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A correlated-cluster model and the ridge phenomenon in hadron–hadron collisions

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ABSTRACT

A study of the near-side ridge phenomenon in hadron–hadron collisions based on a cluster picture of multiparticle production is presented. The near-side ridge effect is shown to have a natural explanation in this context provided that clusters are produced in a correlated manner in the collision transverse plane.

1. Introduction

The study of multiparticle production in high energy hadron collisions has proven to be a useful tool to explore the soft regime of the strong interaction dynamics [1,2]. In particular, particle correlations are known to provide crucial information about the underlying mechanism of the multiparticle production process to be ultimately interpreted in terms of QCD [1,3]. Moreover, the analysis of particle correlations has been shown to reveal signals of non-conventional physics [4,5].

In recent years big efforts have been devoted to the study of two-particle correlations in the search for collective phenomena (see [6] for a review). Two-particle correlations are analyzed in a two-dimensional azimuthal $\Delta\eta$-$\Delta\phi$ phase space, where $\Delta\phi$ and $\Delta\eta$ denote the difference of the azimuthal angle ϕ and the pseudorapidity η of the two selected particles, respectively.[1] Two-particle correlation function is defined as:

$$C(\Delta\eta,\Delta\phi)=\frac{S(\Delta\eta,\Delta\phi)}{B(\Delta\eta,\Delta\phi)}\,, \tag{1}$$

where S and B denote particle pair distributions from the same event and from different events, representing the signal and background contributions, respectively [7].

Typically, a complex structure is observed for different energies and types of colliding objects. On the one hand, there is a narrow peak centered at $(\Delta y \simeq 0,\ \Delta\phi \simeq 0)$ due to high transverse momentum (p_T) clusters and jets, whereas a broader away-side Gaussian-type structure arises from the decay of lower p_T clusters, resonances and fragmentation, including Bose–Einstein correlations. Besides, an enhancement of two-particle correlations is also found at $\Delta\phi \simeq \pi$. Because of its extended shape as seen in the $\Delta\eta$-$\Delta\phi$ plot, it is usually referred to as the *away-side ridge*. This

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[1] We consider a right-handed coordinate system with the z axis along the beams' direction. Cylindrical coordinates are used in the transverse plane, ϕ being the azimuthal angle. The pseudorapidity is defined in terms of the polar angle θ as $\eta = -\ln\tan(\theta/2)$. The (longitudinal) rapidity is defined as $y=\frac{1}{2}\ln\left(\frac{E+p_L}{E-p_L}\right)$ and coincides with the pseudorapidity for massless particles. Here, p_L is the longitudinal (along the beam axis) component of the measured particle moment.

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effect can be explained due to particle correlations coming from momentum conservation in back-to-back jets. Another interesting structure is observed, namely the long-range ($|\Delta\eta| \leq 5$) near-side ($\Delta\phi \simeq 0$) correlations yielding a *near-side ridge*, whose study is the main objective of this work.

Long-range correlations are usually attributed to a collective hydrodynamical flow, and therefore the ridge structure is expected in nuclear collisions due to, e.g., an initial anisotropy that is imprinted on the azimuthal-angle distributions of final-state particles through the collective expansion of the medium [6]. Another possible explanation is given by the color-glass condensate, where the two-gluon density is enhanced at small $\Delta\phi$, which still needs a collective flow boost to reproduce the observed ridge [8]. Both mechanisms have, however, some shortcomings such as a locally thermalized medium, which is required for the hydrodynamic flow in order to account for the near-side ridge phenomenon [9]. Finally, a third kind of explanation considers jet-medium interactions where semihard partons can induce local fluctuations by energy loss in high density soft-parton fields, yielding azimuthal asymmetries manifesting as a ridge structure [10].

Unexpectedly, a long-range ridge structure has also been observed in proton-nucleus [11,12] and, particularly, in proton–proton (pp) [13,12] collisions, still requiring a definitive explanation. The similarities between the correlations found in small systems and heavy-ion collisions suggest a common origin. Meanwhile, if hydrodynamics is successfully applied in the case of heavy ions, the hydrodynamical explanation of the ridge effect in pp collisions, even for high-multiplicity events where the effect has been actually observed, seems still unclear (however, see [14]). This quite unexpected phenomenon still requires a careful study to establish its physical origin.

Finally, it is worthwhile mentioning that the experimental analyses of azimuthal anisotropy usually involve a Fourier series containing harmonics [15] up to fifth order, $\sum_{n=0}^{5} a_n \cos(n\Delta\phi)$, where the coefficients a_n have to be interpreted and estimated by theoretical models.

In this Letter, we study two-particle correlations by invoking a simple two-step scenario for multiparticle production: the resulting multiplicity is given by the convolution of the distribution of particle emission sources (clusters/fireballs/semihard jets...) with the fragmentation/decay of the sources. A correlated-cluster model (CCM) is developed and applied throughout, where both the clusters and the final-state particles are considered to be emitted according to Gaussian distributions in rapidity and azimuth, encoding short-range and long-range correlations in both variables. Our ultimate goal is to provide a common (*effective*) framework for both proton and heavy-ion collisions to deal with the near-side ridge phenomenon. Compact expressions are provided for $C(\Delta\eta, \Delta\phi)$.

2. Definitions and notation

The general inclusive two-particle correlation function is defined through the Lorentz invariant inclusive differential single and double cross sections $\sigma_{\rm in}^{-1} E d^3\sigma/d^3p$ and $\sigma_{\rm in}^{-1} E_1E_2 d^6\sigma/d^3p_1d^3p_2$, respectively. Here, $\sigma_{\rm in}$ denotes the inelastic cross section, E and p denote the total energy and moment of particles, while the subscripts 1 and 2 refer to the two considered particles. As usual in this kind of analysis [1], we will not distinguish between different species of particles, focusing only on charged particles.

In terms of the rapidity (y) and the azimuthal angle (ϕ), the one-particle density $\tilde{\rho}$ and the two-particle density $\tilde{\rho}_2$ are defined through

$$\tilde{\rho}(y, \vec{p}_T) = \frac{1}{\sigma_{\rm in}} \frac{d^3\sigma}{d^3p} = \frac{1}{\sigma_{\rm in}} \frac{d^3\sigma}{dy d^2p_T} = \frac{1}{2\sigma_{\rm in}} \frac{d^3\sigma}{dy d\phi dp_T^2}\,, \tag{2}$$

$$\tilde{\rho}_2(y_1, \vec{p}_{T1}, y_2, \vec{p}_{T2}) = \frac{1}{\sigma_{\rm in}} \frac{d^6\sigma}{d^3p_1d^3p_2} = \frac{1}{\sigma_{\rm in}} \frac{d^6\sigma}{dy_1 d^2p_{T1} dy_2 d^2p_{T2}} = \frac{1}{4\sigma_{\rm in}} \frac{d^6\sigma}{dy_1 d\phi_1 dp_{T1}^2 dy_2 d\phi_2 dp_{T2}^2}\,,$$

where p_T (p_T^2) denotes the (square modulus of the) particle transverse momentum.

From the above expressions, let us define the normalized two-particle correlation function as

$$C(1,2) = \frac{\tilde{\rho}_2(1,2)}{\tilde{\rho}(1)\tilde{\rho}(2)}\,, \tag{3}$$

where the indices 1 and 2 stand for the set of kinematic variables of the first and second particles of the pair, respectively.

On the other hand, it is customary integrating over p_T^2 on a suitable range denoted by Ω_T (determined by experimental cuts on events), and the single and two-particle densities become

$$\rho(y,\phi) = \int_{\Omega_T} dp_T^2\, \tilde{\rho}(y, \vec{p}_T),$$

$$\rho_2(y_1,\phi_1,y_2,\phi_2) = \int_{\Omega_T} dp_{T1}^2 dp_{T2}^2\, \tilde{\rho}_2(y_1, \vec{p}_{T1}, y_2, \vec{p}_{T2})\,. \tag{4}$$

In order to match our theoretical approach to the definition of Eq. (1), where the (pseudo)rapidity and azimuthal differences are involved, we identify the two-particle distribution of the uncorrelated pairs by means of the two Dirac δ-functions, i.e.

$$b(\Delta y, \Delta\phi) = \int dy_1 dy_2 d\phi_1 d\phi_2\, \rho(y_1,\phi_1)\rho(y_2,\phi_2) \times \delta(\Delta y - y_1 + y_2)\delta(\Delta\phi - \phi_1 + \phi_2)\,, \tag{5}$$

such that $\Delta y = y_1 - y_2$ and $\Delta\phi = \phi_1 - \phi_2$.

In its turn, the pair distribution of correlated pairs can be identified with

$$s(\Delta y, \Delta\phi) = \int dy_1 dy_2 d\phi_1 d\phi_2\, \rho_2(y_1,\phi_1,y_2,\phi_2) \times \delta(\Delta y - y_1 + y_2)\delta(\Delta\phi - \phi_1 + \phi_2)\,, \tag{6}$$

where again the Dirac δ-functions are incorporated.

Then, the normalized correlation function is redefined in the following way,

$$C(\Delta y, \Delta\phi) = \frac{s(\Delta y, \Delta\phi)}{b(\Delta y, \Delta\phi)}\,, \tag{7}$$

being suitable for a comparison with the experimental results obstained using Eq. (1).

3. Two-particle correlations in a cluster model

It is usually accepted that particle production in soft hadronic interactions occurs via an intermediate step of decaying ancestors/clusters/fireballs yielding final-state particles [1,3]. It should be noted, however, that the "cluster" concept has to be understood in a broad sense, i.e. a group of particles with some correlated properties.

The Independent Cluster Model (clusters are produced in a non-correlated way) has been widely applied to the study of hadron collisions (see [17,18] and references therein). In [19], the near-side ridge formation is studied in the framework of the so-called Correlated Emission Model for heavy ion collisions. In this model, a semihard parton, emitted in the primary collision, scatters when traversing the medium yielding a local energy flow. The correlation between both (semihard parton and local flow) enhances the effect of soft emission, leading to the ridge formation. On the other hand, our approach, based on correlated cluster production, can be viewed as a rather model-independent approach to ridge formation, useful to deal with *pp* collisions too.

After integrating over the (square) cluster transverse momentum along the whole kinematically-allowed range, the single and two-cluster densities read

$$\rho^{(c)}(y_c,\phi_c)=\int dp_{Tc}^2\tilde{\rho}^{(c)}(y_c,\vec{p}_{Tc}),$$

$$\rho_2^{(c)}(y_{c1},\phi_{c1},y_{c2},\phi_{c2}) = \int dp_{Tc1}^2 dp_{Tc2}^2\tilde{\rho}_2^{(c)}(y_{c1},\vec{p}_{c1},y_{c2},\vec{p}_{c2})\,. \quad (8)$$

Now we define particle densities depending on rapidity and azimuthal variables, coming from the decay of a single cluster with rapidity y_c and azimuthal angle ϕ_c leading to final-state particles:

$$\rho^{(1)}(y,\phi;y_c,\phi_c)=\int_{\Omega_T} dp_T^2\tilde{\rho}^{(1)}(y,\vec{p}_T;y_c,\phi_c)\,, \quad (9)$$

$$\rho_2^{(1)}(y_1,\phi_1,y_2,\phi_2;y_c,\phi_c) = \int_{\Omega_T} dp_{T1}^2 dp_{T2}^2\tilde{\rho}_2^{(1)}(y_1,\vec{p}_{T1},y_2,\vec{p}_{T2};y_c,\phi_c)\,, \quad (10)$$

where Ω_T again refers to the selected transverse momentum range of final-state particles.

Thus, the single particle density can be expressed as the convolution of the cluster density and the particle density from a single cluster, i.e.

$$\rho(y,\phi)=\int dy_c d\phi_c\rho^{(c)}(y_c,\phi_c)\rho^{(1)}(y,\phi;y_c,\phi_c)\,. \quad (11)$$

Notice that all the kinematic variables appearing in the above expressions (cluster and particle rapidities and azimuthal angles) are measured in the Laboratory Reference Frame (LRF) which coincides with the center-of-mass frame of the hadron–hadron collision.

To the extent that $\rho^{(1)}(y,\phi;y_c,\phi_c)$ is flat in the central rapidity region and small transverse cluster momenta, we have $\rho(y,\phi)=\langle N_c\rangle\bar{\rho}^{(1)}$, where $\bar{\rho}^{(1)}$ stands for an average particle density for single cluster decay. Hence the resulting single-particle density would be flat too and its height to be proportional to the mean number of clusters per collision.

However, this approximation is quite rough as we are extending our study to the (pseudo)rapidities $|y|\lesssim 5$ and possibly to large cluster transverse momenta. To this end, we introduce a function $E_1(y,\phi)$, which keeps the expected dependence on the rapidity and azimuthal variables of the emitted particles, so that the single-particle density becomes:

$$\rho(y,\phi)=\langle N_c\rangle\bar{\rho}^{(1)}E_1(y,\phi)\,. \quad (12)$$

The $E_1(y,\phi)$ function is normalized in such way that

$$\int dy\,d\phi\,E_1(y,\phi)=1\,,$$

where integration is taken over the full available kinematic range of both the variables.

Similarly, we introduce the product of the two single-particle distributions representing the mixed-event background, i.e.

$$\rho_{\rm mixed}(y_1,\phi_1,y_2,\phi_2) = \rho(y_1,\phi_1)\rho(y_2,\phi_2) = \langle N_c\rangle^2\bar{\rho}^{(1)2}E_1(y_1,\phi_1)E_1(y_2,\phi_2)\,, \quad (13)$$

which suggests to define

$$E_b(y_1,\phi_1,y_2,\phi_2)=E_1(y_1,\phi_1)E_1(y_2,\phi_2)\,. \quad (14)$$

The two-particle density can also be written as

$$\rho_2(y_1,\phi_1,y_2,\phi_2) = \int dy_c\phi_c\rho^{(c)}(y_c,\phi_c)\rho_2^{(1)}(y_1,\phi_1,y_2,\phi_2;y_c,\phi_c) + \int dy_{c1}dy_{c2}d\phi_{c1}d\phi_{c2}\rho_2^{(c)}(y_{c1},\phi_{c1},y_{c2},\phi_{c2}) \times\rho^{(1)}(y_1,\phi_1;y_{c1},\phi_{c1})\rho^{(1)}(y_2,\phi_2;y_{c2},\phi_{c2})\,. \quad (15)$$

The first term on the r.h.s. corresponds to the emission of secondaries from a single cluster while the second term corresponds to the emission of the two particles from two distinct clusters.

Therefore, we conclude for the two-particle density:

$$\rho_2(y_1,\phi_1,y_2,\phi_2) = \langle N_c\rangle\bar{\rho}^{(1)2}E_s^{\rm SR}(y_1,\phi_1,y_2,\phi_2) + \langle N_c(N_c-1)\rangle\bar{\rho}^{(1)2}E_s^{\rm LR}(y_1,\phi_1,y_2,\phi_2)\,, \quad (16)$$

where $E_s^{\rm SR}(y_1,\phi_1,y_2,\phi_2)$ stands for the short range (pseudo)rapidity correlations and $E_s^{\rm LR}(y_1,\phi_1,y_2,\phi_2)$ stands for the long range correlations stemming from the two integrals of Eq. (15), respectively.

Needless to say, the above expression is mainly intended to describe the near-side effect using the CCM. Other kind of correlations (like particles inside jets or the away-side ridge) fall off the above description and will not be considered in this Letter.

3.1. Factorization hypothesis

Taking into account that the rapidity y and the azimuthal variable ϕ are orthogonal variables, we tentatively assume that both $E_b(y_1,\phi_1,y_2,\phi_2)$ and $E_s(y_1,\phi_1,y_2,\phi_2)$ can be factorized as

$$E_b(y_1,\phi_1,y_2,\phi_2)=E_b^L(y_1,y_2)\cdot E_b^T(\phi_1,\phi_2)\,,$$
$$E_s(y_1,\phi_1,y_2,\phi_2)=E_s^L(y_1,y_2)\cdot E_s^T(\phi_1,\phi_2)\,, \quad (17)$$

where the superscripts L and T denote the longitudinal and transverse parts, respectively.

Moreover, as usual in cluster models, we shall adopt Gaussian distributions in rapidity and azimuthal spaces for both cluster density and particle density from clusters,[2] as developed below.

On average, clusters should be isotropically produced in the transverse plane of the primary hadronic collision (even though anisotropy would be present in event-by-event fluctuations). Thus, only dependence on the rapidity variable remains in the single-cluster density,

$$\rho^{(c)}(y_c,\phi_c)\sim\exp\left[-\frac{y_c^2}{2\delta_{cy}^2}\right], \quad (18)$$

[2] For an isotropically decaying cluster with rapidity y_c, the single (massless) particle density can be written as $\rho^{(1)}(y;y_c)\sim\cosh^{-2}(y-y_c)$. As it is well known [16], it can be well approximated by a Gaussian of width $\delta_y\lesssim 0.9$.

where δ_{cy} denotes the rapidity correlation length for cluster production. On account of the plateau structure of multiplicity distribution in pseudorapidity phase space, one may assume that the dependence of $\rho^{(c)}(y_c, \phi_c)$ on y_c is rather weak, i.e. $\delta_{cy}^2 \gg 1$. Indeed, one can empirically expect that δ_{cy} is basically determined by the rapidity plateau length of the single-particle distribution (for all charged particles).

Now we turn to the single particle density $\rho^{(1)}(y, \phi; y_c, \phi_c)$. Since clusters are produced with some non-null (transverse) momentum, the initial isotropic distribution will be transformed into an elliptic shape depending on the cluster and emitted particle transverse velocities. (We note again that ϕ stands throughout for a variable in the LRF, which coincides with the center-of mass of the pp colliding system.) Hence a dependence on the cluster azimuthal angle ϕ_c should remain in $\rho^{(1)}(y, \phi; y_c, \phi_c)$.

As shown in the Appendix, such elliptic shape can be approximately expressed in terms of a Gaussian for highest boosted particles. Therefore one gets

$$\rho^{(1)}(y, \phi; y_c, \phi_c) \sim \exp\left[-\frac{(y - y_c)^2}{2\delta_y^2}\right] \exp\left[-\frac{(\phi - \phi_c)^2}{2\delta_\phi^2}\right]. \tag{19}$$

The parameter δ_y, usually referred to as the cluster decay "width", characterizes the (pseudo)rapidity separation of particles emitted in a single cluster decay; it has been experimentally measured to be $\lesssim 1$ rapidity units [17]. Therefore, it turns out that $\delta_{cy}^2 \gg \delta_y^2$, in accord with our previous discussion.

For small azimuthal angles with respect to the cluster direction in the transverse plane

$$\delta_\phi \sim \frac{1}{v_T \gamma_T} \tag{20}$$

stands for the cluster decay "width" regarding the azimuthal angle instead of the rapidity variable. Since the near-side ridge effect shows up for particles with transverse momentum of order of 1 GeV, we will take $\delta_\phi \simeq 0.14$ radians as a reference value (corresponding to a pion with $p_T = 1$ GeV emitted at rest in the cluster reference frame). At low p_T, δ_ϕ becomes large and the angular distribution remains almost flat (thereby the ridge disappears). Conversely, at higher p_T values δ_ϕ will decrease leading to a more pronounced peak at $\phi \simeq \phi_c$. Note also that one expects $\delta_{c\phi}^2 \gg \delta_\phi^2$ since the particles from the boosted clusters should be more collimated in azimuth than the clusters themselves (the latter being quite more massive).

On the other hand, we further assume that the clusters are emitted in a correlated manner both in rapidity and azimuth. Thus, the two-cluster density is given by

$$\rho_2^{(c)}(y_{c1}, \phi_{c1}, y_{c2}, \phi_{c2}) \sim \exp\left[-\frac{(y_{c1} + y_{c2})^2}{2\delta_{cy}^2}\right] \exp\left[-\frac{(\phi_{c1} - \phi_{c2})^2}{2\delta_{c\phi}^2}\right], \tag{21}$$

where δ_{cy} and $\delta_{c\phi}$ stand for the rapidity and azimuthal correlation lengths, respectively. Let us remark that Eq. (21) can be regarded as a parameterization especially suitable to determine the near-ridge effect using the CCM. The physical origin of such azimuthal correlations among clusters has to be provided by specific models.

The underlying physical picture corresponds to cluster pairs emitted mostly with opposite rapidities but in the same hemisphere defined by the cluster velocity in the transverse plane. The correlation strengths are determined by the δ_{cy} and $\delta_{c\phi}$ parameters (not to be confused with δ_y and δ_ϕ) respectively.

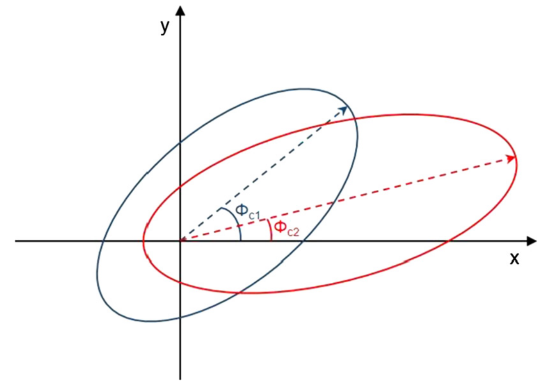

Fig. 1. Illustrative picture of two clusters produced in a primary hadron collision at the origin of the transverse plane with azimuthal angles ϕ_{c1} and ϕ_{c2}, decaying into final state particles. Elliptic shapes are due to Lorentz boosts, although they could become somewhat distorted since they correspond to the projection on the transverse plane of boosted distributions in a three-dimensional space.

The condition on the rapidity can be seen as a consequence of longitudinal momentum conservation. Away-side particles can carry the remaining momentum due to a non-exact momentum balance by the two clusters along the beams' direction. The azimuthal condition is implemented in this version of the CCM by hand[3] but should be attributed to a dynamical mechanism developed in a concrete model. As shown below, the requirement of azimuthal cluster correlations is definitely needed in order to account for the near-side ridge effect according to the CCM.

In Fig. 1 we illustrate the particle emission from two clusters produced in the same primary hadron collision leading to different elliptic shapes due to different Lorentz boosts. Clusters are assumed to be correlated both in rapidity and azimuth according to Eq. (21).

Now we integrate over rapidities and azimuthal angles using the Dirac's δ-functions. Lastly, we end up with a correlation function depending on $e(\Delta y, \Delta\phi)$ yielding a "residual" dependence on the rapidity and azimuthal variables:

$$s(\Delta y, \Delta\phi) = \langle N_c \rangle \bar{\rho}^{(1)2} e_s(\Delta y, \Delta\phi), \tag{22}$$

where $e_s(\Delta y, \Delta\phi)$ is defined as:

$$e_s(\Delta y, \Delta\phi) = \int dy_1 dy_2 d\phi_1 d\phi_2 \delta(\Delta y - y_1 + y_2) \times \delta(\Delta\phi - \phi_1 + \phi_2) E_s(y_1, \phi_1, y_2, \phi_2). \tag{23}$$

Since one can reasonably expect that $\delta_{cy}^2 \gg \delta_y^2$, $\delta_{c\phi}^2 \gg \delta_\phi^2$ (as it is argued just above), we write the short-range and long-range pieces of the two-particle density as (see Appendix for e-functions):

$$s^{SR}(\Delta y, \Delta\phi) = \langle N_c \rangle \bar{\rho}^{(1)2} e_s^{SR}(\Delta y, \Delta\phi), \tag{24}$$

where

$$e_s^{SR}(\Delta y, \Delta\phi) \sim \exp\left[-\frac{(\Delta y)^2}{4\delta_y^2}\right] \exp\left[-\frac{(\Delta\phi)^2}{4\delta_\phi^2}\right]. \tag{25}$$

On the other hand,

$$s^{LR}(\Delta y, \Delta\phi) = \langle N_c(N_c - 1) \rangle \bar{\rho}^{(1)2} e_s^{LR}(\Delta y, \Delta\phi), \tag{26}$$

where

[3] On the other hand, the Gaussian function $\exp[-(\phi_{c1} - \phi_{c2} - \pi)^2/2\delta_{c\phi}^2]$ would correspond to the back-to-back cluster emission in the transverse plane rather related to the away-side ridge effect.

$$e_s^{LR}(\Delta y, \Delta\phi) \sim \exp\left[-\frac{(\Delta\phi)^2}{2(2\delta_\phi^2+\delta_{c\phi}^2)}\right]. \quad (27)$$

Let us observe that for a Poissonian distribution of clusters $\langle N_c(N_c-1)\rangle = \langle N_c\rangle^2$.

Regarding the uncorrelated pairs, one finds

$$b(\Delta y, \Delta\phi) = \langle N_c\rangle^2 \bar{\rho}^{(1)2} e_b(\Delta y, \Delta\phi)\,. \quad (28)$$

Upon integration over both rapidities, with Δy fixed, one readily gets

$$e_b(\Delta y, \Delta\phi) \sim \exp\left[-\frac{(\Delta y)^2}{4(\delta_y^2+\delta_{cy}^2)}\right]. \quad (29)$$

Note that there is no azimuthal dependence in the above expression.

4. Interpretation of the near-side ridge effect according to CCM

In this section we show that the near-side effect emerges quite naturally from Eqs. (24) to (29) according to the CCM. To this end, we write

$$C(\Delta y, \Delta\phi) = \frac{s^{SR}(\Delta y, \Delta\phi) + s^{LR}(\Delta y, \Delta\phi)}{b(\Delta y, \Delta\phi)} = 1 + \frac{h^{SR}(\Delta y, \Delta\phi)}{\langle N_c\rangle} + \frac{\langle N_c(N_c-1)\rangle}{\langle N_c\rangle^2} h^{LR}(\Delta\phi)\,, \quad (30)$$

where

$$h^{SR}(\Delta y, \Delta\phi) = \frac{e_s^{SR}(\Delta y, \Delta\phi)}{e_b(\Delta y, \Delta\phi)} = \exp\left[-\frac{\delta_{cy}^2}{4\delta_y^2(\delta_y^2+\delta_{cy}^2)}(\Delta y)^2\right] \times \exp\left[-\frac{(\Delta\phi)^2}{4\delta_\phi^2}\right], \quad (31)$$

and

$$h^{LR}(\Delta y, \Delta\phi) = \frac{e_s^{LR}(\Delta y, \Delta\phi)}{e_b(\Delta y, \Delta\phi)} \simeq \exp\left[\frac{(\Delta y)^2}{4(\delta_y^2+\delta_{cy}^2)}\right] \times \exp\left[-\frac{(\Delta\phi)^2}{2(2\delta_\phi^2+\delta_{c\phi}^2)}\right], \quad (32)$$

where the latter smoothly increases with the (pseudo)rapidity separation Δy provided that $\delta_{cy}^2 \gg 1$, as discussed above.

For $\delta_{cy}^2 \gg \delta_y^2$ we find

$$h^{SR}(\Delta y, \Delta\phi) = \exp\left[-\frac{(\Delta y)^2}{4\delta_y^2}\right] \exp\left[-\frac{(\Delta\phi)^2}{4\delta_\phi^2}\right] \quad (33)$$

and

$$h^{LR}(\Delta y, \Delta\phi) \simeq \exp\left[-\frac{(\Delta\phi)^2}{2(2\delta_\phi^2+\delta_{c\phi}^2)}\right] \quad (34)$$

since there is an almost complete cancellation of the rapidity dependence; only the dependence on the azimuthal difference survives the ratio leading to the rise of a ridge at small $\Delta\phi$. The physical reason is that δ_{cy} is provided by the plateau length in the multiplicity pseudorapidity distribution (hence large), while $\delta_{c\phi}$ is assumed to be quite smaller, as shown below.

Indeed, the angular dependence of the second-order Fourier harmonic contribution ($\cos(2\Delta\phi)$) to the near-side correlation function can be approximated by the Gaussian of Eq. (34) for small $\Delta\phi$. By Taylor-expanding the exponential about $\Delta\phi \simeq 0$, and requiring $\cos(2\,\Delta\phi) \simeq \exp\left[-(\Delta\phi)^2/2(2\delta_\phi^2+\delta_{c\phi}^2)\right]$, we get

$$2\delta_\phi^2 + \delta_{c\phi}^2 \simeq 0.25. \quad (35)$$

For high p_T enough (therefore δ_ϕ^2 small) the above condition leads to

$$\delta_{c\phi} \lesssim 0.5 \text{ (radians)},$$

in good agreement with findings from [19,20] and experimental measurements [21]. Let us stress that the long-range correlations in h^{LR} are consequence of the azimuthal cluster correlations shown above, whereas the (pseudo)rapidity correlations play a minor role in the near-side ridge effect, as also pointed out in [20].

Finally, note that the weight of h^{LR} relative to the other terms of Eq. (30) should increase as $\langle N_c\rangle$ increases. This can explain why the near-side ridge effect shows up at larger multiplicity (hence larger $\langle N_c\rangle$).

5. Summary

We have studied the near-side ridge effect observed in nuclear and hadronic collisions at RHIC and LHC. The study is carried out in the context of the correlated cluster model (CCM) using Gaussian distributions in azimuth and rapidity both for clusters and final state hadrons, encoding short-range and long-range correlations.

In order to reproduce the ridge phenomenon representing two-particle correlations at small azimuthal difference $\Delta\phi$ over a wide (pseudo)rapidity range, clusters have necessarilly to be emitted in a correlated way in azimuthal space, but not in rapidity space. Let us stress that correlations among particles in single-cluster decays are not enough to account for the near-side ridge effect. Moreover, a relation for the second-order Fourier harmonic (related to the elliptic flow in a hydrodynamical scenario) is obtained in agreement with experimental results. Although the physical origin of cluster correlations could vary depending on the nature of the colliding bodies, the CCM provides a common framework to explain the ridge effect in proton–proton, proton-nucleus and heavy-ion collisions.

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Appendix A. Gaussian distributions and their convolutions

A.1. (Pseudo)rapidity dependence

We will assume throughout that both clusters and particles stemming from clusters obey Gaussian distributions in rapidity space, i.e.

$$\rho^{(c)}(y_c, \phi_c) \sim \exp\left[-\frac{y_c^2}{2\delta_{cy}^2}\right],$$

$$\rho^{(1)}(y, \phi; y_c, \phi_c) \sim \exp\left[-\frac{(y-y_c)^2}{2\delta_y^2}\right], \quad (A.1)$$

respectively.

Upon integration over the cluster rapidity y_c, the $E_1^L(y)$ function, introduced in Eq. (12), reads

$$E_1^L(y) \sim \int dy_c \exp\left[-\frac{y_c^2}{2\delta_{cy}^2}\right]\exp\left[-\frac{(y-y_c)^2}{2\delta_y^2}\right] \sim \exp\left[-\frac{y^2}{2(\delta_y^2+\delta_{cy}^2)}\right]. \tag{A.2}$$

Hence, for two particles emitted from two different clusters one gets for the longitudinal part of the E_b function, introduced in Eq. (13),

$$E_b^L(y_1,y_2) = E_1^L(y_1)\cdot E_1^L(y_2) \sim \exp\left[-\frac{(y_1^2+y_2^2)}{2(\delta_y^2+\delta_{cy}^2)}\right]. \tag{A.3}$$

Upon integration on both rapidities keeping the rapidity interval $\Delta y = y_1 - y_2$ fixed, one gets

$$e_b^L(\Delta y) \sim \exp\left[-\frac{(\Delta y)^2}{4(\delta_y^2+\delta_{cy}^2)}\right]. \tag{A.4}$$

For two particles stemming from the same cluster with rapidity y_c

$$E_s^L(y_1,y_2) \sim \int dy_c \exp\left[-\frac{y_c^2}{2\delta_{cy}^2}\right]\exp\left[-\frac{(y_1-y_c)^2}{2\delta_y^2}\right] \times \exp\left[-\frac{(y_2-y_c)^2}{2\delta_y^2}\right] \sim \exp\left[-\frac{\delta_{cy}^2(y_1-y_2)^2}{2\delta_y^2(\delta_y^2+2\delta_{cy}^2)}\right] \times \exp\left[-\frac{(y_1^2+y_2^2)}{2(\delta_y^2+2\delta_{cy}^2)}\right]. \tag{A.5}$$

After integration using the Dirac's δ-function, the above expression leads to

$$e_s^{SR}(y_1,y_2) \sim \exp\left[-\frac{(\Delta y)^2}{4\delta_y^2}\right]. \tag{A.6}$$

Notice that δ_{cy} drops off in the last expression so that it can be referred to as a short-range correlation (SRC) contribution, as indicated in the superscript.

For two particles with rapidity y_1 and y_2 coming from two different (correlated) clusters with rapidities y_{c1} and y_{c2} respectively, we have (see Eq. (21))

$$E_s^L(y_1,y_2) \sim \int dy_{c1}dy_{c2} \exp\left[-\frac{(y_{c1}+y_{c2})^2}{2\delta_{cy}^2}\right] \times \exp\left[-\frac{(y_1-y_{c1})^2}{2\delta_y^2}\right]\exp\left[-\frac{(y_2-y_{c2})^2}{2\delta_y^2}\right] \sim \exp\left[-\frac{(y_1+y_2)^2}{2(2\delta_y^2+\delta_{cy}^2)}\right]. \tag{A.7}$$

Using again the Dirac δ-function, one gets

$$e_s^{LR}(\Delta y) \sim \text{const.}\,, \tag{A.8}$$

which corresponds to a long-range correlation (LRC), as indicated in the superscript.

In sum, we get two pieces with different behaviours (SRC versus LRC) in rapidity space:

$$e_s^{SR}(\Delta y) \sim \exp\left[-\frac{(\Delta y)^2}{4\delta_y^2}\right];\ e_s^{LR}(\Delta y) \sim \text{const.} \tag{A.9}$$

Had we used uncorrelated cluster production, i.e. $\rho_2^{(c)} \sim \exp\left[-(y_{c1}^2+y_{c2}^2)/2\delta_{cy}^2\right]$, our final results given by Eqs. (33) and (34) would remain the same.

A.2. Azimuthal dependence

Clusters are supposed to be produced isotropically in the transverse plane of the hadron collision. Let ϕ_c be a particular value of the azimuthal variable for a cluster whose decay into particles is also assumed isotropic. Thus the azimuthal distribution $w(\phi^*)$ in the cluster rest frame should be a constant.

Under a Lorentz boost of velocity v_T, the angular distribution in the LRF is given by [22]

$$w(\phi-\phi_c) = \frac{1}{\gamma_T[1-v_T^2\cos^2(\phi-\phi_c)]} f(\phi,g)\,. \tag{A.10}$$

We recall here that ϕ is measured in the LRF, i.e. the angular distribution is boosted by a Lorentz factor $\gamma_T = (1-v_T^2)^{-1/2}$ in the transverse plane. The $f(\phi,g)$ function stands for

$$f(\phi,g) = \frac{g \pm \sqrt{D}}{\pm\sqrt{D}}\,, \tag{A.11}$$

where $D = 1 + \gamma_T^2(1-g^2)\tan^2(\phi-\phi_c)$, with $g = v_T/v_T^*$ denoting the ratio of the cluster transverse velocity v_T in the LRF and the particle transverse velocity v_T^* measured in the cluster rest frame, respectively.

For $g \geq 1$ all particles are emitted along the forward hemisphere (defined by the cluster velocity) in the transverse plane of the LRF. In fact, this should be the case for particles, emitted with transverse momentum in the range $1 \leq p_T \leq 5$ GeV measured in the LRF. Therefore, we will consider that $g \approx 1$ and set $f(\phi,g)$ approximately equal to a constant. Larger transverse momenta of particles would correspond rather to jet production instead of an intermediate-p_T cluster production on which we are focusing in this work.

In order to get simpler expressions at the end, we approximate the azimuthal distribution by a Gaussian for small $\phi-\phi_c$ angles, namely,

$$w(\phi-\phi_c) \approx \exp\left[-\frac{(\phi-\phi_c)^2}{2\delta_\phi^2}\right], \quad \delta_\phi \simeq \frac{1}{v_T\gamma_T}\,. \tag{A.12}$$

It becomes apparent that large cluster transverse velocity leads to small δ_ϕ and thereby a more pronounced peak at $\phi \simeq \phi_c$, in accordance with Eq. (A.10).

Admittedly, cluster emission boosted along the transverse plane has only been considered above, while the general situation should contemplate cluster and particle motions with velocity components along the beam direction as well. However, the main conclusion of the above should remain valid.

In addition to the hypothesis of isotropically decaying clusters in their own rest frame, we will assume axial symmetry for cluster production in the transverse plane, i.e.

$$E_b^T(\phi_1,\phi_2) \sim \text{const.} \to e_b^T(\Delta\phi) \sim \text{const.} \tag{A.13}$$

Thus, the distribution for two particles, emitted from the same cluster with azimuthal angle ϕ_c should obey

$$\int d\phi_c \exp\left[-\frac{(\phi_1-\phi_c)^2}{2\delta_\phi^2}\right]\exp\left[-\frac{(\phi_2-\phi_c)^2}{2\delta_\phi^2}\right] \sim \exp\left[-\frac{(\phi_1-\phi_2)^2}{4\delta_\phi^2}\right] \tag{A.14}$$

for small azimuthal angles. Therefore, regarding the azimuthal dependence we can write

$$e_s^{SR}(\Delta\phi) \sim \exp\left[-\frac{(\Delta\phi)^2}{4\delta_{c\phi}^2}\right]. \tag{A.15}$$

On the other hand, we will assume that clusters are produced in a correlated way according to Eq. (21). Hence for two particles with azimuthal angles ϕ_1 and ϕ_2 coming from two different clusters with azimuthal angles ϕ_{c1} and ϕ_{c2}, we will write

$$E_s^T(\phi_1,\phi_2) \sim \int d\phi_{c1} d\phi_{c2} \exp\left[-\frac{(\phi_{c1}-\phi_{c2})^2}{2\delta_{c\phi}^2}\right] \times \exp\left[-\frac{(\phi_1-\phi_{c1})^2}{2\delta_\phi^2}\right] \exp\left[-\frac{(\phi_2-\phi_{c2})^2}{2\delta_\phi^2}\right] \sim \exp\left[-\frac{(\phi_1-\phi_2)^2}{2(2\delta_\phi^2+\delta_{c\phi}^2)}\right], \tag{A.16}$$

that directly leads to

$$e_s^{LR}(\Delta\phi) \sim \exp\left[-\frac{(\Delta\phi)^2}{2(2\delta_\phi^2+\delta_{c\phi}^2)}\right], \tag{A.17}$$

which corresponds to a LRC, as indicated in the superscript.

A.3. Final expressions

In sum, we find that the SRC and the LRC pieces of the $e_s(\Delta y, \Delta\phi)$ function can be written as

$$e_s^{SR}(\Delta y,\Delta\phi) \sim \exp\left[-\frac{(\Delta y)^2}{4\delta_y^2}\right] \exp\left[-\frac{(\Delta\phi)^2}{4\delta_\phi^2}\right]$$

and

$$e_s^{LR}(\Delta y,\Delta\phi) \sim \exp\left[-\frac{(\Delta\phi)^2}{2(2\delta_\phi^2+\delta_{c\phi}^2)}\right].$$

Note that $e_b(\Delta y, \Delta\phi)$ only retains dependence on the rapidity variable for isotropic cluster production in the transverse plane,

$$e_b(\Delta y,\Delta\phi) \sim \exp\left[-\frac{(\Delta y)^2}{4(\delta_y^2+\delta_{cy}^2)}\right].$$

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