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All-loop gauge couplings from anomaly cancellation in string effective theories

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Abstract

We derive, to all orders in perturbation theory, the E_8 gauge coupling and the modified dilaton-axion Kähler potential for the effective theories of a class of $d = 4$, $N = 1$ heterotic string models. The derivation relies on an extended version of the Green-Schwarz anomaly cancellation mechanism, and exploits target-space duality invariance. Although we deal with field-dependent effective gauge couplings and scales in a non-renormalizable supergravity theory, we derive for them a renormalization group equation as a relation among dynamical fields. When expectation values of these fields are considered, our results agree with those previously obtained in renormalizable theories with $N = 1$ global supersymmetry. We finally comment on possible generalizations of the present results.

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A powerful tool to connect four-dimensional string solutions with physics at experimentally accessible energies is provided by the associated effective theories, which describe the low-energy dynamics of the light string excitations. For potentially realistic models, and neglecting higher derivative terms, these are particular versions of $d = 4$, $N = 1$ supergravity. In the last years, intense theoretical effort has been devoted to the determination of these effective low-energy theories, using both explicit string calculations and general symmetry arguments.

The first effective theories to be studied in detail were those reproducing the *tree-level* string predictions for scattering amplitudes among light states. These *classical* effective theories, however, are not sufficient for a description of low-energy physics. A striking example is provided by the effective gauge couplings g_a, associated with the different factors of the four-dimensional gauge group $G = \prod_a G_a$. At the classical level, the string coupling constant is universal, and is determined (modulo Kac-Moody levels) by the vacuum expectation value of the four-dimensional dilaton field. To reproduce the differences among the low-energy effective gauge couplings, one needs to consider quantum corrections, in analogy with conventional Grand Unified Theories. In the early days of string model building, one-loop estimates of the effective low-energy gauge couplings were obtained by using renormalization group arguments at the level of the effective field theory, i.e. considering only loop diagrams involving light states. For more reliable estimates, however, one cannot neglect the effects of loop diagrams involving the exchange of heavy string modes. Recently, *one-loop* moduli-dependent corrections to gauge coupling constants have been computed at the string level in a large class of four-dimensional heterotic string models [1-8].

In this work, we generalize the previous results by introducing a framework to compute the effective gauge couplings to *all orders* in the string perturbative expansion. We obtain an all-order result under the assumption that no dilaton-dependent correction to the low-energy effective Lagrangian other that the anomaly terms are generated in (string) perturbation theory. Our approach relies on some quantum string symmetries, namely the target-space dualities [9], which must be incorporated in the string-derived effective supergravities. The other important ingredient, which allows us to extract an all-loop result from explicit one-loop calculations, is the intimate connection [10], due to supersymmetry, between the anomaly of the dilatation current (related to the effective β-functions) and the $U(1)$ σ-model anomalies associated to target-space duality transformations. It is precisely this supersymmetric relation between anomalies which causes a formal agreement between our determination of the field-dependent effective gauge couplings, derived for the non-renormalizable effective supergravity theory of strings as a relation among dynamical fields, and previous results obtained in renormalizable theories with $N = 1$ global supersymmetry [11-13], and using instanton calculus [14, 15].

We consider here $d = 4$, $N = 1$ heterotic string models. At the classical level, and in the standard formulation [16,17] with matter described by chiral multiplets only, their effective supergravity theories are characterized by a universal gauge kinetic function [18]

$$f_{ab} = \delta_{ab} S, \tag{1}$$

(we omit Kac-Moody levels, since they are usually equal to 1 in potentially realistic string vacua), by a Kähler potential of the general form [19]

$$K = -\log(S + \overline{S}) + \hat{K}(M, \overline{M}; C, \overline{C}), \tag{2}$$

and by a superpotential $w(C)$. In eqs. (1) and (2), S is the dilaton-axion chiral superfield, whereas M and C represent the moduli and matter chiral superfields, respectively. In the following, we will consider only the moduli-dependent part of $\hat{K}$, freezing all the matter fields to zero. Limiting ourselves to (2,2) models, we can write the general formula [20]

$$\hat{K} = -\log\left[V_{(1,1)} \cdot V_{(2,1)}\right], \tag{3}$$

where $V_{(1,1)}$ and $V_{(2,1)}$ are the volume factors of the internal manifolds for the $(1,1)$ and $(2,1)$ moduli, respectively (the former are associated with deformations of the Kähler class, the latter with deformations of the complex structure).

Under any classical symmetry transformation of the tree-level effective action, the Kähler potential K transforms as follows

$$K \rightarrow K + \varphi + \overline{\varphi}, \tag{4}$$

where φ is a holomorphic function of the chiral superfields. Since the classical supergravity Lagrangian depends on K and w only via the combination $\mathcal{G} = K + \log|w|^2$, the previous transformation can be compensated by a corresponding transformation of the superpotential w

$$w \rightarrow w e^{-\varphi}. \tag{5}$$

It is important to keep in mind, however, that any classical symmetry transformation acts on the fermions Ψ_I of the theory (gauginos λ_a and matter fields ψ_i), appropriately rescaled in order to have canonically normalized kinetic terms, in the form of a chiral rotation

$$\Psi_I \rightarrow \Psi_I e^{-\frac{\xi_I}{4}(\varphi-\overline{\varphi})}, \tag{6}$$

where ξ_I is an appropriate real weight. As a result of this chiral rotation, any given classical symmetry of the theory can be violated at the quantum level, i.e. is potentially anomalous. For example, triangle graphs with external lines corresponding to two gauge bosons of G_a and one (composite) connection associated with the symmetry under consideration, will induce at one-loop anomaly terms of the form

$$-\frac{i}{8}c_a(\varphi-\overline{\varphi})F^a_{\mu\nu}\tilde{F}^{a\,\mu\nu}, \qquad c_a = -\frac{1}{4\pi^2}\sum_I \xi_I \operatorname{Tr} Q_I^2, \tag{7}$$

where Q_I is any of the generators of G_a, in the representation of the fermions Ψ_I. As apparent from the previous formula, the coefficients c_a can be easily calculated in the effective theory once one knows the weights ξ_I. Explicit string calculations are bound to give an identical result, except if some or all anomalies are cancelled by a generalized Green-Schwarz mechanism involving the massless states.

The previous general argument finds a relevant application in the case of target-space duality symmetries, which are known to be good string symmetries to any order of the string (higher genus) perturbation theory [21]. It was recently shown [3,5,6,8] that in any $(2,2)$ symmetric abelian orbifold model, all anomalies with respect to target-space modular invariance are cancelled by the combination of two different mechanisms. The first one is purely field-theoretical, and generalizes [3,4,6] the Green-Schwarz anomaly cancellation mechanism [22]. One introduces in the loop-corrected effective Lagrangian a term of the form

$$-\frac{1}{4}k\epsilon^{\mu\nu\rho\sigma}C_\mu\sum_a\Omega^a_{\nu\rho\sigma}, \tag{8}$$

where C_μ is a composite vector field and $\Omega^a_{\nu\rho\sigma}$ is the Yang-Mills Chern-Simons form. The variation of (8) under a symmetry transformation gives

$$\frac{i}{8}k(\varphi-\overline{\varphi})F^a_{\mu\nu}\tilde{F}^{a\,\mu\nu}, \tag{9}$$

with the constant k independent of the gauge group factor. Since the term (8) is also gauge variant, restoration of gauge invariance requires the presence of a linear supermultiplet [23], the gauge transformation of its antisymmetric tensor component compensating the gauge variation of the Chern-Simons form. Since superstrings possess a single linear multiplet, the constant k

in eq. (8) must be group-independent. The second mechanism is a purely string phenomenon [2,5], which generates in the one-loop effective Lagrangian, after integration of the string massive modes, additional terms of the form

$$-\frac{i}{8}d_a(\delta-\overline{\delta})F^a_{\mu\nu}\tilde{F}^{a\,\mu\nu}, \tag{10}$$

where δ is a holomorphic function of the scalar fields in the chiral multiplets, which under the symmetry transformation varies as

$$\delta \to \delta - \varphi, \tag{11}$$

and the d_a are constants which depend in general on the gauge group factor. In the $(2,2)$ models under discussion, these local anomaly cancelling terms in the effective one-loop Lagrangian involve automorphic functions of the target-space duality group [24,2]. From the field theory point of view, the terms in eq. (10) can be seen as local 'counterterms', necessary for the cancellation of target-space modular anomalies [3,5]. The condition for complete anomaly cancelation is then $d_a + k = c_a$. A fact of main importance, which will be used in the following, is that once the anomaly coefficients are known, then, because of supersymmetry, the expressions of the field-dependent effective gauge coupling constants are completely determined up to field-independent terms. The latter are also fully controlled by the infrared divergences of the theory, due to the massless modes only [25].

To simplify the discussion, we consider here the case of the E_8 gauge coupling constant and anomaly terms in $(2,2)$ symmetric abelian orbifolds with no $N=2$ fixed planes, e.g. Z_3 and Z_7. In this case, only gauginos undergo chiral rotations, and the composite connection associated to target-space modular invariance is actually proportional to the Kähler connection. Moreover, the coefficients d_{E_8} are identically zero [6], and the full anomaly cancellation is achieved by means of the Green-Schwarz mechanism only [3]. To discuss anomalies and effective gauge couplings at once, the superfield formalism proves to be very useful. Using standard techniques [26,27] one can then extract all relevant component expressions.

In the formalism in which the S chiral superfield is replaced by a linear multiplet L via a supersymmetric duality transformation [23], the tree-level effective Lagrangian can be written as a superconformal density [28]

$$\mathcal{L} = -\frac{1}{\sqrt{2}}\left[(S_0\overline{S}_0)^{3/2}e^{-\hat{K}/2}\hat{L}^{-1/2}\right]_D + \left(\left[S_0^3 w\right]_F + \text{h.c.}\right), \tag{12}$$

where $\hat{L} = L - \Omega \equiv L - \sum_a \Omega_a$, Ω_a is the Chern-Simons superfield associated with the gauge group factor G_a and S_0 is the compensating multiplet of supergravity[1]. To see the equivalence with the S-representation, it is sufficient [29] to replace $\hat{L}$ with an unconstrained real superfield V and to introduce the Lagrangian

$$\hat{\mathcal{L}} = \mathcal{L}(\hat{L}\to V) - \left[(S+\overline{S})(V+\Omega)\right]_D. \tag{13}$$

The equation of motion for $(S+\overline{S})$ indicates that $V+\Omega$ is a linear multiplet, and substituting the solution into eq. (13), one goes back to the previous description in terms of the linear multiplet, eq. (12).

Solving the equations of motion with respect to V, one obtains an equivalent form of the Lagrangian in terms of the chiral supermultiplet S. Explicitly, the equation of motion for V reads

$$U \equiv \frac{1}{2}\left(\frac{S_0\overline{S}_0}{2Ve^{\hat{K}/3}}\right)^{3/2} = \frac{S+\overline{S}}{2}. \tag{14}$$

[1] Our conventions are such that the D-density $-(3/2)[S_0\overline{S}_0\Phi(\Sigma,\overline{\Sigma})]_D$ generates an Einstein term of the form $-(1/2)eR|S_0|^2\Phi(\Sigma,\overline{\Sigma})$. We also use the same notation for chiral superfields S_0 or Σ, and for their lowest complex scalar components.

As discussed later, eq. (14) is modified by one-loop and higher-loop effects. Since gauge kinetic terms are of the form

$$\begin{aligned} -[(S+\overline{S})\Omega]_D &= \tfrac{1}{4}[SW^aW^a]_F + h.c. \\ &= -\tfrac{1}{8}(S+\overline{S})F^a_{\mu\nu}F^{a\,\mu\nu} + \ldots, \end{aligned} \tag{15}$$

satisfying eq. (1), we will make the identifications

$$\frac{S+\overline{S}}{2} = \frac{1}{g_0^2}, \tag{16}$$

where g_0 is the unrenormalized gauge coupling, and

$$U = \frac{1}{g_{E_8}^2}, \tag{17}$$

where g_{E_8} is the loop-corrected effective gauge coupling for the factor group E_8. Eq. (14) is the obvious statement that g_0 and g_{E_8} coincide at the tree-level. Also, we will see that the Kähler potential for the dilaton, which at tree-level takes the form

$$K_{dilaton} = -\log(S+\overline{S}) = \log\frac{g_0^2}{2}, \tag{18}$$

or

$$K_{dilaton} = -\log(2U) = \log\frac{g_{E_8}^2}{2}, \tag{19}$$

keeps this last form at one-loop, but in terms of the one-loop effective gauge coupling, and receives corrections at higher orders. We will derive below an exact, all-order formula for $K_{dilaton}$.

To begin, we observe that the classical Lagrangian (12) is invariant under the transformations

$$\begin{cases} \hat{K} & \to \hat{K} + \varphi + \overline{\varphi} \\ S_0 & \to S_0 e^{\varphi/3} \\ w & \to w e^{-\varphi} \\ \hat{L} & \to \hat{L} \end{cases} \tag{20}$$

In particular, target-space duality transformations have an action of this type, with a specific form of φ. Therefore, under target-space duality transformations, S_0 transforms with weight 1/3, whereas its scaling dimension is 1. The combination $S_0\overline{S}_0e^{-\hat{K}/3}$ has scaling dimension 2, and is invariant under target-space duality transformations. Both these properties also hold for $\hat{L}$. Then the combination U defined in eq. (14), which is related both to the E_8 gauge coupling constant and to the dilaton Kähler potential, is at the same time target-space duality invariant and dimensionless.

To prepare the ground for the all-loop result, we first review the determination of the one-loop corrected gauge couplings and dilaton Kähler potential, and the anomaly cancellation mechanism at the one-loop level [3]. In the case of E_8, the triangular graph associated to the target-space duality anomaly can be represented, in superfield language, by a non-local Lagrangian of the form

$$\mathcal{L}^{E_8}_{nl} = \int d^2\theta \frac{1}{4} W^a_{E_8} W^a_{E_8} \frac{1}{16}\frac{\overline{\mathcal{D}}^2\mathcal{D}^2}{\Box}\frac{C(E_8)}{8\pi^2}\hat{K} + \text{h.c.}. \tag{21}$$

Performing a Kähler transformation gives rise to anomalies of the form

$$\frac{C(E_8)}{64\pi^2}\left[(\varphi+\overline{\varphi})F^a_{\mu\nu}F^{a\,\mu\nu} + (\varphi-\overline{\varphi})F^a_{\mu\nu}\tilde{F}^{a\,\mu\nu}\right], \tag{22}$$

which are compensated by the local Green-Schwarz term

$$\mathcal{L}_{GS} = \left[\frac{C(E_8)}{8\pi^2}\hat{L}\hat{K}\right]_D = \left[\frac{3C(E_8)}{8\pi^2}\hat{L}\log e^{\hat{K}/3}\right]_D . \tag{23}$$

This gives the one-loop result discussed in refs. [3,4,6]. Taking into account eq. (23), and performing as before the substitution $\hat{L} \to V$, one obtains

$$\tilde{\mathcal{L}}_1 = \tilde{\mathcal{L}} + \mathcal{L}_{GS}(\hat{L} \to V),$$

and the equation of motion for V becomes

$$U = \frac{S+\overline{S}}{2} - \frac{C(E_8)}{16\pi^2}\hat{K}. \tag{24}$$

We have thus obtained the one-loop correction to eq. (14). Solving the above equation of motion leads to the one-loop corrected effective action. In the resulting S-field formulation, as announced before, the effective E_8 gauge coupling becomes

$$\frac{1}{g^2_{E_8}} = \frac{S+\overline{S}}{2} - \frac{C(E_8)}{16\pi^2}\hat{K}, \tag{25}$$

and the Kähler potential for the dilaton reads

$$K_{dilaton} = -\log\left(S+\overline{S} - \frac{C(E_8)}{8\pi^2}\hat{K}\right) = \log\frac{g^2_{E_8}}{2}, \tag{26}$$

identical to eq. (19). This was the result of ref. [3].

The Green-Schwarz term of eq. (23) can be further completed if we remember that in a general superconformal gauge the compensating multiplet S_0 is also not inert under target-space duality transformations. As a consequence, the Green-Schwarz counterterm should also cancel, simultaneously, the anomaly due to S_0 transformations. This is because physics must be independent of the particular gauge choice for S_0. It is immediate to see that there is only one possible modification of the one-loop Green-Schwarz counterterm, eq. (23), which is simultaneously compatible with target-space duality invariance and superconformal invariance, namely

$$\hat{K} \equiv 3\log\left(e^{\hat{K}/3}\right) \longrightarrow 3\log\left(e^{\hat{K}/3}\hat{L}\right) \equiv 3\log\left(\frac{S_0\overline{S}_0}{2}\right) - 2\log(2U). \tag{27}$$

Indeed, eq. (27) is the most general term compatible with the one-loop scaling symmetry [30,29] and which reproduces the desired anomaly.

We can now write the non-local contribution in a general superconformal gauge as

$$\begin{aligned}\mathcal{L}^{E_8}_{nl}{}' &= \int d^2\theta \frac{1}{4} W^a_{E_8} W^a_{E_8} \frac{1}{16}\frac{\overline{\mathcal{D}}^2\mathcal{D}^2}{\Box}\frac{3C(E_8)}{8\pi^2}\left[\log\left(e^{\hat{K}/3}\hat{L}\right)\right] + \text{h.c.}\\ &= \int d^2\theta \frac{1}{4} W^a_{E_8} W^a_{E_8} \frac{1}{16}\frac{\overline{\mathcal{D}}^2\mathcal{D}^2}{\Box}\frac{3C(E_8)}{8\pi^2}\left[\log\left(\frac{2e^{\hat{K}/3}\hat{L}}{S_0\overline{S}_0}\right) + \log\left(\frac{S_0\overline{S}_0}{2}\right)\right] + \text{h.c.}.\end{aligned} \tag{28}$$

In the first expression, $\hat{L}$ corrects the quantum dimension of $e^{\hat{K}/3}$ without spoiling the transformation properties of $\hat{K}$ under target-space duality. In the second expression, we have rearranged the terms to evidentiate the scale-invariant and duality-invariant combination $2U$ defined in (14) [after replacement of $\hat{L}$ by V]. The $\log(S_0\overline{S}_0/2)$ term contains the violation of duality as well as scaling invariances.

Using the same scaling argument, the local counterterm of eq.(23) must be modified as follows

$$\mathcal{L}'_{GS} = \left[\frac{C(E_8)}{8\pi^2} 3\hat{L} \log\left(e^{\hat{K}/3}\hat{L}\right)\right]_D$$

$$= \left[\frac{C(E_8)}{8\pi^2} 3\hat{L}\left\{\log\left(\frac{2e^{\hat{K}/3}\hat{L}}{S_0\overline{S}_0}\right) + \log\left(\frac{S_0\overline{S}_0}{2}\right)\right\}\right]_D . \tag{29}$$

The second term in the last member precisely cancels the anomaly induced by the non-local terms.

The connection between the cancellation of target-space modular anomalies and the E_8 β-function is now transparent. The β function is the dilatation anomaly obtained by varying the compensator field S_0 (see, for example, refs. [11,27]). However, this is related to the duality anomaly, since S_0 transforms under duality. To move from the linear multiplet formulation to the S-field formulation, after the correction given in eq. (29), one has to write

$$\tilde{\mathcal{L}}_{all} = \hat{\mathcal{L}} + \mathcal{L}'_{GS}(\hat{L} \to V). \tag{30}$$

Solving the equation of motion for V, one finds

$$U = \frac{S+\overline{S}}{2} + \frac{C(E_8)}{8\pi^2}\left[\log U + c - \frac{3}{2}\log(S_0\overline{S}_0)\right], \tag{31}$$

where

$$c = -\frac{3}{2} + \frac{5}{2}\log 2. \tag{32}$$

Identifying as before U with $\frac{1}{g^2_{E_8}}$, where $g^2_{E_8}$ is the corrected E_8 effective coupling constant, one finds

$$\frac{1}{g^2_{E_8}} = \frac{S+\overline{S}}{2} + \frac{C(E_8)}{8\pi^2}\left[-\log g^2_{E_8} + c - \frac{3}{2}\log(S_0\overline{S}_0)\right]. \tag{33}$$

From the previous formula one can immediately derive the effective β-function for the E_8 gauge group factor. Remembering that $S_0\overline{S}_0 \sim M^2$ sets the unit of measure for masses, we can write

$$\beta(g_a) = -\frac{1}{g_a}\frac{dg_a^2}{d[\log(S_0\overline{S}_0)]}, \tag{34}$$

and therefore, in the case of E_8,

$$\beta(g_{E_8}) = g^3_{E_8}\frac{-\frac{3C(E_8)}{16\pi^2}}{1-\frac{C(E_8)}{8\pi^2}g^2_{E_8}}, \tag{35}$$

which coincides with the formula of refs. [12, 14]. One can also write the all-loop corrected Kähler potential for the dilaton field,

$$K_{dilaton} = \log\left[\frac{g^2_{E_8}}{2}\left(1+\frac{C(E_8)}{16\pi^2}g^2_{E_8}\right)^{-3}\right], \tag{36}$$

where now g_{E_8} is the solution of eq. (33).

The relation between the fields U and $(S+\overline{S})/2$, which we have associated with the all-loop corrected and uncorrected E_8 gauge couplings in the effective supergravity theory of $(2,2)$ four-dimensional string models, turns out to be exactly the relation between $1/g^2$ and $1/g_0^2$ in renormalizable, globally supersymmetric $N=1$ Yang-Mills theories. In principle, there is no reason why it should be so, since we are working in a non-renormalizable gauge theory coupled to gravity, so a plausible explanation is required. In string theory, and in the corresponding effective supergravity theories, couplings and scales are dynamical variables, so relations between them must be interpreted as field equations. This is the meaning of our eq. (33). On the other hand, the all-loop renormalization group equation obtained in global supersymmetry is a consequence of the general structure of the anomaly supermultiplet. This is the reason why the two results formally agree.

In eq. (33), the running of the gauge coupling is encoded in S_0, because S_0 itself determines the energy scale of the theory. In order to define a modular invariant effective gauge coupling, one should fix the superconformal gauge with a condition of the form

$$S_0\overline{S}_0 = M^2 e^{\hat{K}/3}, \tag{37}$$

where M^2 is an arbitrary modular invariant function, which in particular can be taken to be a constant. To connect the modular invariant effective gauge coupling defined by eqs. (33,37) with the running gauge coupling defined in the renormalizable, low-energy effective theory in a given renormalization scheme (for example, $\overline{MS}$ or $\overline{DR}$), one should impose on the two couplings an appropriate boundary condition, as discussed for example in refs. [1,3,7]. This would also allow to define, as usual, a modular invariant, renormalization group invariant scale Λ_{E_8}, analogous to Λ_{QCD}, which plays a role in the discussion of possible non-perturbative phenomena like gaugino condensation.

The previous result is valid only for the E_8 gauge group factor, where only gaugino fields contribute to the modular anomaly, and in the absence of anomaly cancellation mechanisms other than the Green-Schwarz one. In more general cases, one has also to consider the associated massless charged matter fields, which are subject to a rescaling under duality transformations, and the composite connections associated to duality transformations are not simply the Kähler connection but also involve the metric of the matter fields. Similarly, in globally supersymmetric theories the β function formula contains also the anomalous dimensions of the matter fields, and mixing is possible in the presence of different gauge group factors. Here we will not examine these complications, work along this line is in progress and will be presented elsewhere.

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