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DETERMINATION OF THE STRONG COUPLING CONSTANT FROM e^+e^- COLLISIONS AT LEP

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ABSTRACT

The latest update of the strong-interaction coupling constant α_s obtained from measurements done by the four LEP collaborations ALEPH, DELPHI, L3, and OPAL is reported. Condensed to one single number, this gives $\alpha_s(m_Z) = 0.119 \pm 0.006$.

1. Introduction

In a recent pre-LEP review[1] about quantum chromodynamics (QCD) it was stated that the running of α_s was not convincingly demonstrated by experimental results available at that time. A measurement at LEP of $\alpha_s(m_Z) \approx 0.11 \pm 0.01$ 'would be a very quantitative and accurate test of QCD...'. The four LEP experiments, ALEPH, DELPHI, L3, and OPAL, have used the Z decays as a QCD laboratory and are even more precise: $\alpha_s(m_Z) = 0.119 \pm 0.006$.

2. α_s from the ratio of hadronic to leptonic Z-decay widths

The determination of α_s from the ratio of hadronic to leptonic Z-decay widths, $R_Z = \Gamma_h/\Gamma_\ell$, is based on the QCD correction, δ_{QCD}, to this ratio:

$$R_Z = R_Z^0(1 + \delta_{QCD})$$

with $\delta_{QCD} = \alpha_s/\pi + 1.41(\alpha_s/\pi)^2$ in second order and for massless quarks. Including the third-order term[2] and quark-mass effects and top-mass-dependent terms, δ_{QCD} becomes[3]:

$$\delta_{QCD} = 1.05(\alpha_s/\pi) + 0.9(\alpha_s/\pi)^2 - 13(\alpha_s/\pi)^3 \; ;$$

R_Z^0 is the Standard Model prediction for R_Z in the absence of strong interactions ($\alpha_s = 0$) and is 19.98 ± 0.03 for $m_t = 130^{+120}_{-50}$ GeV, $m_H = 200^{+800}_{-150}$ GeV.[4]*

Recent R_Z measurements published[5-8] by the LEP Collaborations are

$$21.00 \pm 0.20\,, \quad 20.70 \pm 0.29\,, \quad 20.93 \pm 0.22\,, \quad 20.95 \pm 0.22\,,$$

yielding a weighted average of 20.92 ± 0.11 corresponding to

$$\alpha_s(m_Z) = 0.139 \pm 0.016(\text{exp.}) \pm 0.004(m_t, m_H)\,.$$

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*Numerical value from Ref. 5.

The experimental error of 12% reflects the 0.5% error of R_Z due about equally to statistics ($\sim$ 70,000 leptonic Z decays) and event selection uncertainties. It is of great importance to reduce these uncertainties in dedicated studies, since a comparison of this α_s result, which is practically free from hadronization effects, with the one from jet studies represents crucial tests of QCD and/or of the electroweak theory.

3. α_s from the ratio of hadronic to leptonic tau-decay widths

Another method to obtain α_s essentially free from hadronization effects is to derive it from the ratio of hadronic to leptonic tau-decay rates, $R_\tau = \Gamma_{\tau\to \mathrm{h}}/\Gamma_{\tau\to\ell\nu}$. This ratio has been calculated to third order in α_s, like R_Z, and shown to have negligible uncertainties due to non-perturbative QCD effects.[9]

The L3 Collaboration[10] have used their leptonic tau branching ratios B_e, B_μ, to obtain

$$R_\tau = (1 - B_e - B_\mu)/B_e = 3.64^{+0.25}_{-0.23}$$

and derive

$$\alpha_s(m_\tau) = 0.34^{+0.07}_{-0.09}\,.$$

Extrapolating from $\alpha_s(m_\tau)$ to $\alpha_s(m_Z)$ they find[11] $\alpha_s(m_Z) = 0.118^{+0.007}_{-0.012}$. Uncertainties due to the extrapolation prescription are still under study.[12] They might become important since experimentally this is the most precise method to obtain $\alpha_s(m_Z)$; combining all leptonic tau branching ratios reported recently[13] would already result in an experimental uncertainty of $\Delta\alpha_s(m_Z) \leq 2\%$.

The difference between $\alpha_s(m_\tau)$, 0.34, and $\alpha_s(m_Z)$, 0.139, can be considered as evidence for the running of α_s.

4. α_s from hadron distributions in Z decays

4.1. The problem: How to get from the initial $q\bar{q}$ pair to hadrons

Every property of the hadronic final state of e^+e^- annihilations which is sensitive to hard-gluon emission by quarks has been used to measure the quark–gluon coupling strength: jettiness (especially the differential two-jet rate D_2), global event shape variables [thrust T, oblateness O, C-planarity (parameter)], jet invariant masses in one or both event hemispheres, and energy-weighted angle differences between single hadrons (for definitions, see for instance Kunszt et al.[14]). The distributions of these quantities are the result of a process which after the e^+e^- annihilation into a $q\bar{q}$ pair can be divided into three phases: gluon radiation, hadronization, and particle decays, of which only the last one is experimentally known. Gluon radiation can be calculated approximately by perturbative QCD either to $O(\alpha_s^2)$ 'exact' ('matrix element' models, '$\leq$ 4-parton final state') or in 'leading log approximation' ('parton shower' model, '9–10-parton final state'). Hadronization (transition from partons to hadrons) is described rather well phenomenologically by string and cluster fragmentation schemes (non-perturbative QCD).

The main uncertainty in extracting α_s from these final-state hadron distributions comes from the unknown higher (> 2) order contributions to the 3-jet cross-section, which are accounted for by the (unphysical) renormalization scale μ. See Ref. 14 for a general discussion. Recent experimental results have been reviewed in several reports.[15] Most of the data in the present review were reported[16] to LP–HEP '91.

4.2. α_s from jet rates

Final-state hadrons are combined into jets by an invariant-mass algorithm, most frequently the so-called JADE jet finder[17]: for every event of visible energy $E_{\rm vis}$ the scaled invariant 'mass' of each pair of particles i, j, with energies E_i, E_j, and angle θ_{ij} between them,

$$y_{ij} = 2\frac{E_i E_j}{E^2_{\rm vis}}(1 - \cos\theta_{ij}) \ ,$$

is calculated, and the pair with smallest y_{ij} is replaced by a pseudoparticle with 4-momentum $p_k = p_i + p_j$. In successive steps more and more pseudoparticles are combined until y_{ij} of all pairs is larger then the jet-resolution parameter y_c; the remaining pseudoparticles are the jets. Other combination schemes (called E, E0, p, p0) either give masses to the particles i, j, or add 3-momenta or energies instead of 4-momenta.[14] OPAL[18] have studied systematically the influence of the jet-finding scheme on the α_s determination from jet production rates and found hadronization corrections and uncertainties different for each scheme, but α_s values consistent with each other, hence no additional 'recombination scheme uncertainty'.

The smallness of hadronization corrections ($\leq 5\%$) to jet-rate calculations (see Fig. 1) is one of the reasons for the E0(equivalent to JADE) scheme being the preferred jet finder.

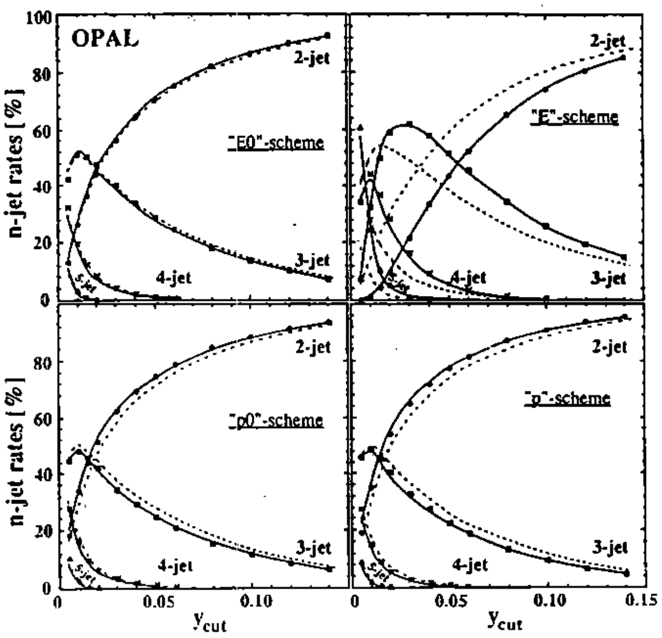

Fig. 1 n-jet production rates (measured by OPAL) compared with QCD calculations before and after hadronization (from Bethke[15]). The points are data, and the lines are JETSET partons (dotted) and hadrons (continuous).

The differential 2-jet distribution

$$D_2(y_c) = [R_2(y_c) - R_2(y_c - \Delta y_c)]/\Delta y_c ,$$

which measures the distribution of the y_c value of the events, for which the jet multiplicity changes from 3 to 2, has been used by OPAL,[18] ALEPH,[19] and DELPHI[20] to extract α_s. The results are shown in Table 1, together with the L3 result[21] obtained from the 3-jet rate R_3.

Table 1: α_s from Jet Rates

Expt.	α_s	$\Delta\alpha_s$(exp.)	$\Delta\alpha_s$(had.)	$\Delta\alpha_s$(scale)
ALEPH	0.121	0.004	0.007	0.008
DELPHI	0.114	0.004	0.003	0.006
L3(R_3)	0.115	0.005	0.003	0.010
OPAL	0.118	0.003	0.004	0.007
Average	0.117	0.004	0.004	0.008

Further evidence for the running of α_s is given by comparing 3-jet production rates $R_3 = \sigma(\text{3-jet})/\sigma(\text{tot})$ at different energies E_{cm}, shown in Fig. 2. The mean value for the four LEP experiments is $R_3 = 18.3 \pm 0.3\%$.

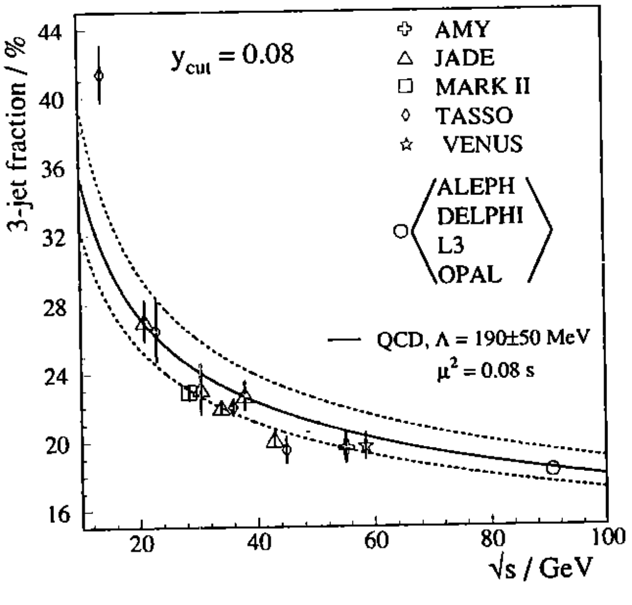

Fig. 2 Energy dependence of 3-jet event production rate (from Hebbeker[16])

The L3 Collaboration[22] have used b-quark tagging to measure the 3-jet rate R_3 due to Z → $b\bar{b}g$ compared with that of the other flavours. The (preliminary) result

$$\alpha_s(\mathrm{b})/\alpha_s(\mathrm{udsc}) = R_3^{\mathrm{b}}/R_3^{\mathrm{udsc}} = 1.00 \pm 0.05(\mathrm{stat.}) \pm 0.06(\mathrm{syst.})$$

is a test of the flavour independence of α_s.

4.3. α_s *from energy–energy correlations*

Energy–energy correlations (EEC) are a histogram of the angles between any pair of hadrons in a hadronic event weighted by the normalized product of their energies. EEC and their asymmetries (AEEC) have been studied by all LEP experiments. Their α_s results are given in Table 2.[20,23–25]

Table 2: α_s from Energy–Energy Correlations

Expt.	EEC				AEEC			
	α_s	± exp.	± theor.	± scale	α_s	± exp.	± theor.	± scale
ALEPH	0.118	0.002	0.005	0.006				
				0.009				
DELPHI	0.116	0.002	0.002	0.006	0.106	0.003	0.005	0.002
L3	0.121	0.004	0.006	0.009	0.115	0.004	0.008	0.002
				0.006			0.006	0.000
OPAL	0.124	0.006	0.007	0.007	0.117	0.008	0.006	0.000
							0.002	0.000
Average	0.119	0.004	0.004	0.007	0.113	0.005	0.007	0.002

ALEPH[23] have reduced hadronization uncertainties by computing EEC not for single particles but for clusters of neighbouring particles (CEEC). The results are shown in Fig. 3. The preclustering is done using the JADE algorithm. This method determines α_s essentially from the angular and energy distribution of jets, and hence complements the method of using only the jet rates. ALEPH did not use AEEC, since systematic effects remain large (> 15%).

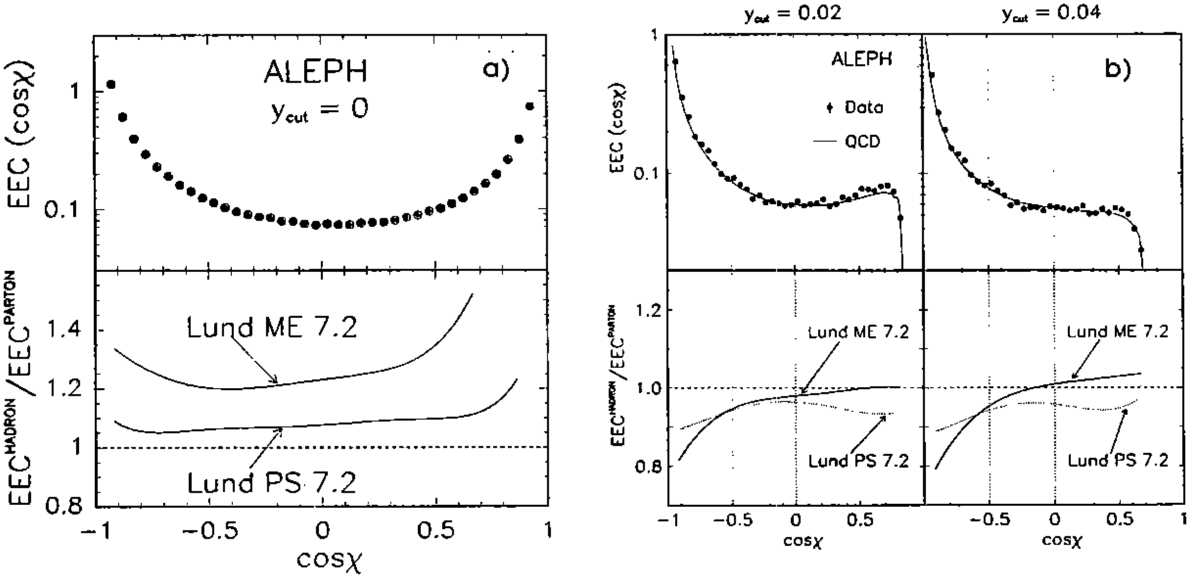

Fig. 3 Measured energy–energy correlations without (a) and with (b) preclustering (see text)

4.4. α_s from global event shapes

Global event-shape analyses with 'infrared-safe' quantities are more reliable at LEP than at the lower PETRA and PEP energies. OPAL's results[26] have been analysed in detail[27] and are summarized as $\alpha_s = 0.12 \pm 0.01$. ALEPH[23] have applied their preclustering method to extract α_s from thrust T, planarity C, and oblateness O, and combined the (correlated) results with the CEEC result, finding $0.117 \pm 0.005^{+0.006}_{-0.009}$.

DELPHI have included event-shape analyses in a combined study of all 3-jet-sensitive variables.[20] The result is shown in Fig. 4, displaying two series of fits made with two different methods for hadronization corrections.

They find 'reasonable' averages of 0.113 and 0.109 from the PS and ME fits, respectively, and give as a final result:

$$\alpha_s = 0.111 \pm 0.002\ (\text{exp.}) \pm 0.003\ (\text{had.})^{-0.006}_{-0.005}\ (\text{scale}) = 0.111^{+0.007}_{-0.006}.$$

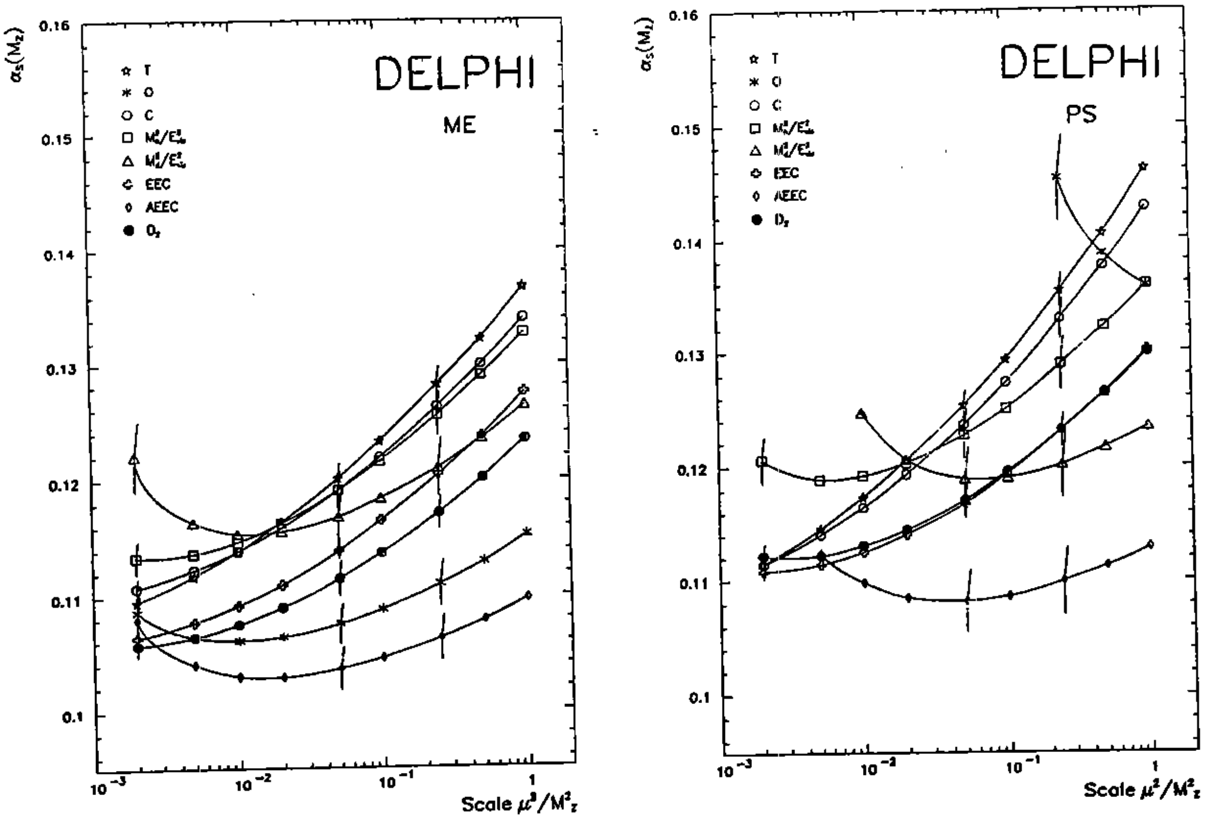

Fig. 4 DELPHI analysis of all 3-jet-sensitive variables in terms of α_s as a function of the renormalization scale for two different hadronization methods

4.5. Summary of α_s results from hadron distributions

As a summary of the studies of hadron distributions in Z decays to determine α_s, Table 3 presents the average or best values as given by the four LEP experiments.

Table 3: α_s from Hadron Distributions

Expt.	α_s	$\Delta\alpha_s$
ALEPH	0.117	+0.008 −0.010
DELPHI	0.111	+0.007 −0.006
L3	0.115	±0.009
OPAL	0.118	±0.008
Average	0.115	±0.008

The errors are dominated by theoretical uncertainties, mainly due to unknown higher orders (5–10%), less to hadronization ($\sim$ 3%); experimental uncertainties average between 2 and 3%.

It has recently been proposed[28] to avoid the arbitrariness in varying the renormalization scale μ in order to 'reduce uncertainties' or improve fit results by exponentiating the leading infrared divergences and thus summing most of the perturbative higher-order effects on the hadron distributions. The α_s values accordingly corrected are 30–50% closer to each other and on the average 10–15% lower than from standard analyses at scale $\mu = m_Z$. However, the average is consistent with the one in Table 3: $\alpha_s(m_Z) = 0.118 \pm$ 0.004 (had.) $\pm$ 0.005 (theor.). The sensitivity of the thrust distribution to μ has also been reduced by resumming leading and next-to-leading logarithms.[29]

5. Summary and Conclusions

A weighted average of the α_s values from R_Z, R_τ, and hadron distributions yields

$$\alpha_s(m_Z) = 0.119 \pm 0.006 \qquad (0.120 \pm 0.007 \text{ without } \alpha_s \text{ from } R_\tau)\ .$$

Flavour independence of α_s has been verified for b-quarks within 8%. The running of α_s has been confirmed.

The largest uncertainty in the α_s determination from hadron distributions is due to the unknown higher orders in the QCD matrix element calculations. The uncertainty can be somewhat reduced by exponentiating (most of) the higher orders, but calculating $O(\alpha_s^3)$ matrix elements is certainly an important project in establishing the validity of the Standard Model.

Increasing the precision on α_s from R_Z to $\Delta\alpha_s \approx 0.005$ requires 2–3 × 10^6 Z decays and a dedicated experimental analysis of Γ_h/Γ_ℓ.

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