



LOW ENERGY PION-NUCLEON SCATTERING AND THE  
NUCLEON SIGMA TERM <sup>1)</sup>

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**ABSTRACT**

The relation of the  $\sigma$ -term to low energy  $\pi N$  scattering is analyzed with particular emphasis on the critical parameters which need to be better determined to improve its precision. The primary quantities which must be improved and directly determined experimentally so as to sharpen the discussion are the  $\pi^- p$  and  $\pi^- d$  scattering lengths from pionic atoms on the one hand, and the isoscalar  $\pi N$  s-wave range term on the other. The latter requires accurate experiments on differential cross-sections and analyzing power in the pion kinetic energy range 50-100 MeV aimed at determining the s wave energy dependence well enough. In the discussion we make extensive use of the simple approximate relation between the sigma-term and the isoscalar s-wave effective range parameter.

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## 1. The Sigma-Term Problem

From the analysis of  $\pi N$  scattering one can deduce a quantity called the nucleon  $\sigma$ -term  $\sigma_{NN}$ , which can be identified as a measure of the amount of a condensate of  $u$  and  $d$  quarks and antiquarks in the nucleon by the relation:

$$\sigma_{NN} = \frac{m_u + m_d}{2M} \langle N | \bar{u}u + \bar{d}d | N \rangle . \quad (1)$$

A related quantity can also be derived from the mass splittings of the hyperons from the experimental values of  $M_\Lambda$ ,  $M_\Sigma$  and  $M_\Xi$ , but the condensate now also contains the strange quark condensate. Under the assumption, which at first glance seems very natural, that nucleons have no strange quark condensate

$$\langle N | \bar{s}s | N \rangle = 0 \quad (2)$$

one then deduces from the mass formula

$$\sigma_{NN}(\text{mass formula}) \simeq 35 \text{ MeV}. \quad (3)$$

This is in flagrant conflict with the value deduced from  $\pi N$  scattering

$$\sigma_{NN}(\pi N \text{ scattering}) \simeq 60 \text{ MeV}. \quad (4)$$

This is a quite a surprise. There are three main possible ways out of the dilemma.

1. A substantial strange quark component in the proton with  $\langle N | \bar{s}s | N \rangle \neq 0$ . If we measure the fraction of the strange quark condensate by the quantity  $y$  defined as

$$y = 2 \frac{\langle N | \bar{s}s | N \rangle}{\langle N | \bar{u}u + \bar{d}d | N \rangle}, \quad (5)$$

then the value of the  $\sigma$ -term is expected to be

$$\sigma_{NN} = \frac{\sigma_{NN}(\text{no strangeness})}{1 - y}. \quad (6)$$

The value of 60 MeV requires about 20 to 25% of strange quarks in the proton meson cloud. This explanation was originally advocated by Donoghue and Nappi[1] and has been vigorously pursued in the literature in the last years by various authors. Such a large value for the strange condensate runs of course against our naive and immediate expectation that the proton is basically a non-strange object. If this can be shown to be a correct conclusion, we have thus learned something quite important. There are various problems with so much strangeness, however. The most important one is that the large mass of the strange quark has the consequence that an important part of the nucleon mass must be ascribed to it[2][3].

2. The extrapolation procedure to the soft point may not be sufficiently well under control. This issue will be discussed in this workshop by Dr. Sainio [4]. Here I will give a pedagogical exposition of the key low-energy  $\pi N$  scattering parameters which determine the  $\sigma$ -term so that you can judge the essential physics involved.

3. Finally, there may be experimental uncertainties that are not fully recognized, and key experiments that need to be performed. Which is the important experimental input in the determination, at which points can systematic errors in the analysis bias the results, and which are the experiments most vitally needed to achieve accuracy?

## 2. Definition of the Nucleon $\sigma$ -Term

The  $\sigma$ -term is defined as the nucleon expectation value of the commutator of the axial charge and its time-derivative taken in the isospin symmetric combination

$$\sigma_{NN} = -i \langle N | [Q_5, \dot{Q}_5] | N \rangle^{(+)} . \quad (7)$$

Here,  $Q_5(t)$  is the axial charge which, in complete analogy with the ordinary charge, is the volume integral of the axial charge density  $A_o$

$$Q_5(t) = \int dx A_o(x, t). \quad (8)$$

If the axial current is conserved, as it is in the case of exact chiral symmetry, we have the axial continuity equation

$$\partial_\mu A_\mu = 0, \quad (9)$$

from which we conclude immediately, as for the ordinary charge, that the axial charge is conserved in the chiral limit:

$$\dot{Q}_5(t) = 0. \quad (10)$$

Consequently it follows from the definition of the  $\sigma$ -term that it also vanishes in the same limit:

$$\sigma_{NN}(\text{chiral limit}) = 0. \quad (11)$$

This tells us that the  $\sigma$ -term differs from zero only because chiral symmetry is broken because the  $u$  and  $d$  quark masses  $m_{u,d} \neq 0$  or, alternatively, because  $m_\pi \neq 0$ . Consequently, the  $\sigma$ -term is basically a small quantity of order

$$\sigma_{NN} = O(m_\pi^2). \quad (12)$$

## 3. Model for Chiral Symmetry Breaking

Inside QCD the customary way to break chiral symmetry is explicitly via the quark masses. The chiral symmetry-breaking part of the Lagrangian is then the mass term

$$L' = \bar{q} M q \equiv m_u \bar{u} u + m_d \bar{d} d + m_s \bar{s} s . \quad (13)$$

with the  $u$  and  $d$  quark masses

$$m_{u,d} \neq 0. \quad (14)$$

To lowest order, the nucleon consists of the simple 3-quark configuration  $|qqq\rangle$ , but it is dressed up by  $\bar{q}q$  pairs which, to a nuclear physicist, look very familiar as being analogous to the usual particle-hole expansion in nuclei. The nucleon wave-function in quark terms has then the schematic structure

$$|N\rangle = |qqq\rangle + |(\alpha \bar{u} u + \beta \bar{d} d + \gamma \bar{s} s) qqq\rangle + .. \quad (15)$$

The higher order terms represent the quark condensate' or the meson cloud' of the nucleon. If we now substitute this symmetry-breaking Lagrangian into the definition for  $\sigma_{NN}$ , the time-derivative of the axial charge no longer vanishes. After some algebra, one finds

$$\sigma_{NN} = \frac{m_u + m_d}{2M} \langle N | \bar{u} u + \bar{d} d | N \rangle . \quad (16)$$

This is the form very often given in the literature instead of the basic definition.

#### 4. The $\sigma$ -Term and $\pi N$ Scattering

Consider the scattering of a pion from a nucleon as schematically defined in Figure 1. The scattering amplitude is a function of the incoming and outgoing pion momenta  $q_\mu$  and  $q'_\mu$  as well as the nucleon 4-momenta  $p_\mu$  and  $p'_\mu$ . In other words, it depends on the usual relativistic variables  $s$  and  $t$  together with  $q^2$  and  $q'^2$ . These variables are clumsy for the discussion of low-energy pion-nucleon physics, as the nucleon mass is large compared to the pion mass and to the pion momenta. One therefore introduces another invariant quantity  $\nu$  instead of  $s$ , such that  $\nu$  is nearly identical to the pion energy in the center-of-mass system. It is convenient in the discussion of the soft limit, characteristic of the  $\sigma$ -term, to subtract the pseudo-vector Born amplitude according to the figure (it does not matter if one instead subtracts the pseudoscalar Born terms, but the mathematics is simpler if one takes off the pseudovector one). This defines the amplitude  $\bar{F}$  which is the one we will work with:

$$\bar{F} \equiv F - F_{Born}^{(pseudovector)}. \quad (17)$$

$$\nu = \frac{PQ}{M};$$

$$P_\mu = \frac{(p+p')_\mu}{2};$$

$$Q_\mu = \frac{(q+q')_\mu}{2};$$

$$t = (q - q')^2;$$

$$\nu_B = \frac{t - q^2 - q'^2}{4M} = -\frac{qq'}{2M}.$$

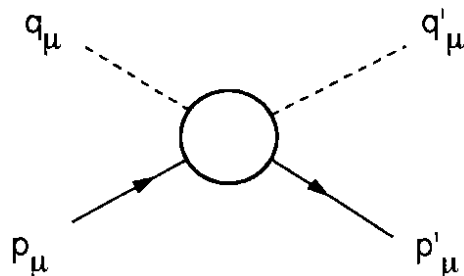


Fig. 1

Using the PCAC relation [5] one can now demonstrate that  $\sigma_{NN}$  is exactly equivalent to the isoscalar spin-averaged  $\pi N$  scattering amplitude at the 'soft point' with  $q_\mu = q'_\mu = 0$ .

$$\sigma_{NN} = -f_\pi^2 \bar{F}^{(+)}(soft). \quad (18)$$

This is all well and good, but the problem now is how to connect this soft point, for which the pion is unphysical with  $\nu = t = 0$ ;  $q^2 = q'^2 = 0$ , to real physical  $\pi N$  scattering which has  $q^2 = q'^2 = m_\pi^2$ . The standard trick is to make use of a physical point at which the value of scattering amplitude with  $q^2 = q'^2 = m_\pi^2$  approximately reproduces the soft value. Although this point is also unphysical, it has the virtue that the analytical methods for extrapolating scattering amplitudes can be applied in a relatively direct way. The remaining problem is then to achieve as good a connection between this value and the true value for the  $\sigma$ -term. A special point with such properties has been known for quite some time and is referred to as the Cheng-Dashen point [6]. It corresponds to the following values for the variables:

$$\nu = 0; \quad t = +2m_\pi^2; \quad q^2 = q'^2 = m_\pi^2. \quad (19)$$

At the Cheng-Dashen point we have the following approximate relation:

$$f_\pi^2 \bar{F}^{(+)}(Cheng - Dashen) \equiv \Sigma \simeq \sigma_{NN}. \quad (20)$$

Until recently it was generally believed that the two quantities  $\Sigma$  and  $\sigma_{NN}$  differed by only 5% or so, i.e. by a few MeV, but a recent more detailed investigation by Gasser and Leutwyler [3] indicates that the correction is much larger, over 10 MeV. The new value deduced from experiments is now  $45 \pm 7$  MeV, which is in much better agreement with the predicted value 35 MeV. This is caused by the  $2\pi$  channel and the details will be reported on by Dr. Sainio[4].

What is so special about the Cheng-Dashen point? It is well known that at this point the dynamical contributions from the  $\Delta$  isobar are eliminated to high precision. The origin is readily seen by examining the amplitude not in terms of  $\nu$  and  $t$  but in terms of the the  $\pi N$  scattering angle  $\theta$  and momentum  $\mathbf{q}$  in the center-of-mass system. The scalar product  $\mathbf{q} \cdot \mathbf{q}' \simeq 0$  at this point, which means that

$$\cos \theta \simeq 0. \quad (21)$$

Therefore, there are approximately no  $p$ -wave contributions at the Cheng-Dashen point, i.e. no  $\Delta$  contribution. This important feature of no  $p$  waves with  $\mathbf{q} \cdot \mathbf{q}' = 0$  is characteristic of all the four so-called special points: the soft point has the momenta  $\mathbf{q}$  and  $\mathbf{q}'$  both vanishing; the so-called Adler point has one of the momenta  $\mathbf{q}$  and  $\mathbf{q}'$  vanishing; the Cheng-Dashen point has  $\cos \theta$  vanishing and the threshold point has of course no  $p$  wave, so  $\mathbf{q} = \mathbf{q}' = 0$ . This suggests that it should be valuable to examine the relation of  $\sigma_{NN}$  to the low-energy  $s$  wave scattering parameters as well as the constraints which present experiments directly impose on these quantities. In order to be quantitative, let us examine the structure of the isoscalar low-energy  $\pi N$  amplitude in the center-of-mass system [7][8]

$$\bar{F}^{(+)} = 4\pi \frac{\sqrt{s}}{M} (\bar{a}^{(+)} + \bar{b}^{(+)} \mathbf{q}^2 + \dots \bar{c}^{(+)} \mathbf{q} \cdot \mathbf{q}' \dots). \quad (22)$$

Here the scattering length is very small

$$\bar{a}^{(+)} \simeq 0.$$

The effective range parameter  $\bar{b}^{(+)}$  turns out to be mainly responsible for the value of the  $\sigma$ -term. It is moderately large and only moderately well determined. In fact, it is not measured, but theoretically deduced from experiments using dispersion theory.

Finally, the  $p$  wave scattering volume  $\bar{c}^{(+)}$  is large and dominates much of low-energy pion scattering. It is reliably deduced from the theoretical dispersion analysis, but is of only secondary importance for the  $\sigma$ -term.

If we now make a very rough and cavalier extrapolation to the Cheng-Dashen point which has  $\nu = 0$ ;  $\cos \theta \simeq 0$  and  $\mathbf{q}^2 \simeq -m_\pi^2$ , we have roughly <sup>2)</sup>

$$\Sigma \simeq \sigma_{NN} \simeq f_\pi^2 4\pi (\bar{a}^{(+)} - \bar{b}^{(+)} m_\pi^2 + \dots). \quad (23)$$

2) I have given a more exact expression in eq. (16) in the cited paper [7]

$$\Sigma \simeq \sigma_{NN} \simeq f_\pi^2 4\pi (\bar{a}^{(+)} - [\bar{b}^{(+)} m_\pi^2 - (m_\pi / (M + m_\pi) \bar{c}^{(+)} \dots]).$$

This takes into account the  $m_\pi/M$  correction from the large  $p$  wave amplitude  $c_0$ . A nearly identical expression has been independently obtained by Gasser [9] as part of a sum-rule for a special value of an arbitrary parameter ( $t = 0$ ). It has been stated erroneously by Höhler [10], and several other authors subsequently, that as early as 1980 Olsson and Osypowski [11] introduced the range term in this context. They only

Let us see how that looks in the standard dispersion relation approach. Here, one uses the extrapolation of the amplitude  $\bar{D}^{(+)}(\nu^2, t)$  which only contains even powers of the variable  $\nu$ . Koch [12][13] has given a low-energy expansion in  $\nu^2$  and  $t$  with numerical coefficients which depend on the specific analysis but which give an idea about the magnitude of the various terms. It is reproduced in the table below.

1	$t$	$\nu^2$	$\nu^2 t$	$\nu^4$	$t^2$
$-1.46 \pm 0.10$	1.14	1.12	0.17	0.20	0.036

Let us now regroup these terms using Breit-frame variables for the pion momenta and scattering angles. The Breit-frame is the coordinate system with no energy loss, which for an infinitely heavy scatterer coincides exactly with the center-of-mass frame. For the  $\pi N$  system, which has the nucleon much heavier than the pion, it is nearly coincident with the center-of-mass frame, but there are some corrections which cannot be neglected. In the Breit frame the relativistic variables equivalent to  $\nu$  and  $t$  are:

$$\mathbf{q}_B^2 \equiv \nu^2 - m_\pi^2. \quad (24)$$

$$(\mathbf{q}_B \cdot \mathbf{q}'_B - \mathbf{q}_B^2) \equiv q_B^2 \cos \theta_B \equiv \frac{t}{2} + \nu^2 - m_\pi^2. \quad (25)$$

The identical table in these new variables takes the following form:

1	$\mathbf{q}_B \cdot \mathbf{q}'_B$	$\mathbf{q}_B^2$	$(\mathbf{q}_B \cdot \mathbf{q}'_B)(\mathbf{q}_B^2)$	$(\mathbf{q}_B^2)^2$	$(\mathbf{q}_B \cdot \mathbf{q}'_B)^2$
$-0.14 \pm 0.10$	2.22	-1.10	0.05	0.00	0.14

In this identical regrouping of the expansion there are only two important coefficients: the Breit-frame  $p$  wave parameter and the the  $s$  wave range parameter. In addition, the scattering length plays a rather important role, since it has a large uncertainty. This confirms the qualitative argument about the importance of the  $s$  wave range parameter I made above.

Traditionally all of the low-energy quantities are obtained from partial-wave dispersion relations rather than from direct measurements; this situation is presently changing with the direct determination of the  $\pi N$  scattering lengths from the pionic hydrogen atom, as we have heard from Dr. Badertscher [14][15]. Very recently there has been an interesting theoretical progress in this area: Gasser *et al.* [16][17] have shown that all the 6  $s$  and  $p$  wave amplitudes can be determined from 6 near forward dispersion relations with the additional knowledge of two threshold constants:  $a_0^+$  and  $a_{1+}^+$ , which is more direct than previously. Still, the  $\Sigma$  term is not determined by immediately identifiable and measurable physical quantities with the exception of the scattering length. It is this fact that brought me to developing the physical picture linking it to the range term [8]. Höhler [10] has criticized this approach as being less accurate than the dispersion extrapolation and with no advantage. His argument is essentially that the low-energy parameters are derived using the dispersion approach anyway. Therefore, if the range parameter or a closely related  $s$  wave quantity cannot be directly obtained experimentally, then he is right and my approach is superfluous. I will now show that this is not the case.

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parametrize the low-energy physics and the parameter enters as part of a dispersive treatment.

The Isoscalar scattering length and range term are clearly the key quantities to measure. How are these obtained and how accurately? According to Höhler's monograph on  $\pi N$  Scattering [13] one has by the analyticity evaluation of the data before 1983 and using only data above 90 MeV the values:

$$a^{(+)} = -0.010 (3) m_{\pi}^{-1}; b^{(+)} = -0.044 (7) m_{\pi}^{-3}. \quad (26)$$

The uncertainties of each add in quadrature and each unit represents approximately 1 MeV in the  $\sigma$ -term. At the time the uncertainty from experiments in the  $\sigma$ -term was evaluated to be about 7 MeV, not counting systematics. The main uncertainty is thus associated with the range term. The isoscalar scattering length  $\bar{a}^{(+)}$  is best determined from the  $1s$  level shift in pionic hydrogen or deuterium and not from scattering experiments. This determines the  $\pi^{-}p$  and the  $\pi^{-}d$  scattering lengths. In the first case, the value of  $a^{(+)}$  is obtained as the difference of two large numbers

$$a^{(+)} = a^{\pi^{-}p} - a^{(-)}. \quad (27)$$

Here the isovector  $\pi N$  scattering length is either obtained directly from the well understood forward dispersion relation with  $a^{\pi^{-}p} = 0.091 (2) m_{\pi}^{-1}$  or from the Panofsky ratio (which requires the precision evaluation of the photo-production amplitude) with  $a^{\pi^{-}p} = 0.088 (2) m_{\pi}^{-1}$ . Personally I prefer the first one as the more direct one. The first measurements of the pionic hydrogen  $1s$  energy shift by Bovet *et al.* [18] gave quite surprising results far outside the range of expected values:  $a^{\pi^{-}p} = 0.060(6) m_{\pi}^{-1}$  corresponding to  $a^{(+)} = -0.021 (6) m_{\pi}^{-1}$ . This was at variance with the Bovet group's own measurement of the  $1s$  energy shift in pionic deuterium as well as with all the nuclear physics experience with this key quantity and in particular with data on the  $1s$  shift in pionic  ${}^3\text{He}$ . This problem now seems resolved with the new measurement by Beer *et al.* [15], which gives  $a^{\pi^{-}p} = 0.086 m_{\pi}^{-1}$ , so that

$$a^{(+)} = 0.086 (4) - 0.091 (2) = -0.005 (4) m_{\pi}^{-1},$$

in agreement with the analyticity value to two standard deviations. The uncertainty in the  $\sigma$ -term from this source is 4 MeV and the experiment can be improved further. About 2 MeV comes from the subtraction procedure. The precision in the isovector scattering length can hardly be much improved, since it is deduced to 2% already and the validity of isospin symmetry and Coulomb corrections will be delicate. The  $\pi^{-}$  deuteron scattering length is a better prospect for improved precision and should be experimentally easier. The price to pay in that case is that one must live with a strong-interaction double scattering correction which is supposed to be well under control. The advantage of the deuteron is that the leading term is directly proportional to  $a^{(+)}$ . The delicate procedure of taking the difference between two large quantities is then circumvented:

$$a(\pi^{-}d) = 2a^{(+)} + (\text{double scattering}) \simeq -0.021 - 0.026.$$

The correction term is not very large. It is well described both by a 3-body Faddeev approach as well as by double scattering theory. Therefore, a 5% precision should readily be obtainable in the term  $a(\pi^{-}d)$ . At this level, dispersive corrections due to absorptive effect can no longer be neglected and these are less controllable. A precision experiment

on the deuteron will require additional theoretical analysis, but at the moment this is not the issue. The deuteron permits a precision of  $\pm 0.001$  in  $a^{(+)}$  or 1 Mev in the  $\sigma$ -term before systematics set in, twice as accurate as the value possible using hydrogen. Such a measurement would of course be valuable for the applications of pions to nuclear physics, since the isoscalar scattering length is also a key quantity in the pion optical potential. The priority is therefore the pionic deuteron level shift as the superior method, and only then the pionic hydrogen shift. With the recent remeasurement of pionic hydrogen the uncertainty in  $\sigma$ -term is no longer dominated by the the scattering length but by the the range term.

The ideal experiment for determining the range term as directly as possible would in principle be elastic scattering of (tagged)  $\pi^0$  from protons, since this directly gives the isospin symmetric amplitude. While this is not a totally inconceivable enterprise with storage rings like CELSIUS and  $4\pi$  detectors, the difficulties are such that it is presently far preferable to accept the onus of constructing the correct combination of amplitudes by combining  $\pi^+$  and  $\pi^-$  proton scattering. The price is that the cancellation at threshold of the  $\pi^+p$  and  $\pi^-p$  scattering lengths is no longer automatically included in the experiments.

The range parameter  $b^{(+)}$  is presently deduced by dispersion techniques to be

$$b^{(+)} = -0.044 (7) m_{\pi}^{-3},$$

i.e. with an uncertainty that contributes 6 Mev to  $\sigma_{NN}$ . A major cause of the uncertainty is the normalization of the total cross-section in the  $\Delta$  region which is about 1.5%. This normalization is determined by one single experiment from the early '70s by Carter *et al.* [19]. Although this experiment as such is excellent, it certainly requires a modern check in view of its central importance. On the other hand, we have already reached a level at which isospin violation effects begin to turn up. This will sharply limit the possibility of improving the precision in  $b^{(+)}$  using this technique. This points to the necessity of determining  $b^{(+)}$  or a related quantity as directly as possible. Since the  $\pi^0$  option is not suitable, this means that elastic  $\pi^{\pm}$  scattering must be exploited. The  $s$  wave scattering will approximately contain the combination

$$a + bq^2.$$

Here, the two terms do not produce physically distinguishable effects except for the energy variation. Since the scattering lengths will be determined with high precision, the point will be to make the sensitivity to  $b$  sufficient. At low energy, the range term is small compared to the scattering length and scattering experiments add little new. In the 50 MeV region and above, the range term is comparable to the scattering length. Above 100 MeV the deviation from the scattering length value becomes more substantial, but the extrapolation to the threshold region is more delicate. The schematic differential cross-section for unpolarized targets has the following form in the low-energy region, where the amplitudes are nearly real:

$$\frac{d\sigma}{d\Omega} \simeq [(a + bq^2 + cq \cdot q' \dots]^2 + \text{spin flip term}. \quad (28)$$

This means that the interference term  $2(a + bq^2)cq^2 \cos \theta$  carries the main information on the range term. The correction terms do not cause much problem. The relative



importance of the different terms in the spin-averaged amplitudes are readily apparent using the Koch 80 solution [13] as a guide (note that  $q^2 = 1$  corresponds to 50 MeV pions):

$$\pi^0 p: -0.010 - 0.045q^2 + 0.020q^2 \cos \theta. \quad (29)$$

$$\pi^- p: -0.082 - 0.031q^2 + 0.034q^2 \cos \theta. \quad (30)$$

$$\pi^+ p: -0.101 - 0.058q^2 + 0.384q^2 \cos \theta. \quad (31)$$

Clearly, neutral pions would be ideal in principle. Not only are there no Coulomb corrections, but the  $s$  wave term is dominated by the wanted isoscalar range term. The quantity

$$2(a + bq^2)c,$$

which is the coefficient of the  $\cos \theta$  term in the angular distribution can be used directly to determine the coefficient  $b$  in the different cases. Both for neutral and positive pions the  $p$  coefficient  $c$  is under good control and has rather small uncertainties; for negative pions the situation is less clear. The  $p$  wave coefficient is then small, no longer  $\Delta$  dominated and with considerable cancellations.

For  $\pi^0$  scattering an angular coefficient determined to better than 13% will match the present 7 MeV uncertainty in the  $\sigma$ -term.

For charged-pion scattering the range terms contribute half of their precision to the  $\sigma$ -term. The  $\pi^+ p$  range term must thus be extracted to a precision of better than 10 to 15%, i.e. 0.005 to 0.008  $m_\pi^{-3}$  units, in the 50 to 100 MeV range in experiments determining the  $\cos \theta$  term. This is perfectly reasonable and which can be improved. The problem is the  $\pi^- p$  range term. Here, the uncertainty in the  $p$  wave coefficient makes the procedure dubious. The phenomenological uncertainty in this term is optimistically at least 12% and very likely the double. This means that even with a perfectly determined interference term it contributes 4 to 5 MeV to the error of the  $\sigma$ -term. In real life the  $\pi^-$  data contribute even more to the error. This unfortunate circumstance necessitates the measurement of the analyzing power in the  $\pi^- p$  scattering, since this is the quantity that carries the missing information. In this case the trouble is that the energy of the pion must be kept rather low, since otherwise the analyzing power will be dominated by the interference of  $p$  waves, which do not carry the information we are after. To be successful the energy in the polarization experiments must be about 50 to 70 MeV. This makes, on the one hand, the range term large enough, and on the other one, keeps the  $p$  wave contribution small enough. A precision in the analyzing power of 20% or better suffices to improve the value of the  $\pi^- p$  range term and to improve the  $\sigma$ -term.

A totally different road has been taken in the last couple of years based on the idea that the Coulomb interference can be used to determine the forward amplitude directly by a relatively minor extrapolation. In this case it is also possible to observe the  $\pi^\pm$  interference under identical conditions, and deduce directly the average of the  $\pm$  amplitude, i.e. the isoscalar amplitude. The first measurement of this amplitude at 50 MeV [20] together with the isoscalar forward dispersion relation made it evident that the  $\pi^- p$  atom data of Bovet *et al.* [18] were erroneous. What are the prospects for improvements using such experiments. As for all scattering data, there is no chance to compete with the upcoming precision determinations of the scattering lengths from atoms. They must

be judged on the information they can give on the  $s$  wave energy variation, i.e. the range parameter  $b^{(+)}$ . The measured amplitude is essentially proportional to

$$4\pi[a^{(+)} + (b^{(+)} + c^{(+)})q^2] = 4\pi[-0.010 + (-0.046 + 0.201)q^2] m\pi^{-1}.$$

To pin down  $b^{(+)}$  to beyond the present 15% level the amplitude must be determined to a precision  $0.09 q^2 m_\pi^{-1}$  or  $0.6 q^2 \text{ GeV}^{-1}$ . This is close to the present precision in the experiment of Wiedner *et al.* [20][21], so there is some hope of possible improvement. However, it means that the total forward amplitude is already determined to 5% precision, so the room for improvement may not be great.

In conclusion, there have recently been advances in the determination of the  $\sigma$ -term both on the theoretical and experimental side. The critical re-examination of the extrapolation procedure to the soft point by Gasser and Leutwyler has brought down the deduced value by nearly 12 MeV to 45 MeV[3]. This is only 2 standard deviations from the theoretically predicted value of 35 MeV. There is no longer a flagrant discrepancy between experiment and theory and the need for an unusual strangeness content in the proton is now much less pressing. Still, the issue of the value of the  $\sigma$ -term remains an important one. The crucial experiments to improve it are first the level shifts in hydrogenic deuterium and hydrogen, which will give very accurate values for the scattering lengths. The second are associated with the direct determination of the isoscalar range term which is needed to a precision better than 15%. Part of this information can be obtained from  $\pi^+p$  scattering in the 50 to 100 MeV range on an unpolarized target. It will be difficult to extract the complementary information from the  $\pi^-p$  unpolarized scattering. This calls for measurements of the analyzing power in the 50-70 MeV range, in particular for  $\pi^-p$  scattering. An alternative method is the Coulomb interference measurement of forward-scattering amplitudes, but the required precision is then higher. Only an accurate determination of the  $\sigma$ -term will permit a serious comparison of prediction to reality.

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