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TRANSITION FORM–FACTORS IN π^0, η AND η' COUPLINGS TO $\gamma\gamma^*$

Ll. AMETLLER
Departament de Física i Enginyeria Nuclear, UPC,
08800 Vilanova (Barcelona), Spain

J. BIJNENS
Theory Division, CERN, CH-1211- Geneva - 23, Switzerland

A. BRAMON
Grup de Física Teòrica, Universitat Autònoma de Barcelona,
08193 Bellaterra (Barcelona), Spain

and

F. CORNET
Departamento de Física Teórica y del Cosmos,
Universidad de Granada, 18071 Granada, Spain

ABSTRACT

Recent measurements of the transition form–factors for the $P\gamma\gamma^*$ vertices, with $P = \pi^0$, η and η', are compared with different models. These include Vector–Meson Dominance (VMD), constituent–quark loops (QL), the QCD–inspired interpolation by Brodsky–Lepage (BL) and Chiral Perturbation Theory (ChPT). General agreement is observed and differences –due to SU(3)-breaking– are stressed and discussed.

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Experimental data for the two–photon transitions $\gamma\gamma^* \to \pi^0$, η and η' have been recently obtained and discussed [1, 2]. They involve (at least) one space–like photon, γ^*, with squared four–momentum $q^2 = -Q^2 < 0$. This completes and confirms older results concerning time–like photons ($q^2 > 0$) obtained from η, $\eta' \to \gamma\gamma^* \to \gamma\mu^+\mu^-$ decays [3, 4] and solves the chaotic situation related to the $\pi^0\gamma\gamma^*$ vertex [4, 5]. One usually fits the observed q^2 dependence in the different $P\gamma\gamma^*$ transitions by means of a normalized, single–pole term with an associated mass Λ_P, i.e.,

$$F_P(q^2) = F(\Lambda_P, q^2)/F(\Lambda_P, 0) = (1 - q^2/\Lambda_P^2)^{-1} \simeq 1 + q^2/\Lambda_P^2 \equiv 1 + b_P q^2, \tag{1}$$

where in the last steps (for small q^2) we have introduced the slope $b_P \equiv 1/\Lambda_P^2 = < r_P^2 > /6$ related to the size of the pseudoscalar meson P. The available experimental data [1, 2, 3] for $\Lambda_{\pi^0,\eta,\eta'}$ and their averaged values [2] are summarized in Table 1. The amplitude for a generic $P \leftrightarrow \gamma\gamma^*$ process is then

$$A(P \leftrightarrow \gamma\gamma^*) = \pm i F(\Lambda_P, q^2)\epsilon_{\mu\nu\alpha\beta}\varepsilon^\mu k^\nu \varepsilon^{*\alpha} q^\beta \tag{2}$$

with $k^2 = 0$ ($q^2 \neq 0$) for the real (virtual) photon with polarization ε (ε^*).

Theoretically, $P\gamma\gamma$ transitions involving on–mass–shell photons, $k^2 = q^2 = 0$, contain valuable information on the quark–content (or mixing) of the η, η' mesons. Concerning this point, the situation is quite satisfactory and general agreement has been achieved [2, 4, 6]. This implies

$$\begin{aligned} \eta &= \cos\theta\eta_8 - \sin\theta\eta_1 = \cos\beta(u\bar{u} + d\bar{d})/\sqrt{2} - \sin\beta s\bar{s} \\ \eta' &= \sin\theta\eta_8 + \cos\theta\eta_1 = \sin\beta(u\bar{u} + d\bar{d})/\sqrt{2} + \cos\beta s\bar{s} \\ \theta &= \beta - \arctan\sqrt{2} \simeq -\,\mathrm{arccot}\, 2\sqrt{2} \simeq -19.5^\circ. \end{aligned} \tag{3}$$

The q^2 dependence observed in $P\gamma\gamma^*$ transitions can then be viewed as a tool for understanding light–quark dynamics. With this aim several models have been discussed. The purpose of this note is to compare the experimental measurements of Λ_P quoted in Table 1 with the predictions of the most succesful and/or traditional models. These include conventional ideas related to Vector–Meson dominance (VMD) or constituent–quark loops (QL) and QCD–inspired approaches such as the Brodsky–Lepage interpolation formula (BL) or Chiral Perturbation Theory (ChPT).

Using VMD one immediately obtains [7, 8]

$$F^{VMD}(\Lambda_P, q^2) = \sum_V \frac{g_{PV\gamma}}{f_V} \frac{M_V^2}{M_V^2 - q^2}, \tag{4}$$

where the sum includes the three lightest vector–mesons $V = \rho^0$, ω and φ with SU(3)–symmetric couplings to the photon (f_V) and to $P\gamma$ ($g_{VP\gamma}$). Λ_V is then related to the vector–meson masses, M_V, thus introducing the only source of SU(3)–breaking (apart from mixing) through [5] $M_\rho \simeq M_\omega \simeq \lambda M_\varphi$, with $1/\lambda \simeq 1.30$. More explicitly, one obtains

$$\begin{array}{l} \Lambda^2_\pi \simeq M^2_{\rho,\omega}, \quad \Lambda^2_\eta = \dfrac{5\cos\beta - \sqrt{2}\sin\beta}{5\cos\beta - \sqrt{2}\lambda^2\sin\beta} M^2_{\rho,\omega}, \quad \Lambda^2_{\eta'} = \dfrac{5\sin\beta + \sqrt{2}\cos\beta}{5\sin\beta + \sqrt{2}\lambda^2\cos\beta} M^2_{\rho,\omega} \\ \Lambda_\pi = 0.78 \text{ GeV}, \quad \Lambda_\eta = 0.96\ \Lambda_\pi = 0.75 \text{ GeV}, \quad \Lambda_{\eta'} = 1.06\Lambda_\pi = 0.83 \text{ GeV}, \end{array} \tag{5}$$

where the numerical values of the last row follow from Eq.(3) and Ref.[5] and have been collected in Table 2.

The QL predictions for the $P\gamma\gamma^*$ form–factors are easily obtained computing the q^2 dependence generated by a triangle –loop of constituent quarks of masses m_q and charges e_q. One obtains [7, 8]

$$F^{QL}(\Lambda_P, q^2) = \sum_q \frac{g_{Pq\bar{q}}}{m_q} e_q^2 \left(\frac{1}{\lambda_q} \arcsin\lambda_q\right)^2, \quad \lambda_q^2 \equiv \frac{q^2}{4m_q^2}, \tag{6}$$

where the $Pq\bar{q}$ couplings are SU(3)–symmetric and breaking appears only through the constituent quark masses $m_u = m_d = \lambda' m_s$, with $1/\lambda' \simeq 1.40$. More explicitly, one has

$$\begin{array}{l} \Lambda^2_\pi = 12 m^2_{u,d}, \quad \Lambda^2_\eta = \dfrac{5\cos\beta - \sqrt{2}\lambda'\sin\beta}{5\cos\beta - \sqrt{2}\lambda'^3\sin\beta} 12 m^2_{u,d}, \quad \Lambda^2_{\eta'} = \dfrac{5\sin\beta + \sqrt{2}\lambda'\cos\beta}{5\sin\beta + \sqrt{2}\lambda'^3\cos\beta} 12 m^2_{u,d} \\ \Lambda_\pi = 0.80 \text{ GeV}, \quad \Lambda_\eta = 0.96\ \Lambda_\pi = 0.77 \text{ GeV}, \quad \Lambda_{\eta'} = 1.06\Lambda_\pi = 0.84 \text{ GeV}, \end{array} \tag{7}$$

where we have used Eq.(3) and a somewhat small constituent–mass ($m_{u,d} \simeq 0.23$ GeV) in order to agree reasonably with the data and also with the VMD results [5].

The latter agreement is a manifestation of the old idea of quark–hadron or Q^2 duality already checked in [7, 8] for $\eta \to \gamma\gamma^*$. Here, we have extended its validity to the SU(3) breaking contributions exploiting the approximate equalities $\lambda \simeq \lambda'$ and $M_V^2 \simeq 12 m_q^2$ between VMD and QL parameters.

The Brodsky–Lepage (BL) interpolation formula [9] for these transition form–factors is extremely simple, namely,

$$F_P^{BL}(\Lambda_P, q^2) = \frac{2\sqrt{2}\alpha}{\Lambda_P}(1 - q^2/\Lambda_P^2)^{-1} \tag{8}$$

where $\Lambda_P = 2\pi f_P$ is related to the pseudoscalar–meson decay constant f_P. It is an elegant expression interpolating two theoretically well–rooted results valid at the extreme energies $q^2 \to 0$ and $Q^2 \to \infty$. In the first case, Current Algebra (CA) unambiguously predicts $F(\Lambda_P, q^2 \to 0) = \sqrt{2}\alpha/\pi f_P$, whereas QCD leads to $F(\Lambda_P, Q^2) = 4\pi\alpha\sqrt{2} f_P/Q^2$, in the opposite and reliable region of asymptotically large Q^2. Our normalization is such that the pion decay–constant $f_{P=\pi} = \sqrt{2}\times 93$ MeV $= 132$ MeV and therefore one has $\Lambda_\pi = 2\pi f_\pi = 0.83$ GeV in the correct range of the experimental values. SU(3)–breaking now proceeds exclusively through $f_\pi \neq f_\eta \neq f_{\eta'}$. The two latter decay–constants are not directly measurable (in contrast with $f_{K^\pm}$ or $f_{\pi^\pm} = f_{\pi^0} = f_\pi$, by isospin) but can be deduced from $\eta, \eta' \to \gamma\gamma$ decays into real photons. One has

$$\begin{aligned} \frac{1}{f_\eta} &= \frac{1}{\sqrt{3}}\Big(\frac{\cos\theta}{f_8} - \frac{\sqrt{8}\sin\theta}{f_1}\Big) = \frac{0.914}{f_\pi} \\ \frac{1}{f_{\eta'}} &= \frac{1}{\sqrt{3}}\Big(\frac{\sin\theta}{f_8} + \frac{\sqrt{8}\cos\theta}{f_1}\Big) = \frac{1.25}{f_\pi}, \end{aligned} \tag{9}$$

where the numerical values follow from Eqs.(3) and the averaged data [5] for π^0, η and $\eta' \to \gamma\gamma$ decays. Indeed, several analyses [6, 10, 11] lead to the values

$$\theta = -20^\circ, \quad f_8 \simeq (1.25 - 1.30) f_\pi, \quad f_1 \simeq 1.1 f_\pi \tag{10}$$

and then to those quoted in Eq.(9). Therefore, one predicts

$$\Lambda_\pi = 2\pi f_\pi = 0.83 \text{ GeV}, \quad \Lambda_\eta = 1.10\Lambda_\pi = 0.91 \text{ GeV}, \quad \Lambda_{\eta'} = 0.80\Lambda_\pi = 0.66 \text{ GeV}, \tag{11}$$

as quoted in Table 2. The qualitative relation $\Lambda_\eta > \Lambda_{\eta'}$ seems unavoidable and contrasts with the experimental data (Table 1) which tend to prefer $\Lambda_{\eta'} > \Lambda_\eta$. This discrepancy is already present in the analysis of Ref[1], where the values $f_\eta = \sqrt{2}(91 \pm 6)$ MeV and $f_{\eta'} = \sqrt{2}(78 \pm 5)$ MeV are deduced from the decay widths into two real photons, contrasting with the values $f_\eta = \sqrt{2}(79 \pm 9)$ MeV and $f_{\eta'} = \sqrt{2}(96 \pm 8)$ MeV also deduced in [1] from the observed q^2 dependence. [Notice that in the original paper of Brodsky and Lepage [9] no effort is made to account for SU(3)-breaking and therefore it is not completely clear how eq. (8) should be applied to η or η' form-factors. In the analysis above we have assumed the simplest possibility, namely, that P refers directly to the physical particles η or η'. Alternatively, one could assume that f_P in (8) refers to f_8 or f_1; in this case, Λ_η and $\Lambda_{\eta'}$ are given by expressions similar to those in eqs. (7), leading to $\Lambda_\eta = 1.23\ \Lambda_\pi$ and $\Lambda_{\eta'} = 1.05\ \Lambda_\pi$, rather far from the experimental data.]

ChPT is particularly appropriate for dealing with $P\gamma\gamma^*$ processes. It is a QCD–inspired model with a lagrangian written in terms of the pseudoscalar meson fields, which are assumed to be the pseudo–goldstone boson fields appearing in the process of dynamical breaking of the chiral symmetry of massless QCD. The lagrangian is the most general one reproducing the symmetries of the original QCD lagrangian. It is expanded in powers of p^2/Λ^2 and m^2/Λ^2, where p is a typical momentum, m is the quark mass and $\Lambda \sim 4\pi f_\pi$ is the scale of chiral symmetry breaking. The relevant lowest order terms of the action are

$$S = \int d^4x L_2 - N_c S_{WZ}, \qquad N_c = 3 \tag{12}$$

with

$$\begin{array}{l} L_2 = \frac{1}{8}f^2 tr(D_\mu \Sigma D^\mu \Sigma^\dagger + \chi^\dagger \Sigma + \Sigma^\dagger \chi) \\ S_{WZ} = \frac{i}{48\pi^2}\int d^4x \epsilon^{\mu\nu\alpha\beta} Z_{\mu\nu\alpha\beta} + \cdots , \end{array} \tag{13}$$

where the dots refer to non–photonic terms of no relevance here and

$$\begin{array}{l} Z_{\mu\nu\alpha\beta} = -ieA_\mu tr[Q(\partial_\nu \Sigma \partial_\alpha \Sigma^\dagger \partial_\beta \Sigma \Sigma^\dagger - \partial_\nu \Sigma^\dagger \partial_\alpha \Sigma \partial_\beta \Sigma^\dagger \Sigma] \\ +2e^2(\partial_\mu A_\nu)A_\alpha tr[Q^2 \partial_\beta \Sigma \Sigma^\dagger + q^2 \Sigma^\dagger \partial_\beta \Sigma + \frac{1}{2} Q \Sigma Q \Sigma^\dagger \partial_\beta \Sigma \Sigma^\dagger + \frac{1}{2} Q \Sigma^\dagger Q \Sigma \partial_\beta \Sigma^\dagger \Sigma]. \end{array} \tag{14}$$

The covariant derivative $D_\mu \Sigma = \partial_\mu \Sigma + ie[Q, \Sigma]A_\mu$ contains the photon field and the quark charge matrix $Q = diag(2/3, -1/3, -1/3)$. The pseudoscalar meson fields are contained in a non linear form in Σ

$$\Sigma = exp\frac{2i}{f}M \ , \tag{15}$$

with

$$M = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} \end{pmatrix} \tag{16}$$

and f is a free constant that, at lowest order, can be identified with the pion decay constant f_π. Under chiral $U(3)_L \times U(3)_R$, Σ transforms as $\Sigma \to U_L \Sigma U_R^\dagger$. The lagrangian L_2 in Eq.(13) introduces a small spontaneous chiral–symmetry breaking through the quark mass matrix M, contained in $\chi = BM + \cdots$, where B is a free constant that can be fixed by relating the quark masses to the pseudoscalar ones.

The term containing two photons in S_{WZ} is the only one contributing at lowest order to the amplitude for $P \leftrightarrow \gamma\gamma^*$. The contribution turns out to be q^2 independent :

$$F^{ChPT}(\Lambda_P, q^2) = \frac{\sqrt{2}C_P \alpha}{\pi f_P} \tag{17}$$

with $C_\pi = 1$, $C_{\eta_8} = 1/\sqrt{3}$ and $C_{\eta_1} = 2\sqrt{2}/\sqrt{3}$. It should be noticed that, since the only sources of $U(3)$ breaking in Eqs.(12) and (13) are the quark masses, all the f_P are the same at this order. As expected from the non-renormalization of the anomaly and explicitly shown in Refs.[10, 12] and [13], loop corrections for real photons do not modify the lowest order result and only amount to the introduction of U(3)–breaking in the values of f_P. The π^0, η and $\eta' \to \gamma\gamma$ decay widths are, then, well understood in terms of the parameters in Eq.(10). Their finite parts can be calculated from the assumption that they are saturated by vector–meson contributions [13]. As a result, one obtains ($\sin\theta = -1/3$)

$$
\begin{aligned}
F_\pi(q^2) &= 1 + (b_L + b_V)q^2 \\
F_\eta(q^2) &= 1 + \Big(\frac{2f_1 + f_8}{2f_1 + 2f_8} b_L + b_V\Big) q^2 \\
F_{\eta'}(q^2) &= 1 + \Big(\frac{f_1 - 4f_8}{f_1 - 8f_8} b_L + b_V\Big) q^2
\end{aligned}
\tag{18}
$$

where the finite part of the loop correction to the slope is given by

$$
b_L = -\frac{1}{24\pi^2 f^2}\left(1 + \log(m_K m_\pi/\mu^2)\right) = +0.32\ \text{GeV}^{-2} \tag{19}
$$

for $\mu^2 \equiv M_V^2 \simeq (9M_\rho^2 + M_\omega^2 + 2M_\varphi^2)/12 = 0.69\ \text{GeV}^2$, which is the relevant mean vector–meson mass for our processes. This same mean–mass fixes the contribution dominated by vector mesons, namely

$$
b_V = 1/\mu^2 = 1.46\ \text{GeV}^{-2}, \tag{20}
$$

which (at the present order) is common to π^0, η and η'. The only sources of SU(3) breaking are, therefore, $f_1 \neq f_8 \neq f_\pi$ and the fact that the loop correction for π^0 and η_8 (b_L) is twice as large as that for η_1 ($b_L/2$), leading to the different coefficients of b_L in Eqs.(18). From these Eqs. one gets

$$
\Lambda_\pi = (b_L + b_V)^{-1/2} = 0.75\ \text{GeV}, \quad \Lambda_\eta = 1.03\Lambda_\pi = 0.77\ \text{GeV}, \quad \Lambda_{\eta'} = 1.06\Lambda_\pi = 0.79\ \text{GeV} \tag{21}
$$

In summary, all the models considered agree with the correct value for a mean Λ_P, but differ in the breaking pattern when $P = \pi^0$, η or η'. The VMD and QL approaches lead to $\Lambda_\eta < \Lambda_\pi < \Lambda_{\eta'}$ in agreement with the data of Refs. [1, 3]. The BL interpolation formula, instead, implies $\Lambda_{\eta'} < \Lambda_\pi < \Lambda_\eta$, in disagreement with the experimental data. Finally, ChPT predicts $\Lambda_\pi < \Lambda_\eta < \Lambda_{\eta'}$ in agreement with the averaged data. At this stage, it seems reasonably to conclude that accurate experiments (with precision of the order of a few per cent) are required in order to decide on the correct scheme accounting for the $P\gamma\gamma^*$ transition form–factors.

REFERENCES

[1] TPC/2γ Coll. H. Aihara et al., Phys. Rev Lett. **64** (1990) 172.

[2] CELLO Coll. H. J. Behrend et al., Z. Phys. **C49** (1991) 401.

[3] Lepton-G Coll. R. I. Dzhelyadin et al., Phys. Lett. **88B** (1979) 379.
Lepton-G Coll. R. I. Dzhelyadin et al., Phys. Lett. **94B** (1980) 548.

[4] L. G. Landsberg, Phys. Rep. **128** (1985) 301.

[5] Particle Data Group, Phys. Lett. **239B** (1990) 1.

[6] F. J. Gilman and R. Kauffman, Phys. Rev **D36** (1987) 2761.
A. Bramon, Phys. Lett. **B51** (1974) 87.

[7] A. Bramon and E. Massó, Phys. Lett. **104B** (1981) 311.

[8] Ll. Ametller et al., Nucl. Phys. **B228** (1983) 301.
A. Pich and J. Bernabéu, Z. Phys. **C22** (1984) 197.

[9] S. J. Brodsky and G. P. Lepage, Phys. Rev. **D24** (1981) 1808.

[10] J. F. Donoghue, B. R. Holstein and Y. C. R. Lin, Phys. Rev. Lett. **55** (1985) 2766.

[11] K. T. Chao, Phys. Rev. **D39** (1989) 1353.

[12] J. Bijnens, A. Bramon and F. Cornet, Phys. Rev. Lett. **61** (1988) 1453.

[13] J. Bijnens, A. Bramon and F. Cornet, Z. Phys. **C46** (1990) 599.
G. Ecker et al., Nucl. Phys. **B321** (1989) 311.

Table 1

	Λ_{π^0}(GeV)	Λ_η(GeV)	$\Lambda_{\eta'}$(GeV)
Lepton-G (Ref.[3])	-	0.72 ± 0.09	0.77 ± 0.18
TPC/2γ (Ref.[1])	-	0.70 ± 0.08	0.85 ± 0.07
CELLO (Ref.[2])	0.75 ± 0.03	0.84 ± 0.06	0.79 ± 0.04
Average (Ref.[2])	0.75 ± 0.03	0.77 ± 0.04	0.81 ± 0.04

Table 2

	Λ_{π^0}(GeV)	Λ_η(GeV)	$\Lambda_{\eta'}$(GeV)
VMD ([7, 8])	$M_{\rho,\omega} = 0.78$	$0.96\Lambda_\pi = 0.75$	$1.06\Lambda_\pi = 0.83$
Q. M. [$m_u = m_s/1.4 = 0.23$ GeV]	$\sqrt{12}m_u = 0.80$	$0.96\Lambda_\pi = 0.77$	$1.06\Lambda_\pi = 0.84$
Brodsky–Lepage [9]	$2\pi f_\pi = 0.83$	$1.10\Lambda_\pi = 0.91$	$0.80\Lambda_\pi = 0.66$
ChPT [$M_{\overline{V}} = 0.828$ GeV]	$(b_L + b_V)^{-1/2} = 0.75$	$1.03\Lambda_\pi = 0.77$	$1.06\Lambda_\pi = 0.79$

Table Captions

Table 1. Experimental values for the pole–mass Λ_P (in GeV) in the transition form–factors of pseudoscalar mesons $P = \pi^0$, η and η'.

Table 2. Values for $\Lambda_{\pi^0,\eta,\eta'}$ predicted by Vector Meson Dominance (VMD), Quark Model loops (QL), the Brodsky Lepage interpolating formula and Chiral Perturbation Theory (ChPT).