CERN–TH.6182/91

# The Top Quark Mass in $t$–$\bar{t}$ Condensation Models with Enlarged Bound-State Spectrum

Manfred Lindner and Dieter Lüst

*CERN, CH-1211 Geneva 23, Switzerland*

## ABSTRACT

We argue that a new strong force which induces electroweak symmetry breaking via top condensation presumably leads to a rich $t$–$\bar{t}$ bound-state spectrum composed of more than the three Goldstone boson and one Higgs mode. We discuss the dynamical consistency conditions for the enlarged $t$–$\bar{t}$ bound-state spectrum and demonstrate how top mass predictions change when the effects of such an enlarged spectrum on the generalized compositeness conditions are included. It is found that these corrections can be bigger than the corrections found in 2-Higgs and supersymmetric generalizations of the minimal mechanism. As a consequence it is possible to obtain a phenomenologically acceptable top mass.

CERN–TH.6182/91
July 1991

Top quark condensation is an interesting mechanism for the dynamical breaking of the electroweak gauge symmetries. The heaviness of the top quark is then not a problem as e.g. in technicolour theories but a natural consequence of the condensation mechanism. Motivated by the work of Nambu [1] and others [2] a detailed investigation of this scenario was obtained by [3] (BHL), where the minimal top quark condensation model is triggered via the mechanism of Nambu–Jona-Lasinio (NJL) [4] by the following four-Fermi interaction:

$$\mathcal{L}_{4-\mathrm{Fermi}} = G_0(\bar{\psi}^i_L t_R)(\bar{t}_R \psi^i_L). \tag{1}$$

Here $\psi^i_L = (t_L, b_L)$ is the third generation left-handed quark doublet and $t_R$ the corresponding right-handed singlet. $\mathcal{L}_{4-\mathrm{Fermi}}$ induces, for large enough coupling, a non-vanishing condensate of a top anti-top pair. Then the dynamical top quark mass is described by the gap equation,

$$M_t = -\frac{1}{2}G_0\langle\bar{t}t\rangle = 2G_0 N_C M_t \frac{i}{(2\pi)^4}\int^{\Lambda} d^4l(l^2 - M_t^2)^{-1}, \tag{2}$$

which is cut off at the scale of new physics $\Lambda$. The formation of a scalar (S), a pseudoscalar (P) and two flavoured (F) bound states can be seen from the emergence of poles in the corresponding two-fermion scattering amplitudes $\Gamma_{S(P,F)}$. $\Gamma_P$ and $\Gamma_F$ have single poles at $p^2 = 0$ corresponding to massless Goldstone bosons while $\Gamma_S$ has a single pole at $p^2 = 4M_t^2$ implying a composite Higgs boson $H$ of mass $M_H \simeq 2M_t$. The Goldstone boson spectrum is a simple consequence of the global symmetry breaking pattern $U(2)_L \times U(1)_R \to U(1) \times U(1)$ and thus protected from dynamical uncertainties.

The results for the top and Higgs mass can be improved by considering a low-energy effective field theory (which is just the Standard Model with additional boundary conditions) treating the composite Higgs field as elementary degree of freedom. This is especially justified if the gap is big so that the composite scalar is so tightly bound that its structure cannot be resolved at electroweak scales. Then all couplings are momentum dependent, and one has to infer the compositeness condition that the Higgs field becomes static at high energies ($E \sim \Lambda$), i.e. $Z_H \to 0$, thus recovering from the Standard Model Lagrangian together with the equations of motion the structure of the basic four-Fermi Lagrangian (1). Taking into account also effects of virtual Higgs exchange and gauge boson corrections, the renormalization group equations (RGE's) lead to a top quark mass prediction $M_t \simeq 220 - 230$ GeV for $\Lambda \simeq 10^{19} - 10^{15}$ GeV [3]. These results are now barely acceptable on the basis of indirect experimental limits on the top quark mass based on the analysis of radiative corrections to the $\rho$-parameter [5]. Therefore, if these indirect limits are not spoiled by additional unknown effects, the minimal top quark condensation model seems to be disfavoured by the present experimental data.



In this communication we discuss natural extensions of the minimal model which tend to produce lower values for the top quark mass in agreement with the quoted experimental limits. Even a lower scale of new physics $\Lambda$ may be acceptable which would avoid the fine-tuning required for large $\Lambda$. Specifically we describe the effects of additional composite states which are, like the Higgs boson $H$, dynamical $t$–$\bar{t}$ bound states. Such extensions of the minimal model are in fact very natural since the underlying dynamics responsible for the top quark condensate will (without fine adjustments) most likely lead to a broad spectrum of excited bound states with masses of the order of $\Lambda$, just like the hadron dynamics with $QCD$ as underlying force. In fact, such extensions may be viewed as more natural than the extensions to more generations [3], [6], more Higgses [7], or to supersymmetry [8] since the particles involved in symmetry breaking with masses of the order of the electroweak scale are, in this context, only the ground-state spectrum.

There are different arguments for the emergence of additional bound states. First, as in any bound-state problem, one expects without a very delicate choice in the binding force that the ground states, like the light scalar $H$ and the Goldstone bosons $P$, should be accompanied by (radial) excitations $\widetilde{H}$ and $\widetilde{P}$ with identical quantum numbers but much higher masses (presumably of the order of the compositeness scale $\Lambda$). Thus, the two-fermion scattering amplitude $\Gamma_S(p^2)$ will be saturated not only by the lowest excitation but will also contain a sum over the propagators of all possible bound states in this channel:

$$\Gamma_S(p^2) = -\frac{G_0}{2}\left(\frac{f(p^2)}{M_H^2 - p^2} + \frac{\tilde{f}(p^2)}{M_{\widetilde{H}}^2 - p^2} + \dots\right). \tag{3}$$

The first term in this equation corresponds to the leading contribution of the light scalar which is protected by the gap equation to be of the order of the top mass [3]. The other terms are due to subleading (non-planar) diagrams and correspond to the exchange of an excited Higgs $\widetilde{H}$. Since the gap equation cannot be used anymore to rewrite the relevant loop integral, one typically expects $M_{\widetilde{H}}^2 \simeq \mathcal{O}(\Lambda^2)$.

A second argument for the emergence of additional bound states is that it is natural to assume for the microscopic forces a broader variety of four-Fermi interaction terms in addition to the one displayed in eq. (1).* They can give rise to different types of bound

* Such terms arise anyway when the Higgs field is integrated out from the Standard Model with all Yukawa couplings included.

states such as excited vector bosons $\widetilde{W}$ and $\widetilde{B}$. Again such states are expected to be heavy since the gap equation cannot be used when determining their masses by looking at poles in the corresponding fermion scattering amplitudes $\Gamma_{\rm vector}$. In some sense, the states $\widetilde{W}$ and $\widetilde{B}$ could be regarded as higher spin excitation of the ground states $H$, $P$ and $F$.

To be specific, let us consider the following types of four-Fermi interactions which contribute to the fermionic part of the Lagrangian:

$$\begin{aligned}\mathcal{L} = \mathcal{L}_{\rm kin} &+ G_0(\bar\psi^i_L t_R)(\bar t_R \psi^i_L) + G_1(\bar\psi^i_L\gamma^\mu\tau^{ij}_a\psi^j_L)(\bar\psi^i_L\gamma^\mu\tau^{ij}_a\psi^j_L)\\ &+ G_2(\bar\psi^i_L\gamma^\mu\psi^i_L)(\bar t_R\gamma^\mu t_R) + G_3(\bar\psi^i_L\gamma^\mu\psi^i_L)(\bar b_R\gamma^\mu b_R)\\ &+ G_4(\bar\psi^i_L\gamma^\mu\psi^i_L)(\bar\psi^i_L\gamma^\mu\psi^i_L) + G_5(\bar t_R\gamma^\mu t_R)(\bar t_R\gamma^\mu t_R)\\ &+ G_6(\bar t_R\gamma^\mu t_R)(\bar b_R\gamma^\mu b_R) + G_7(\bar b_R\gamma^\mu b_R)(\bar b_R\gamma^\mu b_R),\end{aligned} \tag{4}$$

where we have suppressed all colour indices. Note that by Fierz rearrangement the first and the third four-fermi terms in (4) are equivalent. Therefore, there is no difference between the scalar and the second vector channel in the static limit. This implies that the effective coupling constant $G$, which triggers chiral symmetry breaking in the gap equation, is given by $G = G_0 + 2G_2$. Taking into account colour indices as well, this relation holds only for the colour singlet combination.†

In analogy to the formation of the scalar field bound state $H \sim \bar t_L t_R$ one naturally expects that these new four-Fermi interactions give rise to the dynamical formation of extra vector particles which are also composed of $\bar t$, $t$ and $b_L$. One could try to compute the two-fermion scattering amplitudes $\Gamma_V$ and look for poles in the corresponding channels. Because of the necessary approximations the results would be very rough and not reliable. Therefore we have to assume here that a richer spectrum will emerge and study the effects on the compositeness conditions in the effective Lagrangian formalism. In this case we have to include all the bound states as if they were elementary fields, and to be specific we consider the following Lagrangian where we have added excited scalars and new vectors:

$$\begin{aligned}\mathcal{L} = \mathcal{L}^{\rm fermion}_{\rm kin} &+ \mathcal{L}^{W,B}_{\rm gauge} + Z_H|D_\mu H|^2 + Z_{\widetilde H}|D_\mu \widetilde H|^2 + M^2_H H^\dagger H - M^2_{\widetilde H}\widetilde H^\dagger \widetilde H\\ &- \frac{\lambda}{2}(H^\dagger H)^2 - \frac{\widetilde\lambda}{2}(\widetilde H^\dagger \widetilde H)^2 + g_t(\bar\psi_L t_R H + \text{h.c.}) + \widetilde g_t(\bar\psi_L t_R \widetilde H + \text{h.c.})\\ &- \frac{1}{4}Z_{\widetilde W}\,{\rm Tr}F^{\widetilde W 2}_{\mu\nu} - \frac{1}{4}Z_{\widetilde B}F^{\widetilde B 2}_{\mu\nu} + M^2_{\widetilde W}\widetilde W^2_\mu + M^2_{\widetilde B}\widetilde B^2_\mu\end{aligned} \tag{5}$$

where $\widetilde H, \widetilde B$ and $\widetilde W$ stand for the complete spectrum of bound states in the corresponding channel. The covariant derivative contains both the elementary and composite vector fields: $D_\mu = \partial_\mu - i(g_2 W_{\mu a} + \widetilde g_2 \widetilde W_{\mu a})\tau^a - i(g_1 B_\mu + \widetilde g_1 \widetilde B_\mu)$. Thus $\widetilde W$ and $\widetilde B$ couple like massive gauge fields. Note that we ignore possible small mixing terms among the elementary fields $W, B$ and the composite vectors $\widetilde W$ and $\widetilde B$ as well as mixing among $H$ and $\widetilde H$.

† In the limit where QCD interactions are turned off the colour singlet and colour octet condensates would be degenerate. This degeneracy is lifted in favour of the colour neutral vacuum as in technicolour due to the attraction (repulsion) of the QCD forces in the colour singlet (octet) channel.



With this richer spectrum in mind the conditions which relate to the composite nature of some of the fields must be generalized. In analogy to [3] we must demand that

$$Z_H, Z_{\widetilde H}, Z_{\widetilde W}, Z_{\widetilde B}, \lambda, \widetilde\lambda \to 0 \tag{6}$$

as $\mu \to \Lambda$. In this way $H$, $\widetilde H$, $\widetilde W^a_\mu$ and $\widetilde B_\mu$ become auxiliary fields which can be integrated out via their equations of motion, and the effective Lagrangian (5) becomes identical to the Lagrangian (4) which involves the four-Fermi terms by using the following field identifications

$$\begin{aligned}H &= \frac{g_t}{M^2_H}\bar t_L t_R\\ \widetilde H &= \frac{\widetilde g_t}{M^2_{\widetilde H}}\bar t_L t_R\\ \widetilde W^a_\mu &= -\frac{\widetilde g_W}{2M^2_{\widetilde W}}\bar\psi^i_L\gamma^\mu\tau^a_{ij}\psi^j_L\\ \widetilde B_\mu &= -\frac{1}{2M^2_{\widetilde B}}(\widetilde g_{1B}\bar\psi^i_L\gamma^\mu\psi^i_L + \widetilde g_{2B}\bar t_R\gamma^\mu t_R + \widetilde g_{3B}\bar b_R\gamma^\mu b_R),\end{aligned} \tag{7}$$

provided the coupling constants satisfy the following relations:

$$\begin{aligned}&G_0 = \frac{g_t^2}{M^2_H} + \frac{\widetilde g_t^2}{M^2_{\widetilde H}}, \quad G_1 = -\frac{\widetilde g^2_W}{4M^2_{\widetilde W}}, \quad G_2 = -\frac{\widetilde g_{1B}\widetilde g_{2B}}{2M^2_{\widetilde B}}, \quad G_3 = -\frac{\widetilde g_{1B}\widetilde g_{3B}}{2M^2_{\widetilde B}},\\ &G_4 = -\frac{\widetilde g^2_{1B}}{4M^2_{\widetilde B}}, \quad G_5 = -\frac{\widetilde g^2_{2B}}{4M^2_{\widetilde B}}, \quad G_6 = -\frac{\widetilde g_{2B}\widetilde g_{3B}}{2M^2_{\widetilde B}}, \quad G_7 = -\frac{\widetilde g^2_{3B}}{4M^2_{\widetilde B}}.\end{aligned} \tag{8}$$

To see what the effect of such extra states is, we have to determine the enlarged set of coupled $\beta$-functions and solve the RGE flow with the generalized boundary conditions. The thresholds of the additional heavy excited states will be dealt with by turning off the corresponding terms in the $\beta$-functions once they freeze out below their masses. As we go to lower energies (but still much higher than the electroweak scale) we recover the normal Standard Model. But the existence of very heavy additional states affects the low energy predictions as we integrate the high energy boundary conditions downwards. In fact, one could say that the prediction of BHL contains the *assumption* that the Standard Model is the complete spectrum, which may not be true. Here we argue that one can expect a richer spectrum which will influence the predictions. Specifically, the existence of the additional $t$-$\bar{t}$ bound states implies a modification of the $\beta$-functions of the elementary gauge bosons as well as of the top-quark Yukawa coupling, since we treat the composite states below the compositeness scale $\Lambda$ as point-like degrees of freedom whose interactions are described by the Lagrangian (5):

$$\begin{aligned}
8\pi^2\frac{dg_1^2}{dt} &= \left(\frac{20}{9}N_F+\frac{1}{6}(1+N_{\widetilde{H}})\right)(g_1^2+\sum_{i=1}^{N_{\widetilde{B}}}\widetilde{g}_{Bi}^2)g_1^2,\\
8\pi^2\frac{dg_2^2}{dt} &= \left(\frac{4}{3}N_F+\frac{1}{6}(1+N_{\widetilde{H}})\right)(g_2^2+\sum_{i=1}^{N_{\widetilde{W}}}\widetilde{g}_{Wi}^2)g_2^2-\frac{22}{3}g_2^4,\\
8\pi^2\frac{dg_3^2}{dt} &= (\frac{4}{3}N_F-11)g_3^4,\\
8\pi^2\frac{dg_t^2}{dt} &= \left(\frac{9}{2}(g_t^2+\sum_{i=1}^{N_{\widetilde{H}}}\widetilde{g}_{ti}^2)-\frac{17}{12}(g_1^2+\sum_{i=1}^{N_{\widetilde{B}}}\widetilde{g}_{Bi}^2)-\frac{9}{4}(g_2^2+\sum_{i=1}^{N_{\widetilde{W}}}\widetilde{g}_{Wi}^2)-8g_3^2\right)g_t^2\ ,
\end{aligned}\tag{9}$$

where we have allowed for more than one additional bound state in each channel, i.e. there is a spectrum of $N_{\widetilde{H}}$ scalars $\widetilde{H}$, $N_{\widetilde{B}}$ isospin singlets $\widetilde{B}$, and $N_{\widetilde{W}}$ isospin triplets $\widetilde{W}$, which contribute to the $\beta$-functions only above their mass thresholds. We have however assumed that all the couplings of $\widetilde{B}$ in (7) are identical.

Since we are dealing with an effective field theory below the compositeness scale $\Lambda$, the coupling constants of the bound states also evolve according to the renormalization group equations. For the corresponding $\beta$-functions we obtain:

$$\begin{aligned}
8\pi^2\frac{d\widetilde{g}_{Bi}^2}{dt} &= \left(\frac{20}{9}N_F+\frac{1}{6}(1+N_{\widetilde{H}})\right)(g_1^2+\sum_{j=1}^{N_{\widetilde{B}}}\widetilde{g}_{Bj}^2)\widetilde{g}_{Bi}^2,\\
8\pi^2\frac{d\widetilde{g}_{Wi}^2}{dt} &= \left(\frac{4}{3}N_F+\frac{1}{6}(1+N_{\widetilde{H}})\right)(g_2^2+\sum_{j=1}^{N_{\widetilde{W}}}\widetilde{g}_{Wj}^2)\widetilde{g}_{Wi}^2-\frac{22}{3}\widetilde{g}_{Wi}^4,\\
8\pi^2\frac{d\widetilde{g}_{ti}^2}{dt} &= \left(\frac{9}{2}(g_t^2+\sum_{j=1}^{N_{\widetilde{H}}}\widetilde{g}_{tj}^2)-\frac{17}{12}(g_1^2+\sum_{j=1}^{N_{\widetilde{B}}}\widetilde{g}_{Bj}^2)-\frac{9}{4}(g_2^2+\sum_{j=1}^{N_{\widetilde{W}}}\widetilde{g}_{Wj}^2)-8g_3^2\right)\widetilde{g}_{ti}^2.
\end{aligned}\tag{10}$$

Note that we do not explicitly include coloured bound states which are expected to be present in many models as in the scenario discussed in [9]. However, a possible additional



colour degree of freedom for $\widetilde{H}$, $\widetilde{B}$ and $\widetilde{W}$ is already parametrized by $N_{\widetilde{H}}$, $N_{\widetilde{B}}$ and $N_{\widetilde{W}}$ when one calculates the contribution of these states to $g_t$. Therefore the effects of coloured states will be contained in our predictions for $M_t$. Of course coloured bound states would modify the running of the strong coupling constant $g_3$.

The compositeness conditions (6), which are equivalent to the requirement that the extra fields become auxiliary fields, translate after a simple field redefinition of $H$, $\widetilde{H}_i$, $\widetilde{B}_i$ and $\widetilde{W}_i$ into the following boundary conditions on the effective running coupling constants at the scale $\Lambda$:

$$g_t,\ \widetilde{g}_{ti},\ \widetilde{g}_{Bi},\ \widetilde{g}_{Wi}\rightarrow\infty. \tag{11}$$

These conditions are not automatically satisfied with an arbitrary number of bound states and arbitrary bound-state masses and couplings. They provide therefore dynamical self-consistency requirements which restrict the form of the bound-state spectrum and the strength of the effective couplings without having to solve the dynamics. Consider for example the $\beta$-functions in (10) of the composite vector bosons $\widetilde{W}_i$. Owing to the negative coefficient in the last term of this equation, one needs a minimal number of $\widetilde{H}$'s and $\widetilde{W}$'s respectively in order to have an increasing coupling constant $\widetilde{g}_{Wi}$ at high momenta. The requirement that $\widetilde{g}_{Wi}$ diverge provides a relation between the minimal number of required composite bound states, the ratio of their masses compared to the scale $\Lambda$, and the values of their coupling constants at their mass thresholds above which these states are effective.

For the vector channels we can solve this problem approximately. If we assume that there are not too many scalars, then the $N_{\widetilde{H}}/6$ contribution to the $\beta$-functions in eqs. (9) and (10) is small and can be omitted for an approximate solution. If we choose the effective couplings of the fields $\widetilde{B}_i$ and $\widetilde{W}_i$ at their thresholds to be identical to the corresponding gauge couplings, $g_1$ or $g_2$, then we find above threshold that $\widetilde{g}_{B_i}$ and $\widetilde{g}_{W_i}$ stay identical to the corresponding gauge coupling. Below threshold these couplings are simply zero. With this assumption about the couplings at threshold we have to solve only the two equations

$$\begin{aligned}
\frac{d}{dt}\left(\frac{1}{g_1^2}\right) &= -\frac{(\frac{20}{9}N_F+\frac{1}{6})\,(N_{\widetilde{B}}(t)+1)}{8\pi^2},\\
\frac{d}{dt}\left(\frac{1}{g_2^2}\right) &= -\frac{(\frac{4}{3}N_F+\frac{1}{6})\,(N_{\widetilde{W}}(t)+1)-\frac{22}{3}}{8\pi^2},
\end{aligned}\tag{12}$$

where $N_{\widetilde{B}}(t)$ and $N_{\widetilde{W}}(t)$ are the number of active vector particles at a given scale. If we choose the spectrum of $N_{\widetilde{B}}$ singlet and $N_{\widetilde{W}}$ triplet states each to be parametrized by a single number $R$ in the following way

$$R=\frac{M_2}{M_1}=\frac{M_3}{M_2}=\ldots=\frac{M_N}{M_{N-1}}=\frac{\Lambda}{M_N}, \tag{13}$$

then we find for $N_{\widetilde{B}}, N_{\widetilde{W}}>0$

$$\begin{aligned}\frac{1}{g_1^2(\Lambda)}&=\frac{1}{g_1^2(SM,\Lambda)}-\frac{(\frac{20}{9}N_F+\frac{1}{6})}{8\pi^2}\frac{(N_{\widetilde{B}}+1)}{2}\ln\left(\frac{\Lambda}{M_{1\widetilde{B}}}\right),\\ \frac{1}{g_2^2(\Lambda)}&=\frac{1}{g_2^2(SM,\Lambda)}-\frac{(\frac{4}{3}N_F+\frac{1}{6})}{8\pi^2}\frac{(N_{\widetilde{W}}+1)}{2}\ln\left(\frac{\Lambda}{M_{1\widetilde{W}}}\right),\end{aligned} \tag{14}$$

where $M_{1\widetilde{B}}$ and $M_{1\widetilde{W}}$ are the lightest singlet and triplet state respectively, and $g_i^2(SM,\Lambda)$ are the couplings evolved to $\Lambda$ with just the Standard Model, i.e.

$$\begin{aligned}\frac{1}{g_1^2(SM,\Lambda)}&=\frac{1}{g_1^2(0)}-\frac{(\frac{20}{9}N_F+\frac{1}{6})}{8\pi^2}\ln\left(\frac{\Lambda}{M_W}\right),\\ \frac{1}{g_2^2(SM,\Lambda)}&=\frac{1}{g_2^2(0)}-\frac{(\frac{4}{3}N_F+\frac{1}{6}-\frac{22}{3})}{8\pi^2}\ln\left(\frac{\Lambda}{M_W}\right),\end{aligned} \tag{15}$$

where $g_1^2(0)=4\pi\alpha_{em}/\cos^2\theta_W\simeq 0.127$, $M_W\simeq 80.1$ GeV and $g_2^2(0)=4\pi\alpha_{em}/\sin^2\theta_W\simeq 0.427$. The compositeness conditions $1/\widetilde{g}_B^2(\Lambda)=1/\widetilde{g}_W^2(\Lambda)=0$ are fulfilled by

$$\begin{aligned}f_{\widetilde{B}}(N_{\widetilde{B}})\ln\left(\frac{\Lambda}{M_{1\widetilde{B}}}\right)&=\frac{8\pi^2}{\frac{20}{9}N_F+\frac{1}{6}}\frac{1}{g_1^2(SM,\Lambda)},\\ f_{\widetilde{W}}(N_{\widetilde{W}})\ln\left(\frac{\Lambda}{M_{1\widetilde{W}}}\right)&=\frac{8\pi^2}{\frac{4}{3}N_F+\frac{1}{6}}\frac{1}{g_2^2(SM,\Lambda)},\end{aligned} \tag{16}$$

where for the spectrum (13) we find $f_{\widetilde{B}}=f_{\widetilde{W}}=(N_i+1)/2$. If instead of the spectrum (13) all extra bound states were turned on simultaneously at the same threshold $M$, then we would find $f_{\widetilde{B}}=f_{\widetilde{W}}=N_i$. Note that by imposing $\widetilde{g}_{Wi}(M_{i\widetilde{W}})=g_2(M_{i\widetilde{W}})$ etc. the coupling constant $g_2$ of the standard $W$-bosons also blows up at $\Lambda$. Then the $W$-bosons become non-propagating degrees of freedom. A possible interesting interpretation of this kind of scenario could be that the $W$-bosons are also $t$–$\bar{t}$ bound states and not elementary particles.

The function $f(N)$ which determines the number of required vector states is plotted in figure (1) versus $\Lambda$ for different mass values $M_1/M_W$ of the lightest states. The right-hand



side of eq. (16) is a number which is entirely determined by the Standard Model evolution of the gauge couplings up to $\Lambda$. Once $\Lambda$ is fixed the left-hand side of eq. (16) determines how the number of states and the mass spectrum have to be balanced. If the assumptions on the couplings and mass spectrum are relaxed then there are still solutions similar to conditions (16). These solutions extend however only into the "neighbourhood" of the conditions derived under these specific assumptions. In particular, it is not possible to satisfy the compositeness conditions with one or only a few states for low values of $\Lambda$. Equivalently, many states are required when the lightest extra bound states are close to $\Lambda$. It is however possible to fulfil the compositeness conditions for the excited states in such a way that the $W$-bosons remain fundamental propagating fields when $\Lambda$ is approached.

Now let us analyse the evolution of the Yukawa coupling constant $g_t$ to obtain the predictions for the top quark mass including the effects of the additional bound states. Again the spectrum has to obey certain dynamical self-consistency conditions for $g_t$ and also for $\widetilde{g}_t$ to diverge at $\Lambda$. Specifically, this requirement now leads to an upper limit on the number of possible composite vector bosons. However it is easy to see that the contribution of the scalars $\widetilde{H}$ is stronger than the contribution of the vectors $\widetilde{W}$ and $\widetilde{B}$ to the running of $g_t$. In figures (2–4) we have plotted various values for $M_t=g_t(M_t)v$, $v\simeq 173$ GeV solving the full set of eqs. (9) and (10) numerically with $\frac{1}{\widetilde{g}_t^2(\Lambda)}=0$ for $\Lambda=10^{15},10^{12},10^{9}$ GeV. Specifically, imposing the spectrum (13) we have computed $M_t$ as a function of $M_1/M_W$ which denotes the masses of the lightest additional vector and scalar bound states. For simplicity we have assumed that the lightest scalar $\widetilde{H}_1$ and the lightest vectors $\widetilde{B}_1$ and $\widetilde{W}_1$ have a common mass $\widetilde{M}_1$. Various cases for the total numbers $(N_{\widetilde{B}},N_{\widetilde{W}},N_{\widetilde{H}})$ of additional states are shown in figures (2–4). As explained before, these numbers are chosen such that $\frac{1}{\widetilde{g}_W(\Lambda)}=\frac{1}{\widetilde{g}_B(\Lambda)}=0$ using the boundary conditions $\widetilde{g}_W(M_1)=g_2(M_1)$, $\widetilde{g}_B(M_1)=g_1(M_1)$. Of course, for $(N_{\widetilde{B}},N_{\widetilde{W}},N_{\widetilde{H}})=(0,0,N_{\widetilde{H}})$ these dynamical consistency requirements are absent, and the scalar spectrum can be chosen arbitrarily. The solid lines in these figures represent the results of ref. [3] corresponding to $(N_{\widetilde{B}},N_{\widetilde{W}},N_{\widetilde{H}})=(0,0,0)$. For each set of numbers $(N_{\widetilde{B}},N_{\widetilde{W}},N_{\widetilde{H}})$ we have considered three different cases depicted as $-,\circ,+$ in the figures which correspond to $\frac{\widetilde{g}_t(\Lambda)}{g_t(\Lambda)}=2,1,\frac{1}{2}$ respectively. The first case represents very strongly coupled excited scalars and seems to be unlikely within a specific dynamical scenario. Finally, figure (5) summarizes the range of our predictions for $M_t$ as a function of the compositeness scale $\Lambda$ for the simplest cases $(N_{\widetilde{W}},N_{\widetilde{B}},N_{\widetilde{H}})=(0,0,1),(0,0,5)$ for different values of $M_{\widetilde{H}}$ with $\frac{\widetilde{g}_t(\Lambda)}{g_t(\Lambda)}=1$. The predictions of [3] are again drawn as a solid line and the full variation under changes of all parameters can be inferred from figures (2–4). This plot clearly shows that phenomenologically acceptable values for $M_t$ can be obtained for scales as $\Lambda \geq 10^{10}$ GeV.* Very low values for $M_t$ can only be obtained by making rather unnatural dynamical assumptions about the form of the bound-state spectrum such as the presence of additional scalars with either very low masses compared to $\Lambda$, or very strong coupling constants. Similarly, it is quite unnatural, though possible, to obtain very high values for the top quark mass (i.e. $M_t >> M_{t,BHL}$) since this would require the existence of many composite vector states with low masses. The results shown in figures (2–5) which are obtained from the RGE's and the boundary conditions (11) require strong couplings close to the compositeness scale $\Lambda$. This leads to scale-dependent uncertainties in the predictions which we estimate to be typically of the order quoted by BHL [3].

In this paper we have discussed the effects of a possibly larger spectrum of bound states in top condensation scenarios. We argued that such a spectrum is very natural since it is typically very hard to obtain a bound-state spectrum which consists just of the ground state. If such excited states exist then they will modify the top mass predictions. The reason is that the boundary conditions of the effective Lagrangian, which are equivalent to the composite nature of the spectrum, have to be imposed at the scale of new physics. When these conditions are transferred into low energy predictions for the top mass, then the presence of additional states will modify the renormalization group running and lead to somewhat different predictions even if the lightest extra state is far too heavy for experimental discovery. Thus, for all practical purposes, the accessible low energy Lagrangian is still the Standard Model and the main consequence of the extra spectrum is a wider range for the numerical value of the top mass. Without knowing the dynamics and the spectrum, the main prediction would thus be $M_t = \mathcal{O}(v)$ and $m_H = \mathcal{O}(2M_t)$. However, one should again emphasize that the extrapolation of the RGE–obtained results is reliable only if the compositeness scale $\Lambda$ is much larger than the weak scale. Otherwise there are large dynamical uncertainties in the predictions for $M_t$. In addition we remark that low values for the masses of the additional bound states could actually change the window for $M_t$, which is based on the analysis of the the $\rho$-parameter [5], since the low-lying additional states could give a significant contribution to the $\rho$-parameter. This might actually admit a scenario where $\Lambda$ is low.

We showed that the compositeness conditions lead to self-consistency conditions which restrict the spectrum and couplings even without specifying the interactions. These lead to relations between the number of extra states, their couplings and masses for the vector states. When the conditions for the vector spectrum were fulfilled then the set of coupled equations for the scalar sector was studied numerically and the resulting range of top mass predictions was obtained.

* As mentioned before, we did not consider the possibility of coloured bound states in the spectrum. If such states were included, the larger degeneracy of states due to the additional colour factor might allow even lower values of $M_t$ and/or $\Lambda$.



Let us briefly comment on the relation of our work to other approaches for extended top quark condensation models. First, four-generation models were already considered in [3] and [6] with the result that $M_{t'} = M_{b'} \simeq 200 - 260$ GeV for $\Lambda \simeq 10^{19} - 10^7$ GeV, while $M_t \leq M_{t'}$. Since LEP has found only three neutrino generations this possibility can only be realized under very special circumstances [10]. Supersymmetric extensions [8] lead to $M_t \simeq 140 - 195$ GeV for $\Lambda \simeq 10^{16}$ GeV and two-Higgs models [7] to $M_t \simeq 210 - 270$ GeV for $\Lambda \simeq 10^{17} - 10^7$ GeV. Thus, for three generations many of these results do not essentially improve the situation on the top quark mass whereas the values for $M_t$ found in the present paper can be considerably smaller than the above-quoted results. It is also important to emphasize that, in contrast to the four-generation, two-Higgs, or supersymmetric models, the addition of new extra heavy bound states does not, strictly speaking, describe an extension of the minimal top quark condensation *model*, but an extension of the minimal top quark condensation *dynamics*. In this sense our results are comparable to the analysis of ref. [11] which discusses the presence of new four-Fermi interactions leading to new colour-triplet bound states. As a consequence of these colour triplet states ref. [11] obtains $M_t \simeq 176$ GeV for $\Lambda \simeq 10^{15}$ GeV, and the result can be understood as an example of the effects of a richer spectrum. Finally, ref. [12] discusses the appearance of additional four-Fermi interactions due to the coupling of chiral fermions to an antisymmetric tensor field.

Very importantly, the results of this paper can also be phrased in terms of a NJL model with higher dimensional operators restricted only by the symmetries of the problem. This approach has been recently advocated by ref. [13]. We do not conclude however that arbitrary predictions for the top mass can be obtained so long as we do not allow for a completely arbitrary spectrum and/or couplings. The reason is simply that there is a dynamical issue in addition to the symmetries of the problem, and only upon assuming ultra-extreme spectra we can reproduce arbitrary values for $M_t$. As we have indicated in our discussion, the presence of very light (compared to $\Lambda$) or very strongly coupled additional $t$–$\bar{t}$ bound states leads to very low values for $M_t$. However, such a scenario seems dynamically very unlikely and would imply, via eq. (8), very large values for the coupling constants $G$ of the effective four-Fermi interactions. This is equivalent to saying that so far we do not know fundamental theories which are able to produce extremely unnatural spectra and unnatural coefficients in front of the higher dimensional terms of a generalized NJL treatment in the context of [13].

## REFERENCES


[1] Y. Nambu, in *New Theories in Physics*, proceedings of the XI International Symposium on Elementary Particle Physics, edited by Z. Ajduk, S. Pokorski and A. Trautman (World Scientific, Singapore, 1989); a version of this work appears in *New Trends in Strong Coupling Gauge Theories*, 1988 International Workshop, Nagoya, Japan, edited by M. Bando, T. Muta and K. Yamawaki (World Scientific, Singapore, 1989); and EFI Report No. 89-08 (1989), unpublished.

[2] A. Miransky, M.Tanabashi and K. Yamawaki, Mod. Phys. Lett. A4 (1989) 1043, Phys. Lett. B221 (1989) 177.

[3] W.A. Bardeen, C.T. Hill and M. Lindner, Phys. Rev. D41 (1990) 1647.

[4] Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122 (1961) 345.

[5] See e.g. P. Langacker, preprint UPR-0435T (1990) and references therein.

[6] W.J. Marciano, Phys. Rev. Lett. 62 (1989) 2793.

[7] M. Luty, Phys. Rev. D41 (1990) 2893.

[8] W.A. Bardeen, T.E. Clark and S.T. Love, Phys. Lett. B237 (1990) 235; W.A. Bardeen, M. Carena, T.E. Clark, K. Sasaki and C.E.M. Wagner, *"Dynamical Symmetry Breaking and the Top Quark Mass in the Minimal Supersymmetric Standard Model"*, preprint FERMILAB-PUB-91/96-T (1991).

[9] M. Lindner and D. Ross, *"Top Condensation from Very Massive Strongly Coupled Gauge Bosons,"* preprint CERN-TH.6179/91 (1991).

[10] C.T. Hill, M. Luty and E.A. Paschos, Phys. Rev. D43 (1991) 3011.

[11] K.S. Babu and R.N. Mohapatra, *"Top Quark Mass in a Dynamical Symmetry Breaking Scheme with Radiative b-quark and τ-lepton Masses,"* preprint UMD-PP-91-129 (1991).

[12] Z. Lalak, B.A. Ovrut and S. Thomas, *"Antisymmetric Tensor Field Coupled to Chiral Matter and Dynamical Symmetry Breaking,"* preprint UPR-0474T (1991).

[13] A. Hasenfratz, P. Hasenfratz, K. Jansen, J. Kuti and Y. Shen, *"The Equivalence of the Top Quark Condensate and the Elementary Higgs Field,"* preprint UCSD/PTH 91-06, AZPH-TH/91-12 (1991).




## Figure Captions

**Figure 1:** The number of required extra vector triplet (solid) and vector singlet (dashed) states as a function of $\Lambda$ from the consistency condition eq. (16) for different values of the lightest extra state, $M_1$.

**Figure 2:** Top mass predictions for different cases satisfying the consistency conditions for $\Lambda = 10^{15}\ GeV$ and different values $M_1$ of the lightest extra states. The numbers in brackets indicate the number of extra vector singlet, vector triplet and scalar states with the spectrum chosen as in eq. (13). The symbols $+, \circ, -$ correspond to different choices of the additional Yukawa couplings (for details see the text).

**Figure 3:** As in 2 for $\Lambda = 10^{12}\ GeV$.

**Figure 4:** As in 2 for $\Lambda = 10^{9}\ GeV$.

**Figure 5:** Top mass predictions as a function of the scale of new physics, $\Lambda$. Shown are the results of ref. [3] as a solid line labeled BHL and the cases $(N_{\tilde{B}}, N_{\tilde{W}}, N_{\tilde{H}}) = (0,0,1)$ for $M_1/M_W = 10^2$ and $M_1/M_W = 10^{10}$ as dashed line, and $(N_{\tilde{B}}, N_{\tilde{W}}, N_{\tilde{H}}) = (0,0,5)$ for $M_1/M_W = 10^2$ and $M_1/M_W = 10^{10}$ as dash-dotted line.

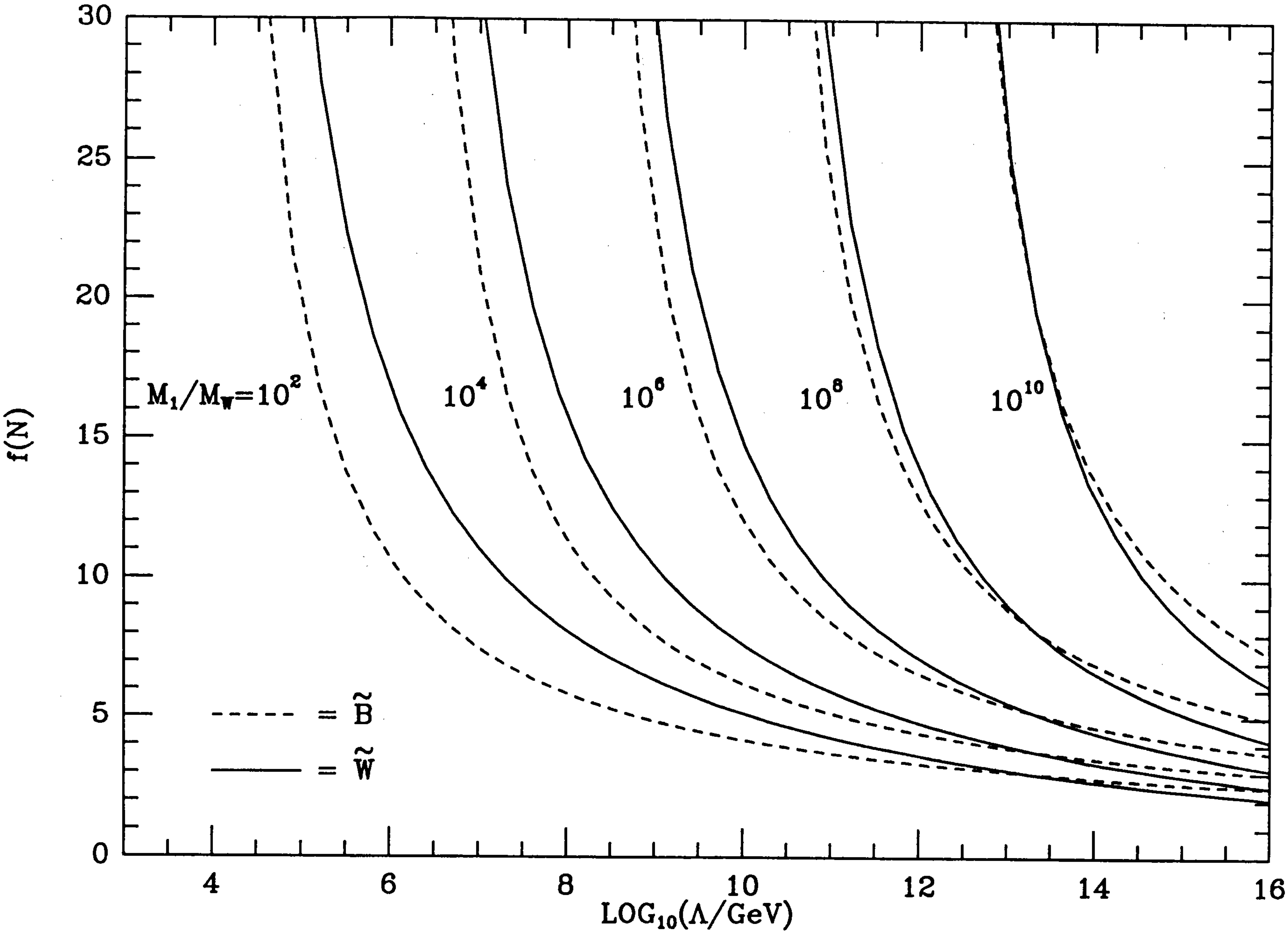


Figure 1

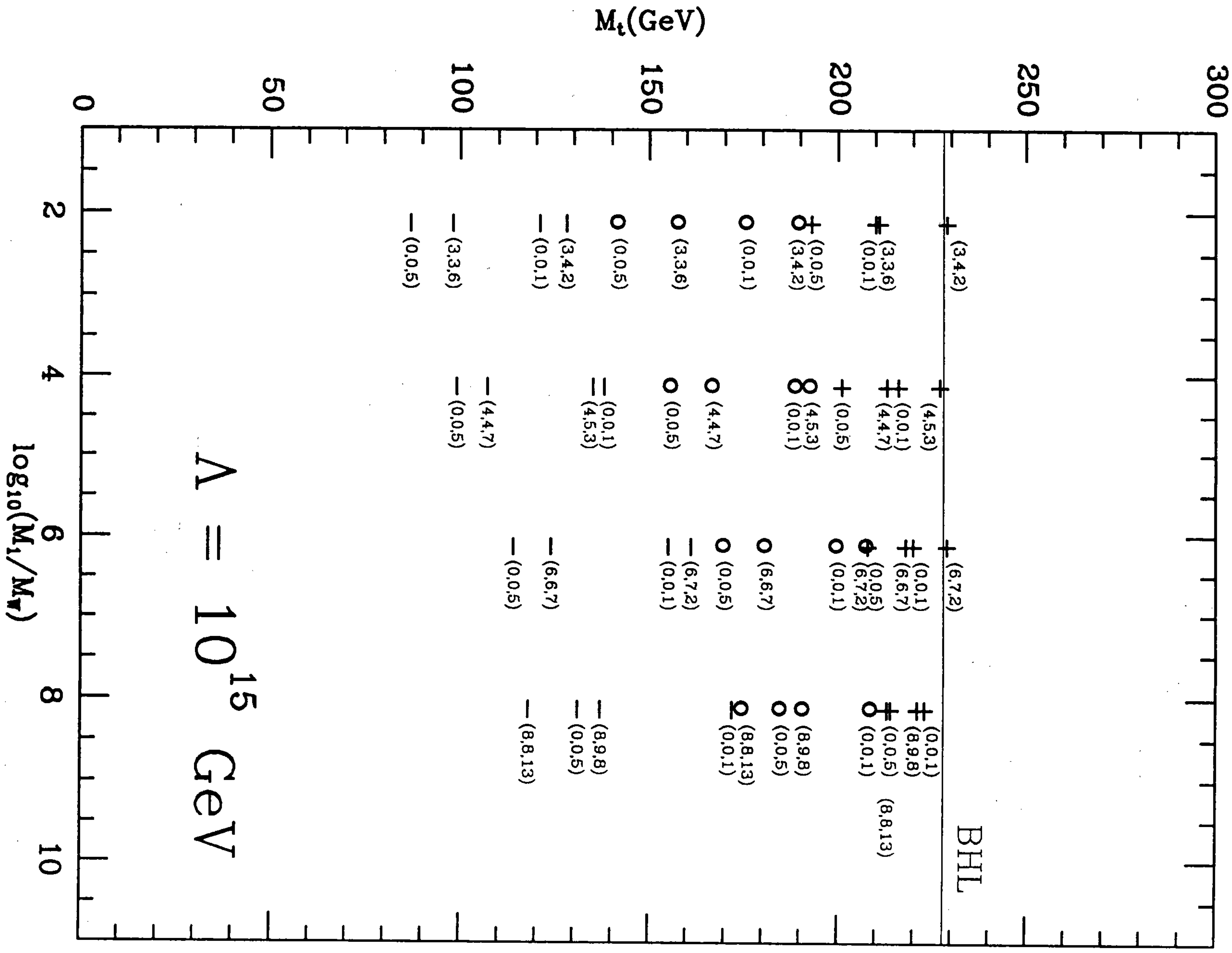


Figure 2

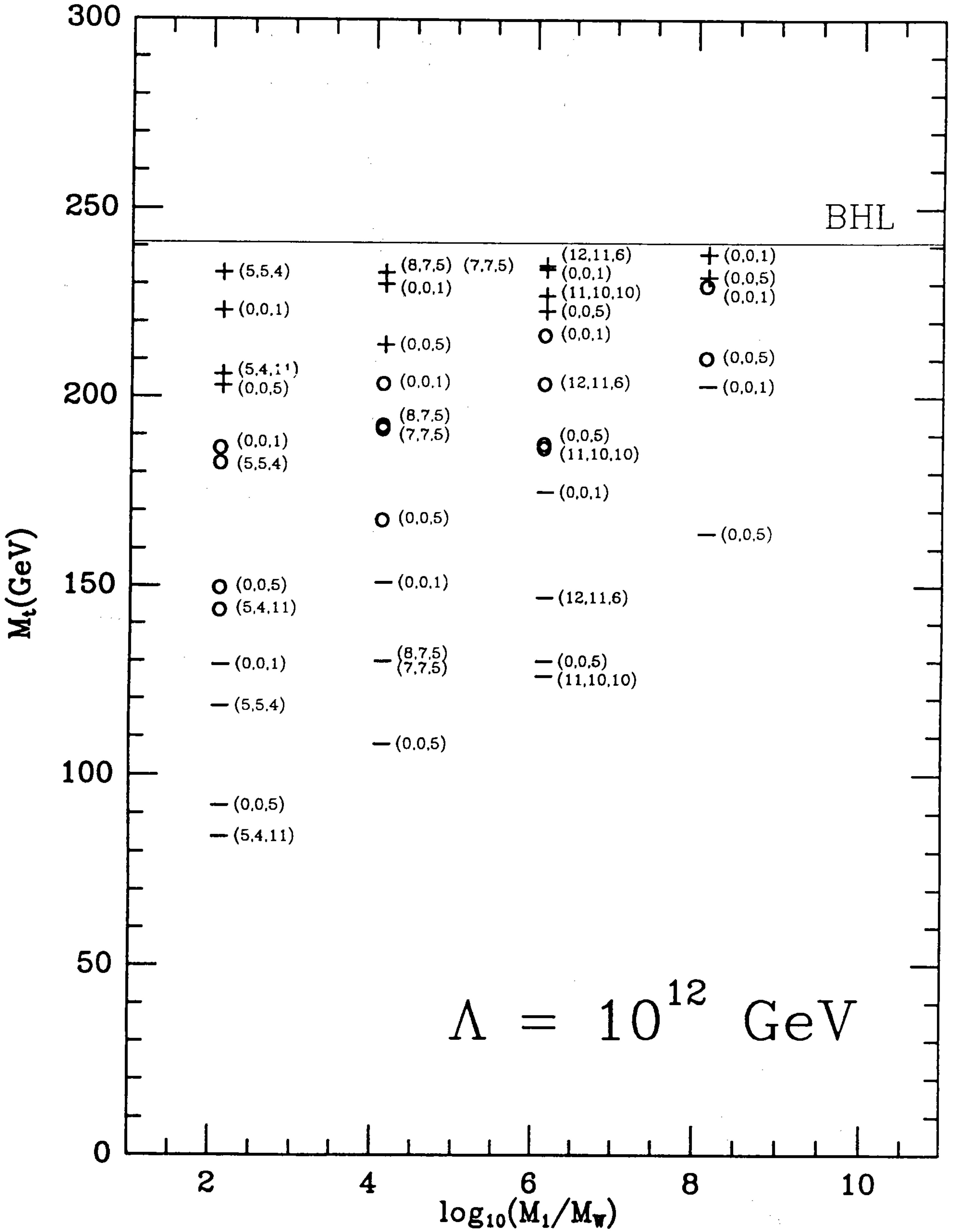


Figure 3

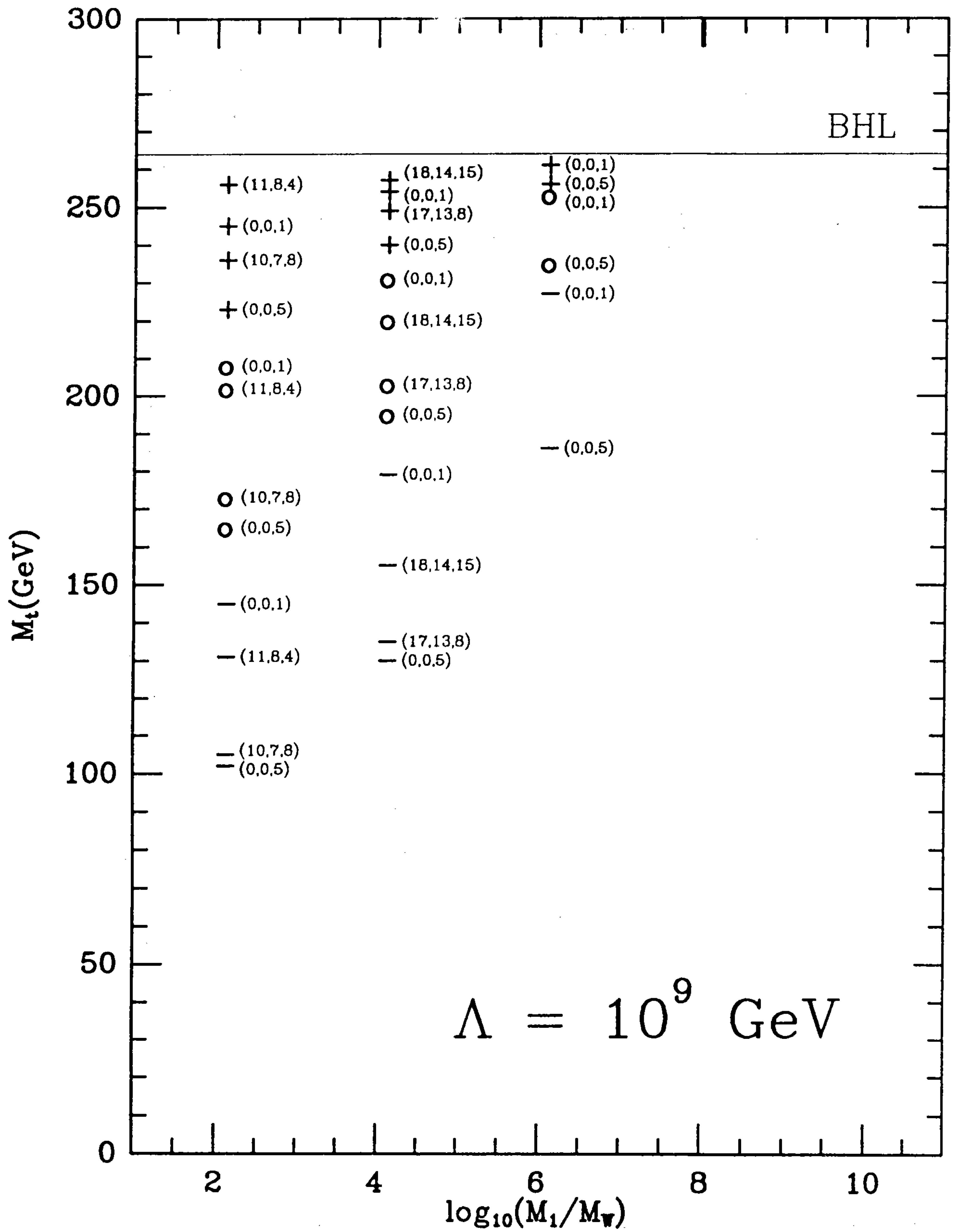


Figure 4

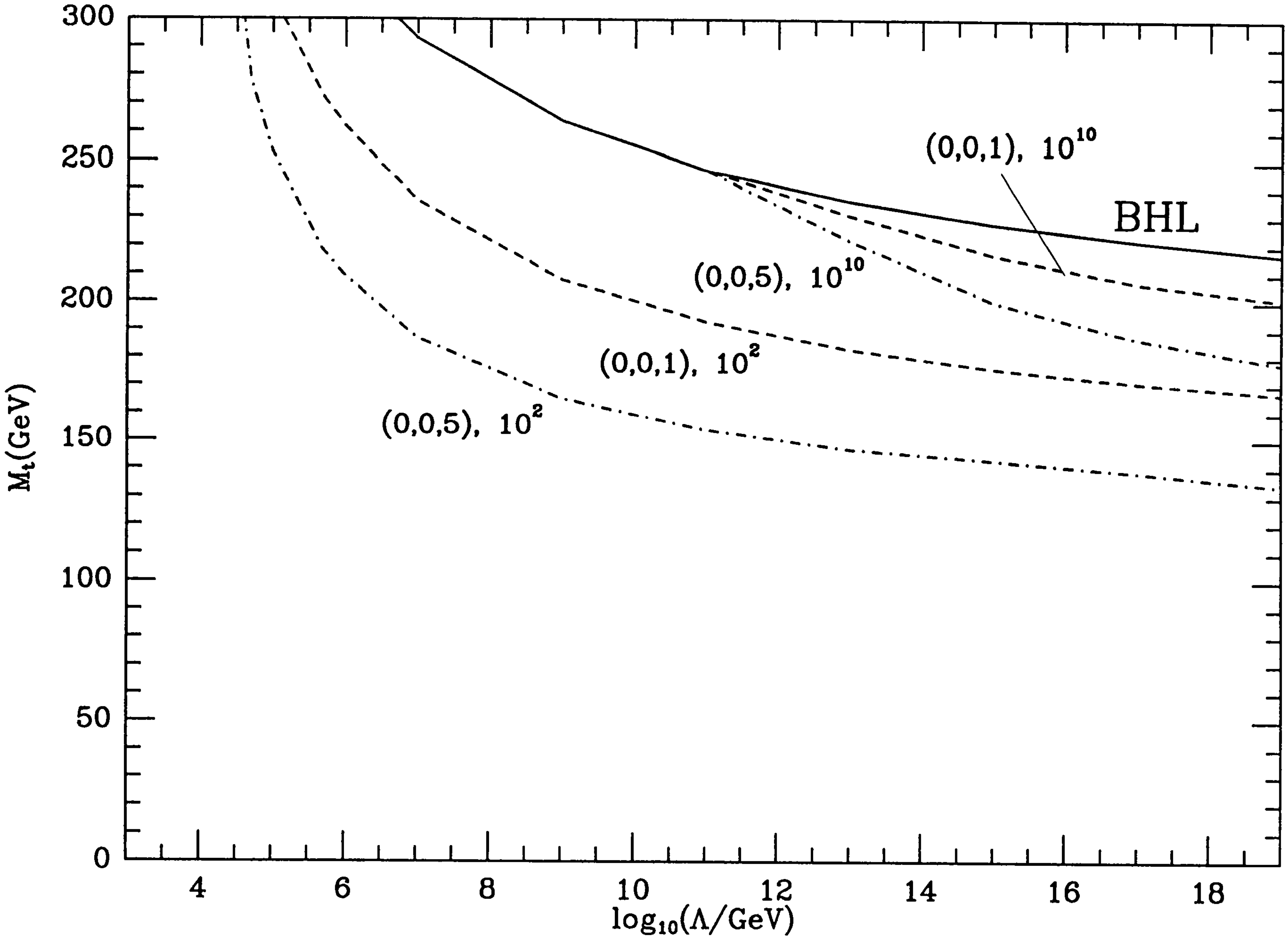


Figure 5

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