux measurement

An important characteristic of the Gargamelle experiment is that the radial distribution of the muon flux inside the shielding is continuously measured at different depths. The muon flux calculated from the π and K production spectra measured at 24 GeV and extrapolated at the actual

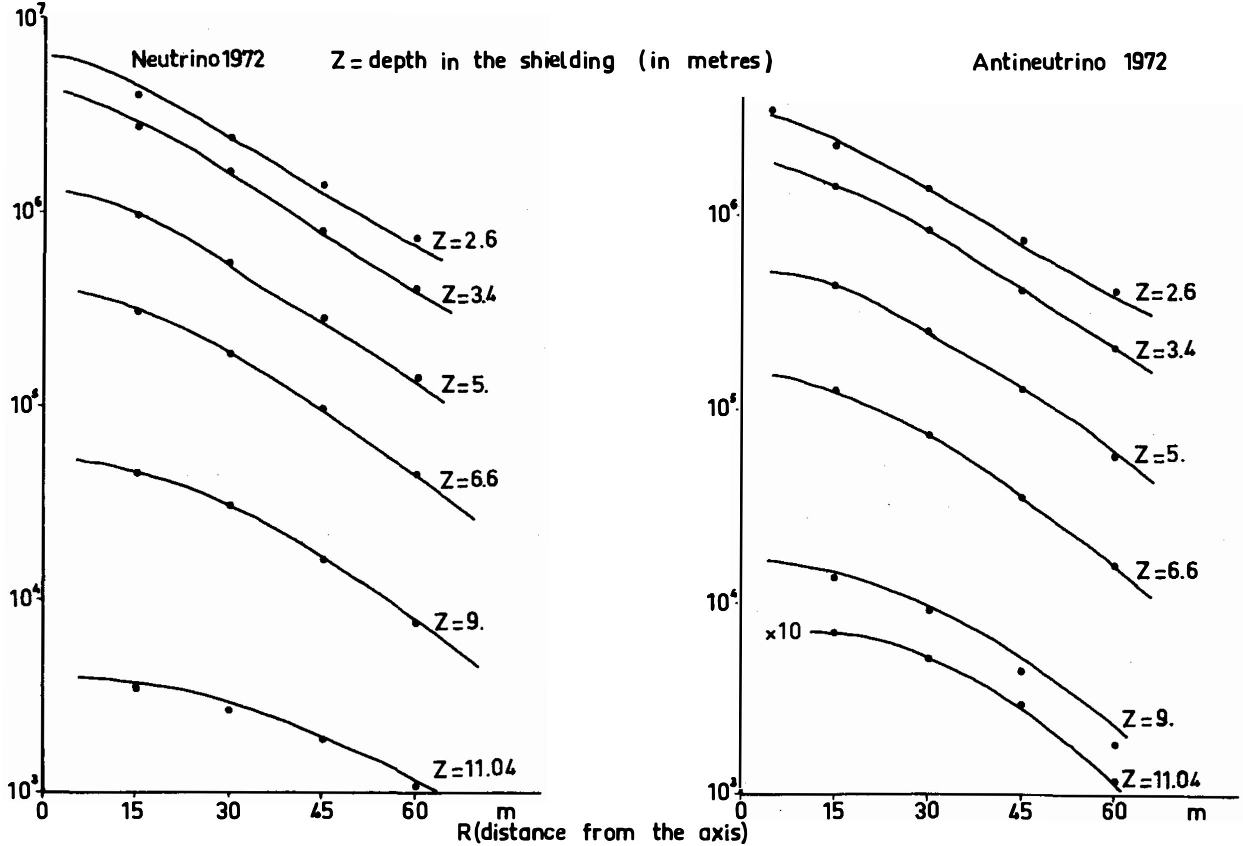


Figure 1_ Comparison of the muon fluxes _Calculated (full curves) _Measured (points)

proton energy of 26 GeV, as well as from the currents in focusing devices, is then compared (in shape) to the muon flux actually measured. The agreement in shape, shown in figure 1, is excellent and the fit gives an error of about 5% on the shape of the $\nu/\bar{\nu}$ energy spectrum. As far as the normalization of the $\nu(\bar{\nu})$ flux is concerned, only the measured muon flux is used (not the number of protons on the target). Between 2 GeV and 6 GeV, the error on the $\nu(\bar{\nu})$ flux is then 9% due to uncertainties in the calibration of muon detectors, and above 6 GeV, it increases up to 12% due to the additional error on the K/π ratio. Below 2 GeV, the errors are bigger since a part of the production spectrum contributing in this region is known only by extrapolation.

B. Contamination of ν or $\bar{\nu}$ due to wrong-sign mesons

The $\bar{\nu}$ flux in the ν beam as well as the ν flux in the $\bar{\nu}$ beam have also been computed by the preceding method ; they have been reduced in the 1972 and 1973 runs by improvements in the focusing system. One finds the following orders of magnitude :

$$\frac{\bar{\nu}\ \text{Flux}}{\nu\ \text{Flux}}\ \text{in}\ \nu\ \text{beam} \sim \begin{cases} 5 \times 10^{-3} & \text{in 1971} \\ 3 \times 10^{-4} & \text{in 1972-1973} \end{cases}$$

$$\frac{\nu\ \text{Flux}}{\bar{\nu}\ \text{Flux}}\ \text{in}\ \bar{\nu}\ \text{beam} \sim \begin{cases} 2.5 \times 10^{-2} & \text{in 1971} \\ 1.5 \times 10^{-3} & \text{in 1972-1973} \end{cases}$$

Consequently, the contribution of events with a wrong-sign muon is small, and their background can be easily subtracted.

C. Muon identification and measurement

Due to the large dimensions of Gargamelle (total length = 4.8 m) compared to the mean hadron collision length in freon CF_3Br ($\sim$ 70cm), the muon is identified in most of our events. There remain two sources of errors, the contribution of each of which is found to be small :

i) About 10% of the events have two muon candidates of the same sign. Using a maximum of likelihood method to calculate a weight for each hypothesis, one finds that in most cases, the most energetic muon candidate has a dominant weight ; this is further confirmed by a similar study in which events with only one muon candidate are weighted in order to compute the contribution of ambiguous events. Thus, if the most energetic muon candidate is chosen, very small corrections have to be applied.

ii) Muonless events such as neutral current candidates or neutron stars can simulate a charged current candidate if a hadron leaves the visible volume (or if a pion stops in the chamber). Their contamination has been calculated by a weighting method similar to the one quoted above and found to be of the order of 2% below 2 GeV and practically negligible at higher energy. The muon energy is measured with an average accuracy of 7%.

D. Hadron energy measurement

The total visible hadron energy is measured with an average accuracy of 15% ; it has to be corrected for the energy of undetected neutrals. The following corrections have been applied :

a) Undetected γ-rays : due to the short radiation length of freon (11cm), only low-energy γ-rays missed on the scan table have to be corrected ; this correction is very small.

b) Neutrons detected by a secondary interaction in the chamber (75% of neutrons) : in order to estimate the neutron energy from the visible energy of the neutron star, a first approximation uses the average value of the inelasticity parameter : $\xi = \frac{E\ visible}{E\ incident}$ of neutron interactions ($<\xi> = 0.3$).

Another method, using kinematics at the neutron interaction vertex, has been developed by the Orsay group. Although it neglects nuclear effects (Fermi motion and hadron reinteractions), and assumes the presence of a neutron in the final state,it yields a distribution of the ξ parameter compatible with the one expected from the data of neutron interactions. For this reason, this method has been used in the analysis presented here ; it has the further advantage of correcting events individually.

c) Undetected neutrons. About 25% of the emitted neutrons escape detection and an average correction has been applied to those events without any detected neutron. Since, on average, secondary neutrons take only 4% of the visible energy in ν events (and 8% in $\bar{\nu}$ events),uncertainties on this correction have always a small effect except in the case of $\bar{\nu}$ elastic events which are treated separately, as far as x' and y distributions are concerned.

d) $\bar{\nu}$ elastic events ($\bar{\nu} + p \rightarrow \mu^+ + n$).

In elastic antineutrino events the neutron energy has been kinematically calculated from the muon energy and angle, taking the Fermi momentum equal to 0. The effect of this approximation is a slight increase of symmetrical errors on variables such as y or x', due to the Fermi motion ; however systematical effects are much reduced in this method.

E. Effects of the Fermi motion on different variables

If p_F is the Fermi momentum, it can be easily shown that, when the neutrino energy is reconstructed as the sum of the muon energy and the secondary hadrons energies, it induces the following symmetrical errors :

$$\frac{\Delta\nu_F}{\nu} = \frac{\Delta x_F}{x} = \sqrt{\frac{\langle P_F{}^2\rangle}{3M^2}} \sqrt{1 + \frac{2Mx}{\nu}}$$

$$\frac{\Delta x'_F}{x'} = \frac{\Delta x_F}{x} \cdot \frac{\nu}{\nu+\frac{M}{2}}$$

$$\frac{\Delta y_F}{y} = \sqrt{\frac{\langle P_F{}^2\rangle}{3M}} \sqrt{2Mx \left(\frac{1}{\nu} - \frac{1}{E}\right)}$$

The error induced on x can be large for $x = 1$ (elastic events) ($\sim$ 50%) ; the error on x' is however smaller, and combined with ordinary measurement errors yields an overall uncertainty $\frac{\Delta x'}{x'} \sim 20\%$. Moreover, the effect of the Fermi motion on $y = \frac{\nu}{E}$ is small $\sim$ 3% due to correlations between

$$\nu = \frac{P_N \cdot q}{M} \quad \text{and} \quad E = \frac{S}{2M}$$

(S = center of mass energy squared)

III. RESULTS

Tests of scale invariance can be divided in the two following classes :

i) Tests in which the asymptotic formula (1) is not directly checked, but in which it is used in expressions integrated over x and y ; predictions on total cross-sections and their ratio, as well as on the mean value of q^2 fall in this category.

ii) More direct tests of formula (1) or of a similar formula, i.e. tests on x' and y distributions

A. Tests on expressions integrated over x and y

a) Total ($\nu \rightarrow \mu^-$) and ($\bar{\nu} \rightarrow \mu^+$) inclusive cross-sections

The well-known results on total cross-sections per nucleon are shown in figure 2. For E greater than 1 GeV one finds, using a linear fit :

$$\sigma^{\nu} = (0.74 \pm 0.02)\, E_{GeV} \times 10^{-38} \text{ cm}^2/\text{nucleon}$$

and

$$\sigma^{\bar{\nu}} = (0.28 \pm 0.01)\, E_{GeV} \times 10^{-38} \text{ cm}^2/\text{nucleon}$$

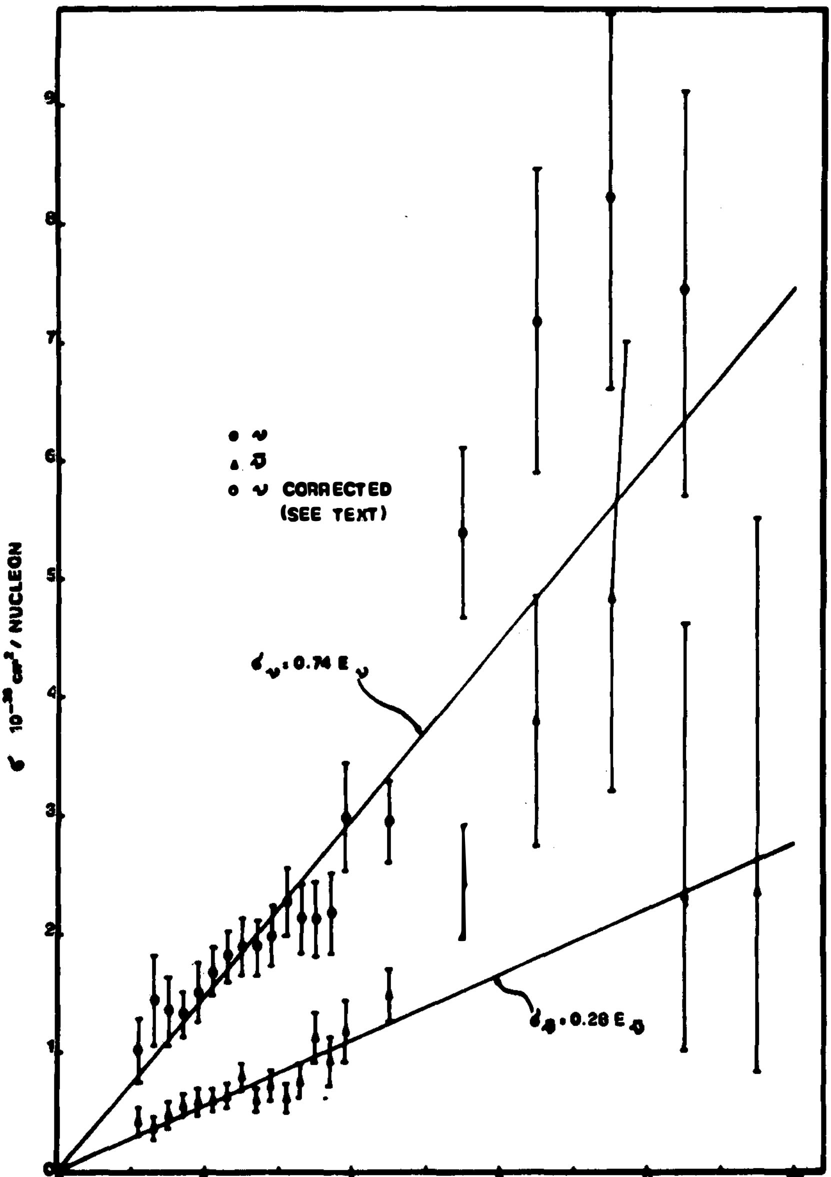
The cross-section ratio R is found to be compatible with a constant independent of incident energy (figure 3) :

$$R = \begin{cases} 0.37 \pm 0.02 & \text{if all the data are used ;} \\ 0.38 \pm 0.02 & \text{if one uses only those events with energy greater than 2 GeV.} \end{cases}$$

Scale invariance and charge symmetry, together with positivity conditions allow to set bounds on R(7):

$$\frac{1}{3} \leqslant R \leqslant 3$$

The fact that R is close to 1/3 leads to stringent bounds on the following expressions :



TOTAL CROSS - SECTIONS

fig. 2

$$0.87 \pm 0.05 \leqslant \left(A = \frac{\int_0^1 2xF_1(x)dx}{\int_0^1 F_2(x)dx} \right) \leqslant 1$$ (in agreement with the Callan-Gross relation)

$$0.87 \pm 0.05 \leqslant \left(B = \frac{-\int_0^1 xF_3 dx}{\int_0^1 F_2(x)dx} \right) \leqslant 0.90 \pm 0.04$$

$$0.49 \pm 0.03 \leqslant \int F_2(x)dx \leqslant 0.51 \pm 0.03$$

The comparison of these results with SLAC electroproduction data on deuterium has been discussed in detail in a preceding talk[7]. It is in good agreement with the predictions of the quark-parton model of Gell-Mann and Zweig with essentially three valence quarks and gluons.

b) Mean value of q^2 versus E

Figures 4a and 4b show the variation of $< q^2>$ with E. They are in good agreement with a linear rise, but the intercept at E = 0 is not negligible ; a linear fit gives :

$$< q^2 >^{\nu} = 0.12 \pm 0.03 + (0.23 \pm 0.01)E_{GeV}$$

$$< q^2 >^{\bar{\nu}} = 0.09 \pm 0.03 + (0.14 \pm 0.015)E_{GeV}$$

for E greater than 1 GeV.

From the slopes, one can deduce the mean value of

$$v = xy = \frac{q^2}{2ME} = \frac{E'(1-\cos\theta)}{M}$$

calculated only from the well-measured muon variables (its energy E' and its angle θ with the incident direction). In the high energy limit, one finds :

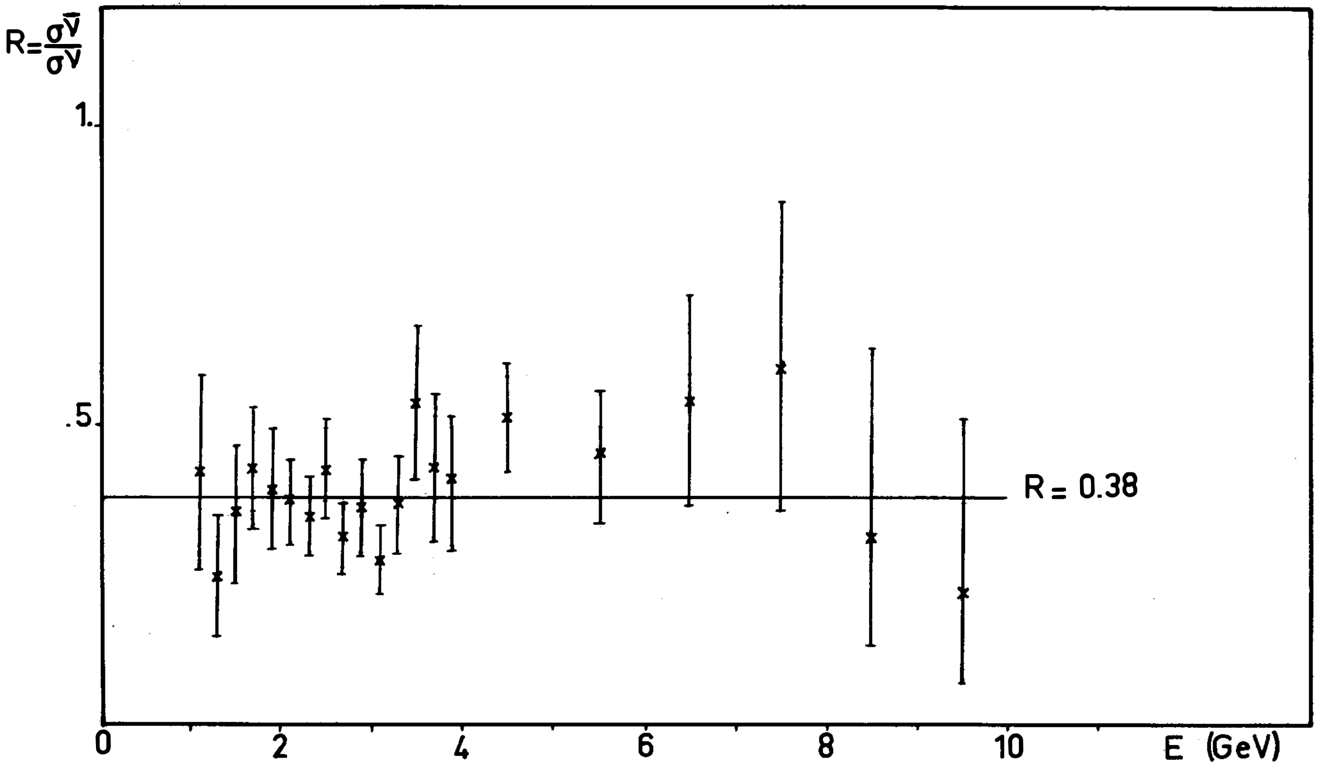
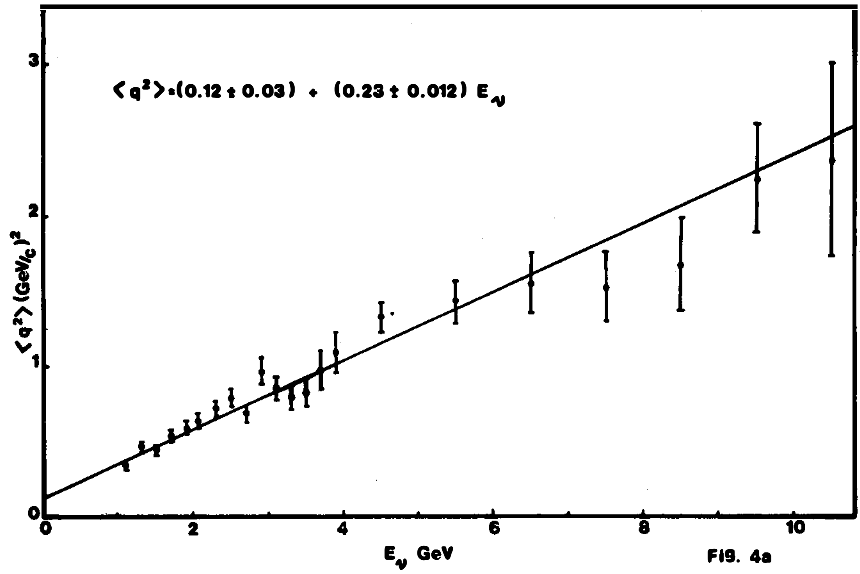
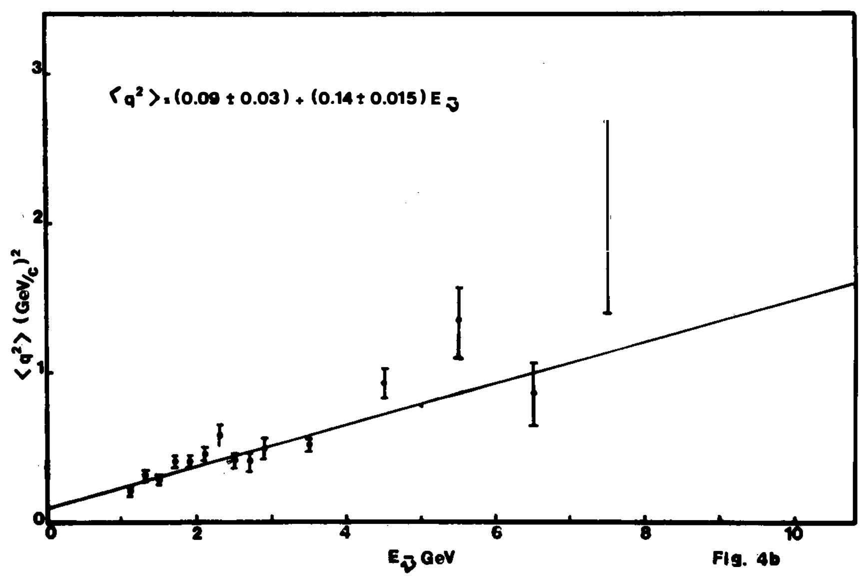


FIGURE 3 _ RATIO $\sigma^{\bar{\nu}}/\sigma^{\nu}$



⟨q²⟩=(0.12 ± 0.03) + (0.23 ± 0.012) E_ν
⟨q²⟩ (GeV/c)²
E_ν GeV
0
1
2
3
0
2
4
6
8
10

Fig. 4a



⟨q²⟩=(0.09 ± 0.03) + (0.14 ± 0.015) E_ν̄
⟨q²⟩ (GeV/c)²
E_ν̄ GeV
0
1
2
3
0
2
4
6
8
10

Fig. 4b

$$\begin{cases} \langle x y \rangle^{\nu} = 0.12 \pm 0.01 \\ \langle x y \rangle^{\bar{\nu}} = 0.07 \pm 0.01 \end{cases}$$

One concludes that tests a) and b) are well satisfied, even at energies of a few GeV.

B. Tests on x' distributions

Even if formula (1) is not valid in a large part of our energy region, the success of tests (a) and (b) supports the hypothesis of "precocious scaling", i. e. of a similar formula, equivalent to (1) in the Bjorken region, but averaging over the contributions of the resonances in the lower energy region. Such a possibility has already been pointed out by Bloom and Gilman (8) for electroproduction data, and also by Rittenberg and Rubinstein (9). Following Bloom and Gilman we use the variable :

$$x' = \frac{q^2}{2M\nu + M^2} = \frac{x}{1 + \frac{M}{2Ey}}$$ (equivalent to x in the Bjorken region)

Then by analogy with formula (1) we define new functions :

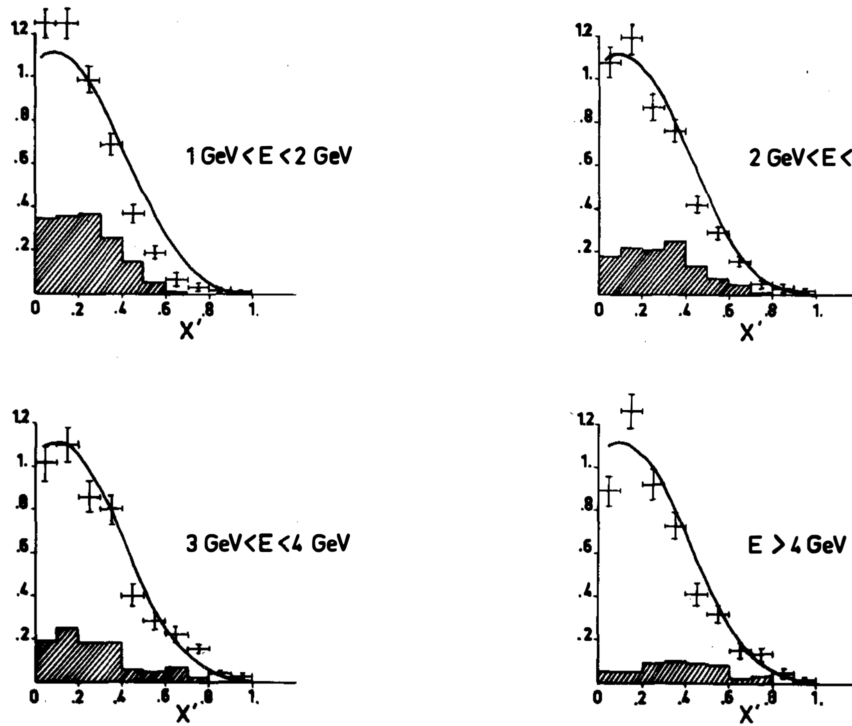
$$(2) \quad \mathcal{F}_2(x') = \frac{3}{4} \frac{\pi}{G^2 ME} \left(\frac{d\sigma^{\nu}}{dx'} + \frac{d\sigma^{\bar{\nu}}}{dx'} \right)$$

$$(3) \quad |x'\mathcal{F}_3(x')| = \frac{3}{2} \frac{\pi}{G^2 ME} \left(\frac{d\sigma^{\nu}}{dx'} - \frac{d\sigma^{\bar{\nu}}}{dx'} \right)$$

Clearly these definitions coincide with those of F_2 anf $|xF_3|$ in the asymptotic region. The results are shown in figure 5 .

Figure 5 : $\mathcal{F}_2(x') = \frac{1}{\frac{G^2ME}{\pi}} \frac{3}{4}\left(\frac{d\sigma^{\nu}}{dx'} + \frac{d\sigma^{\bar{\nu}}}{dx'}\right)$ normalized to the straight line fit to the cross-section

The contribution of elastic cross-sections is shaded



One finds :

i) that the function $\mathcal{F}_2(x')$ calculated by (2) is not sensitive to incident energy ;

ii) that even for energies as low as 1 GeV, it is in good agreement with $\frac{9}{5} F_2^{ed}(x')$ taken from SLAC electroproduction data.

One should note that our experimental errors on x' ($\frac{\Delta x'}{x'} \sim 20\%$, thus $\Delta x' = 0.06$ for $x' = 0.3$) induce only a small distortion of the actual distribution, so that the comparison with the SLAC curve can be made directly.

Results on $\mathcal{F}_2(x')$ and $x'\mathcal{F}_3(x')$ between 1 and 10 GeV are shown in figure 6. The contribution of elastic cross-sections has been shaded in this figure.

C. <u>Tests on y distributions</u>

As is well known(7), if formula (1) holds with

$$A = \frac{\int 2x\ F_1\ dx}{\int F_2\ dx} \qquad \text{and} \qquad B = \frac{-\int xF_3\ dx}{\int F_2\ dx}$$

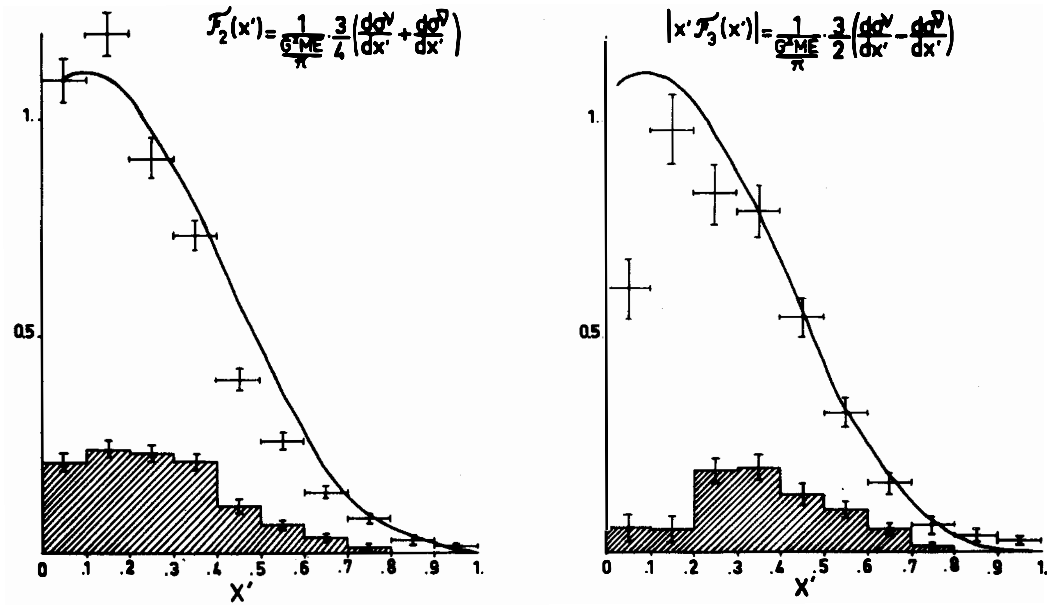
close to unity, as deduced from our result on the cross-section ratio, then one expects :

i) a y distribution close to uniformity in the neutrino sample ;

ii) a y distribution close to $(1 - y)^2$ in the antineutrino sample.

Possible modifications of the asymptotic formula (1) can be of two kinds(7) :

Figure 6 E > 1 GeV



Normalized to the straight line fit to the cross-section

i) modifications of the scaling variables (e.g. $x \to x'$) ;

ii) modifications of the scaling functions ; for example, the scaling function can have the form :

$$\mathcal{F}_2(x') = (1 + \varepsilon\,(x', y))\,\frac{\nu}{M}\,W_2\,(q^2, \nu)$$

provided that $\varepsilon\,(x', y) \to 0$ in the Bjorken limit. Such modifications will affect the y distribution.

Results are shown in figure 7a for neutrino events and in figure 7b for antineutrino events. A small correction has been applied in the last bin ($0.9 < y < 1.$) to take account of the upper kinematic limit of y which is not 1 at our energies :

$$o \leqslant y \leqslant \frac{1}{1 + \frac{Mx}{2E}}$$

One can see that the data agree well with the asymptotic formula for incident energies E greater than 4 GeV ; in this region, our statistics are still significant (i.e. 503 ν-events and 128 $\bar{\nu}$ -events). At lower energies, one finds an excess of events for small values of y. Any attempt to find a new formula for $\frac{d^2\sigma}{dx'dy}$ should reproduce these distribution, taking account of our $\nu(\bar{\nu})$ energy spectrum.

On the other hand, one can notice that "averaging over resonances" has not exactly the same meaning for x' distributions and y distributions. Figure 8 reproduces the kinematical limits in the (x', y) plane, as well as the locations in this plane of several resonances, at different incident energies. It can be seen that for E < 4 GeV, the low y contribution comes essentially from the elastic channel, so that one cannot really speak of "average".

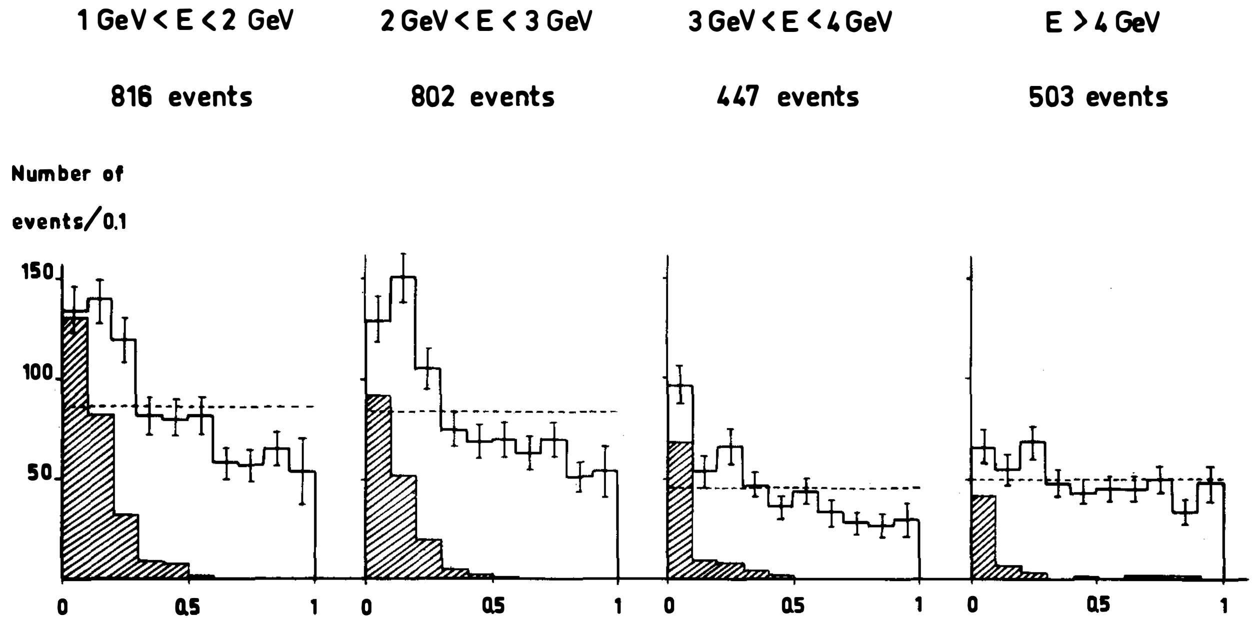


Figure 7 a _ Y DISTRIBUTIONS NEUTRINOS

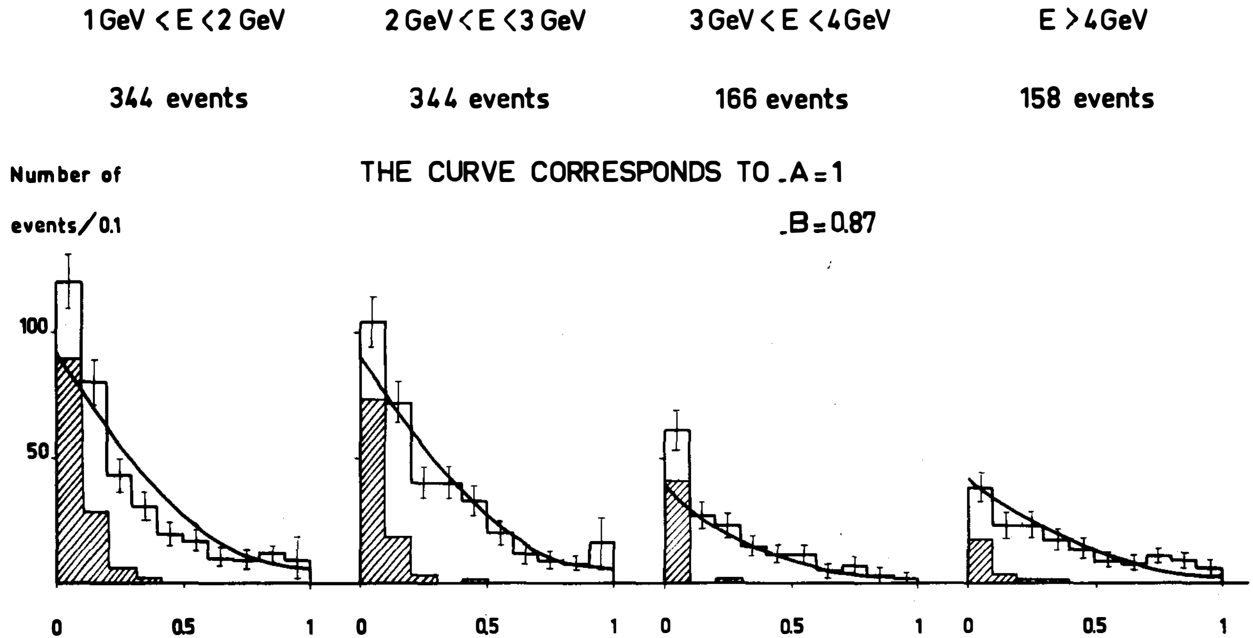


Figure 7b _ Y DISTRIBUTIONS ANTINEUTRINOS

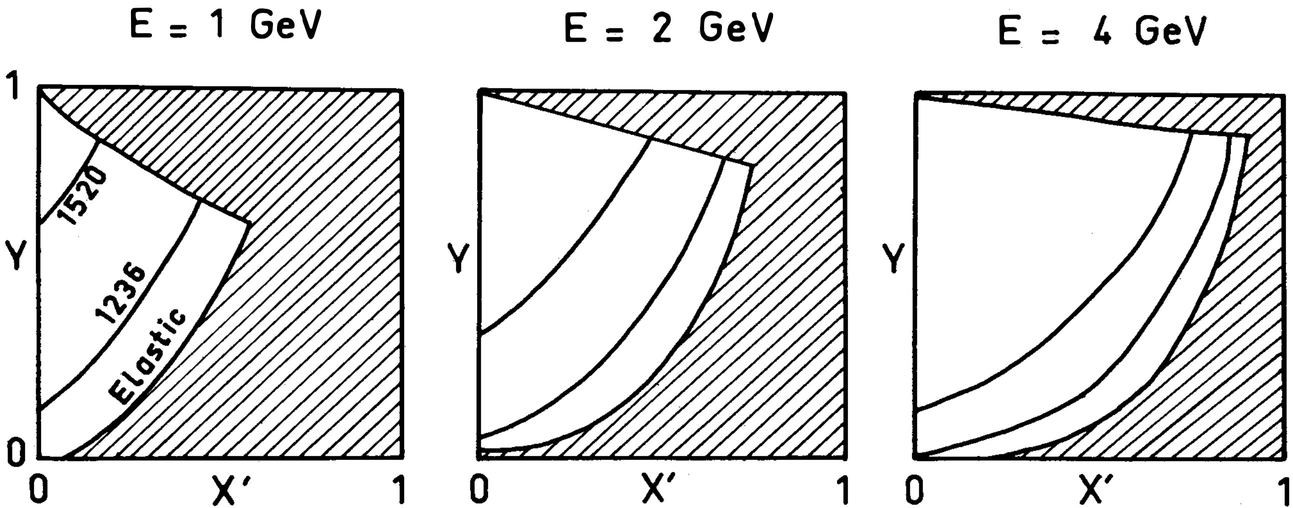


FIGURE 8

IV. *CONCLUSION*

The Gargamelle data for incident energies E greater than 4 GeV are in good agreement with the predictions of scale invariance and charge symmetry. The x' and y distributions are compatible with those expected from the value of the cross-section ratio, and the electroproduction data.

At lower energy, there remain something of the scaling hypothesis, since cross-sections and mean values of q^2 still have a linear behaviour with E, and since x' distributions still reproduce the SLAC asymptotic curve. Data on the y distributions show however that the modifications of the asymptotic formula, necessary to account for low-energy effects are not so simple that the change of variables : $x \to x'$.

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