

**Transition form factors in  $\pi^0$ ,  $\eta$ , and  $\eta'$  couplings to  $\gamma\gamma^*$**

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Recent measurements of the transition form factors for the  $P\gamma\gamma^*$  vertices, with  $P=\pi^0$ ,  $\eta$ , and  $\eta'$ , are compared with different models. These include vector-meson dominance, constituent-quark loops, the QCD-inspired interpolation by Brodsky-Lepage, and chiral perturbation theory. General agreement is observed and differences—due to SU(3) breaking—are stressed and discussed.

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Experimental data for the two-photon transitions  $\gamma\gamma^*\rightarrow\pi^0$ ,  $\eta$ , and  $\eta'$  have been recently obtained and discussed [1,2]. They involve (at least) one spacelike photon  $\gamma^*$  with squared four-momentum  $q^2=-Q^2<0$ . This completes and confirms older results concerning timelike photons ( $q^2>0$ ) obtained from  $\eta, \eta'\rightarrow\gamma\gamma^*\rightarrow\gamma\mu^+\mu^-$  decays [3,4] and solves the chaotic situation related to the  $\pi^0\gamma\gamma^*$  vertex [4,5]. One usually fits the observed  $q^2$  dependence in the different  $P\gamma\gamma^*$  transitions by means of a normalized, single-pole term with an associated mass  $\Lambda_P$ , i.e.,

$$F_P(q^2) = F(\Lambda_P, q^2) / F(\Lambda_P, 0) \\ = (1 - q^2 / \Lambda_P^2)^{-1} \simeq 1 + q^2 / \Lambda_P^2 \equiv 1 + b_P q^2, \quad (1)$$

where in the last steps (for small  $q^2$ ) we have introduced the slope  $b_P \equiv 1 / \Lambda_P^2 = \langle r_P^2 \rangle / 6$  related to the size of the pseudoscalar meson  $P$ . The available experimental data [1–3] for  $\Lambda_{\pi^0, \eta, \eta'}$  and their averaged values [2] are summarized in Table I. The amplitude for a generic  $P \leftrightarrow \gamma\gamma^*$  process is then

$$A(P \leftrightarrow \gamma\gamma^*) = \pm i F(\Lambda_P, q^2) \epsilon_{\mu\nu\alpha\beta} \epsilon^\mu k^\nu \epsilon^{*\alpha} q^\beta \quad (2)$$

TABLE I. Experimental values for the pole mass  $\Lambda_P$  (in GeV) in the transition form factors of pseudoscalar mesons  $P = \pi^0, \eta$ , and  $\eta'$ .

	$\Lambda_{\pi^0}$ (GeV)	$\Lambda_\eta$ (GeV)	$\Lambda_{\eta'}$ (GeV)
Lepton-G [3]		0.72±0.09	0.77±0.18
TPC/2 $\gamma$ [1]		0.70±0.08	0.85±0.07
CELLO [2]	0.75±0.03	0.84±0.06	0.79±0.04
Average [2]	0.75±0.03	0.77±0.04	0.81±0.04

with  $k^2=0$  ( $q^2 \neq 0$ ) for the real (virtual) photon with polarization  $\epsilon$  ( $\epsilon^*$ ).

Theoretically,  $P\gamma\gamma$  transitions involving on-mass-shell photons,  $k^2=q^2=0$ , contain valuable information on the quark content (or mixing) of the  $\eta, \eta'$  mesons. Concerning this point, the situation is quite satisfactory and general agreement has been achieved [2,4,6]. This implies

$$\eta = \cos\theta\eta_8 - \sin\theta\eta_1 = \cos\beta(u\bar{u} + d\bar{d}) / \sqrt{2} - \sin\beta s\bar{s}, \\ \eta' = \sin\theta\eta_8 + \cos\theta\eta_1 = \sin\beta(u\bar{u} + d\bar{d}) / \sqrt{2} + \cos\beta s\bar{s}, \quad (3) \\ \theta = \beta - \arccot\sqrt{2} \simeq -\arccot 2\sqrt{2} \simeq -19.5^\circ.$$

The  $q^2$  dependence observed in  $P\gamma\gamma^*$  transitions can then be viewed as a tool for understanding light-quark dynamics. To this aim several models have been discussed. The purpose of this note is to compare the experimental measurements of  $\Lambda_P$  quoted in Table I with the predictions of the most successful and/or traditional models. These include conventional ideas related to vector-meson dominance (VMD) or constituent-quark loops (QL) and QCD-inspired approaches such as the Brodsky-Lepage (BL) interpolation formula or chiral perturbation theory (ChPT).

Using VMD one immediately obtains [7,8]

$$F^{\text{VMD}}(\Lambda_P, q^2) = \sum_V \frac{g_{PV\gamma}}{f_V} \frac{M_V^2}{M_V^2 - q^2}, \quad (4)$$

where the sum includes the three lightest vector mesons  $V = \rho^0, \omega$ , and  $\varphi$  with SU(3)-symmetric couplings to the photon ( $f_V$ ) and to  $P\gamma$  ( $g_{VP\gamma}$ ).  $\Lambda_V$  is then related to the vector-meson masses  $M_V$ , thus introducing the only source of SU(3) breaking (apart from mixing) through [5]  $M_\rho \simeq M_\omega \simeq \lambda M_\varphi$ , with  $1/\lambda \simeq 1.30$ . More explicitly, one obtains

$$\begin{aligned}
\Lambda_\pi^2 &\simeq M_{\rho,\omega}^2, \quad \Lambda_\pi = 0.78 \text{ GeV}, \\
\Lambda_\eta^2 &= \frac{5 \cos\beta - \sqrt{2} \sin\beta}{5 \cos\beta - \sqrt{2} \lambda \sin\beta} M_{\rho,\omega}^2, \\
\Lambda_\eta &= 0.96 \Lambda_\pi = 0.75 \text{ GeV}, \\
\Lambda_{\eta'}^2 &= \frac{5 \sin\beta + \sqrt{2} \cos\beta}{5 \sin\beta + \sqrt{2} \lambda \cos\beta} M_{\rho,\omega}^2, \\
\Lambda_{\eta'} &= 1.06 \Lambda_\pi = 0.83 \text{ GeV},
\end{aligned} \tag{5}$$

where the numerical values follow from Eq. (3) and Ref. [5] and have been collected in Table II.

The QL predictions for the  $P\gamma\gamma^*$  form factors are easily obtained computing the  $q^2$  dependence generated by a triangle loop of constituent quarks of masses  $m_q$  and charges  $e_q$ . One obtains [7,8]

$$F^{\text{QL}}(\Lambda_P, q^2) = \sum_q \frac{g_{Pq\bar{q}}}{m_q} e_q^2 \left[ \frac{1}{\lambda_q} \arcsin \lambda_q \right]^2, \quad \lambda_q^2 \equiv \frac{q^2}{4m_q^2}, \tag{6}$$

where the  $Pq\bar{q}$  couplings are SU(3) symmetric and breaking appears only through the constituent quark masses  $m_u = m_d = \lambda' m_s$ , with  $1/\lambda' \simeq 1.40$ . More explicitly, one has

$$\begin{aligned}
\Lambda_\pi^2 &= 12m_{u,d}^2, \quad \Lambda_\pi = 0.80 \text{ GeV}, \\
\Lambda_\eta^2 &= \frac{5 \cos\beta - \sqrt{2} \lambda' \sin\beta}{5 \cos\beta - \sqrt{2} \lambda'^3 \sin\beta} 12m_{u,d}^2, \\
\Lambda_\eta &= 0.96 \Lambda_\pi = 0.77 \text{ GeV}, \\
\Lambda_{\eta'}^2 &= \frac{5 \sin\beta + \sqrt{2} \lambda' \cos\beta}{5 \sin\beta + \sqrt{2} \lambda'^3 \cos\beta} 12m_{u,d}^2, \\
\Lambda_{\eta'} &= 1.06 \Lambda_\pi = 0.84 \text{ GeV},
\end{aligned} \tag{7}$$

where we have used Eq. (3) and a somewhat small constituent mass ( $m_{u,d} \simeq 0.23 \text{ GeV}$ ) in order to agree reasonably with the data and also with the VMD results [5].

The latter agreement is a manifestation of the old idea of quark-hadron or  $Q^2$  duality already checked in [7,8] for  $\eta \rightarrow \gamma\gamma^*$ . Here, we have extended its validity to the SU(3)-breaking contributions exploiting the approximate equalities  $\lambda \simeq \lambda'$  and  $M_V^2 \simeq 12m_q^2$  between VMD and QL parameters.

The Brodsky-Lepage (BL) interpolation formula [9] for these transition form factors is extremely simple, namely,

$$F_P^{\text{BL}}(\Lambda_P, q^2) = \frac{2\sqrt{2}\alpha}{\Lambda_P} (1 - q^2/\Lambda_P^2)^{-1}, \tag{8}$$

where  $\Lambda_P = 2\pi f_P$  is related to the pseudoscalar-meson decay constant  $f_P$ . It is an elegant expression interpolating two theoretically well-rooted results valid at the extreme energies  $q^2 \rightarrow 0$  and  $Q^2 \rightarrow \infty$ . In the first case, current algebra (CA) unambiguously predicts  $F(\Lambda_P, q^2 \rightarrow 0) = \sqrt{2}\alpha/\pi f_P$ , whereas, QCD leads to  $F(\Lambda_P, Q^2) = 4\pi\alpha\sqrt{2}f_P/Q^2$ , in the opposite and reliable region of asymptotically large  $Q^2$ . Our normalization is such that the pion decay constant  $f_{P=\pi} = \sqrt{2} \times 93 \text{ MeV} = 132 \text{ MeV}$  and, therefore, one has  $\Lambda_\pi = 2\pi f_\pi = 0.83 \text{ GeV}$  in the correct range of the experimental values. SU(3) breaking now proceeds exclusively through  $f_\pi \neq f_\eta \neq f_{\eta'}$ . The two latter decay constants are not directly measurable (in contrast with  $f_{K^\pm}$  or  $f_{\pi^\pm} = f_{\pi^0} = f_\pi$ , by isospin) but can be deduced from  $\eta, \eta' \rightarrow \gamma\gamma$  decays into real photons. One has

$$\begin{aligned}
\frac{1}{f_\eta} &= \frac{1}{\sqrt{3}} \left[ \frac{\cos\theta}{f_8} - \frac{\sqrt{8} \sin\theta}{f_1} \right] = \frac{0.914}{f_\pi}, \\
\frac{1}{f_{\eta'}} &= \frac{1}{\sqrt{3}} \left[ \frac{\sin\theta}{f_8} + \frac{\sqrt{8} \cos\theta}{f_1} \right] = \frac{1.25}{f_\pi},
\end{aligned} \tag{9}$$

where the numerical values follow from Eqs. (3) and the averaged data [5] for  $\pi^0$ ,  $\eta$ , and  $\eta' \rightarrow \gamma\gamma$  decays. Indeed, several analyses [6,10,11] lead to the values

$$\theta = -20^\circ, \quad f_8 \simeq (0.25 - 1.30)f_\pi, \quad f_1 \simeq 1.1f_\pi \tag{10}$$

and, then, to those quoted in Eq. (9). Therefore, one predicts

$$\begin{aligned}
\Lambda_\pi &= 2\pi f_\pi = 0.83 \text{ GeV}, \\
\Lambda_\eta &= 1.10 \Lambda_\pi = 0.91 \text{ GeV}, \\
\Lambda_{\eta'} &= 0.80 \Lambda_\pi = 0.66 \text{ GeV},
\end{aligned} \tag{11}$$

as quoted in Table II. The qualitative relation  $\Lambda_\eta > \Lambda_{\eta'}$  seems unavoidable and contrasts with the experimental data (Table I) which tend to prefer  $\Lambda_{\eta'} > \Lambda_\eta$ . This discrepancy is already present in the analysis of Ref. [1], where the values  $f_\eta = 91 \pm 6 \text{ MeV}$  and  $f_{\eta'} = 78 \pm 5 \text{ MeV}$  are deduced from the decay widths into two real photons contrasting with the values  $f_\eta = 79 \pm 9 \text{ MeV}$  and  $f_{\eta'} = 96 \pm 8 \text{ MeV}$  also deduced in [1] from the observed  $q^2$  dependence.

ChPT is particularly appropriate for dealing with  $P\gamma\gamma^*$  processes. It is a QCD-inspired model with a Lagrangian written in terms of the pseudoscalar meson fields, which are assumed to be the pseudo-Goldstone-boson fields appearing in the process of dynamical break-

TABLE II. Values for  $\Lambda_{\pi^0, \eta, \eta'}$  predicted by vector-meson dominance (VMD), quark model loops (QL), the Brodsky-Lepage interpolating formula, and chiral perturbation theory (ChPT).

	$\Lambda_{\pi^0}$ (GeV)	$\Lambda_\eta$ (GeV)	$\Lambda_{\eta'}$ (GeV)
VMD [7,8]	$M_{\rho,\omega} = 0.78$	$0.96\Lambda_\pi = 0.75$	$1.06\Lambda_\pi = 0.83$
QL ( $m_u = m_s / 1.4 = 0.23 \text{ GeV}$ )	$\sqrt{12}m_u = 0.80$	$0.96\Lambda_\pi = 0.77$	$1.06\Lambda_\pi = 0.84$
Brodsky-Lepage [9]	$2\pi f_\pi = 0.83$	$1.10\Lambda_\pi = 0.91$	$0.80\Lambda_\pi = 0.66$
ChPT ( $M_V = 0.828 \text{ GeV}$ )	$(b_L + b_V)^{-1/2} = 0.75$	$1.03\Lambda_\pi = 0.77$	$1.06\Lambda_\pi = 0.79$

ing of the chiral symmetry of massless QCD. The Lagrangian is the most general one reproducing the symmetries of the original QCD Lagrangian. It is expanded in powers of  $p^2/\Lambda^2$  and  $m^2/\Lambda^2$ , where  $p$  is a typical momentum,  $m$  is the quark mass and  $\Lambda \sim 4\pi f_\pi$  is the scale of chiral symmetry breaking. The relevant lowest-order terms of the action are

$$S = \int d^4x L_2 - N_c S_{WZ}, \quad N_c = 3, \quad (12)$$

with

$$L_2 = \frac{1}{8} f^2 \text{tr}(D_\mu \Sigma D^\mu \Sigma^\dagger + \chi^\dagger \Sigma + \Sigma^\dagger \chi), \quad (13)$$

$$S_{WZ} = \frac{i}{48\pi^2} \int d^4x \epsilon^{\mu\nu\alpha\beta} Z_{\mu\nu\alpha\beta} + \dots,$$

where the ellipsis refer to nonphotonic terms of no relevance here and

$$\begin{aligned} Z_{\mu\nu\alpha\beta} = & -ie A_\mu \text{tr}[Q(\partial_\nu \Sigma \partial_\alpha \Sigma^\dagger \partial_\beta \Sigma \Sigma^\dagger - \partial_\nu \Sigma^\dagger \partial_\alpha \Sigma \partial_\beta \Sigma^\dagger \Sigma)] \\ & + 2e^2 (\partial_\mu A_\nu) A_\alpha \text{tr}[Q^2 \partial_\beta \Sigma \Sigma^\dagger + Q^2 \Sigma^\dagger \partial_\beta \Sigma + \frac{1}{2} Q \Sigma Q \Sigma^\dagger \partial_\beta \Sigma \Sigma^\dagger + \frac{1}{2} Q \Sigma^\dagger Q \Sigma \partial_\beta \Sigma^\dagger \Sigma]. \end{aligned} \quad (14)$$

The covariant derivative  $D_\mu \Sigma = \partial_\mu \Sigma + ie[Q, \Sigma] A_\mu$  contains the photon field and the quark charge matrix  $Q = \text{diag}(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})$ . The pseudoscalar meson fields are contained in a nonlinear form in  $\Sigma$ ,

$$\Sigma = \exp \left[ \frac{2i}{f} M \right], \quad (15)$$

with

$$M = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} \end{pmatrix} \quad (16)$$

and  $f$  is a free constant that, at lowest order, can be identified with the pion-decay constant  $f_\pi$ . Under chiral  $U(3)_L \times U(3)_R$ ,  $\Sigma$  transforms as  $\Sigma \rightarrow U_L \Sigma U_R^\dagger$ . The Lagrangian  $L_2$  in Eq. (13) introduces a small spontaneous chiral-symmetry breaking through the quark-mass matrix  $M$ , contained in  $\chi = BM + \dots$ , where  $B$  is a free constant that can be fixed relating the quark masses to the pseudoscalar masses.

The term containing two photons in  $S_{WZ}$  is the only one contributing at lowest order to the amplitude for  $P \leftrightarrow \gamma\gamma^*$ . The contribution turns out to be  $q^2$  independent:

$$F^{\text{ChPT}}(\Lambda_P, q^2) = \frac{\sqrt{2} C_P \alpha}{\pi f_P} \quad (17)$$

with  $C_\pi = 1$ ,  $C_{\eta_8} = 1/\sqrt{3}$  and  $C_{\eta_1} = 2\sqrt{2}/\sqrt{3}$ . It should be noticed that, since the only source of  $U(3)$  breaking in Eqs. (12) and (13) are the quark masses, all the  $f_P$  are the same at this order. As expected from the nonrenormalization of the anomaly and explicitly shown in Refs. [10,12,13], loop corrections for real photons do not modify the lowest-order result and only amount to the introduction of  $U(3)$  breaking in the values of  $f_P$ . The  $\pi^0$ ,  $\eta$ , and  $\eta' \rightarrow \gamma\gamma$  decay widths are, then, well understood in terms of the parameters in Eq. (10). Their finite parts can be calculated from the assumption that they are saturated

by vector-meson contributions [13]. As a result, one obtains ( $\sin\theta = -\frac{1}{3}$ )

$$\begin{aligned} F_\pi(q^2) &= 1 + (b_L + b_V) q^2, \\ F_\eta(q^2) &= 1 + \left[ \frac{2f_1 + f_8}{2f_1 + 2f_8} b_L + b_V \right] q^2, \end{aligned} \quad (18)$$

$$F_{\eta'}(q^2) = 1 + \left[ \frac{f_1 - 4f_8}{f_1 - 8f_8} b_L + b_V \right] q^2,$$

where the finite part of the loop correction to the slope is given by

$$b_L = -\frac{1}{24\pi^2 f^2} [1 + \ln(m_K m_\pi / \mu^2)] = +0.32 \text{ GeV}^{-2} \quad (19)$$

for  $\mu^2 \equiv M_V^2 \simeq (9M_\rho^2 + M_\omega^2 + 2m_\phi^2)/12 = 0.69 \text{ GeV}^2$ , which is the relevant mean vector-meson mass for our processes. This same mean mass fixes the contribution dominated by vector mesons, namely,

$$b_V = 1/\mu^2 = 1.46 \text{ GeV}^{-2}, \quad (20)$$

which (at the present order) is common to  $\pi^0$ ,  $\eta$ , and  $\eta'$ . The only sources of  $SU(3)$  breaking are, therefore,  $f_1 \neq f_8 \neq f_\pi$  and the fact that the loop correction for  $\pi^0$

and  $\eta_8 (b_L)$  is twice as large as for  $\eta_1 (b_L/2)$  leading to the different coefficients of  $b_L$  in Eqs. (18). From these equations one gets

$$\begin{aligned}\Lambda_\pi &= (b_L + b_V)^{-1/2} = 0.75 \text{ GeV} , \\ \Lambda_\eta &= 1.03\Lambda_\pi = 0.77 \text{ GeV} , \\ \Lambda_{\eta'} &= 1.06\Lambda_\pi = 0.79 \text{ GeV} .\end{aligned}\tag{21}$$

In summary, all the models considered agree in the

correct value for a mean  $\Lambda_P$ , but differ in the breaking pattern when  $P = \pi^0, \eta, \text{ or } \eta'$ . The VMD and QL approaches lead to  $\Lambda_\eta < \Lambda_\pi < \Lambda_{\eta'}$ , in agreement with the data of Refs. [1,3]. The BL interpolation formula, instead, implies  $\Lambda_{\eta'} < \Lambda_\pi < \Lambda_\eta$ , in disagreement with the experimental data. Finally, ChPT predicts  $\Lambda_\pi < \Lambda_\eta < \Lambda_{\eta'}$  in agreement with the averaged data. At this stage, it seems reasonable to conclude that accurate experiments (with precision of the order of a few percent) are required in order to decide on the correct scheme accounting for the  $P\gamma\gamma^*$  transition form factors.

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