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Synergy of BESIII and LHCb physics programmes

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Abstract

There is potential for BESIII open-charm measurements to have a significant impact on the LHCb physics programme. Despite the general purpose design of the LHCb detector there are certain inputs that can be better determined in other environments or in production mechanisms not accessible at the LHC. With the unprecedented amount of LHCb data that will become available over the one-to-two decades it is necessary to consider where additional inputs are essential, to avoid the situation where the uncertainty on a measurement is dominated by the lack of knowledge of an external input. This document considers the capabilities of the BESIII experiment to provide vital inputs into key LHCb measurements. A number of different potential measurements that could be pursued are discussed.

1 Executive summary

There are at least two vital sets of inputs to the LHCb physics programmes that are most easily accessed using the open-charm data accumulated at BESIII. The first is composed of strong-phase related measurements of neutral D mesons, whose knowledge is vital for a high precision measurement of the CKM angle γ. The use of existing CLEO-c measurements has allowed LHCb to include a wide range of D modes in its determination of the CKM angle γ using $B \to DK$ decays, which together have made possible the currently achieved precision on this fundamental parameter of 8° [1]. These CLEO-c measurements themselves contribute an uncertainty of approximately 2° which, while small compared to the current statistical uncertainties, will soon become dominant as the LHCb rate of data collection increases.

The $\Psi(3770)$ data already accumulated by the BESIII experiment are sufficient to bring the contribution from strong-phase uncertainties to approximately 1°. The goal of the LHCb phase-1 upgrade, due to start collecting data in 2021, is to measure γ with a precision of 1°, which will approximately match the uncertainty on this parameter from indirect constraints from other measurements. Therefore it would be most desirable to have strong-phase measurements from an even larger dataset so that the uncertainty from external inputs does not become dominant. For this reason the existing plan to take another 15-20 fb^{-1} at this resonance would be very important and is highly encouraged. With this amount of data, the strong-phase contribution to the uncertainty on γ will remain sub-dominant throughout the physics programme of the approved LHCb phase-1 upgrade. Plans are also being formulated for a second, phase-2 upgrade, which would accumulate a much larger data set. In this case the input from a 15-20 fb^{-1} $\Psi(3770)$ sample would be essential.

The second family of measurements required for LHCb physics that can be performed at BESIII involves branching fractions of certain charm hadron decays. Improvements in the knowledge of branching fractions for Cabibbo favoured decays of all charm hadrons are crucial since they are already, or will become, the leading uncertainty the measurements of b-hadron branching fractions. An example of this is the current knowledge of the $\Lambda_c \to pK\pi$ branching fraction, which leads to the dominant uncertainty in the LHCb measurement of the ratio $|V_{ub}|/|V_{cb}|$ [2]. Advances in the knowledge of baseline branching fractions will become more important as statistical uncertainties, systematic uncertainties, and uncertainties on theoretical inputs continue to reduce over the next two decades. Furthermore, measurements of ratios of branching fractions in targeted charm-hadron modes, that are difficult to observe directly at LHCb but have the ability to form large backgrounds, can also have a significant impact in reducing uncertainties for interesting LHCb measurements.

The LHCb physics programme would gain significantly by a range of improved and new measurements in these two areas. Further details are given within the note.

2 Introduction

Precision measurements of heavy flavour observables can constrain models of physics beyond the Standard Model and have the potential to allow the observation of the manifestation of new physics particles through their virtual contributions to heavy-quark transitions. While the LHCb experiment has unique capabilities to perform many interesting measurements, a number of them do require some experimental input from other sources. These external inputs fall into two general categories. The first comprises strong phase differences in neutral D-meson decays, knowledge of which is required for precision measurements of the CKM angle γ and also for indirect CP-violation measurements in charm mixing. The second comprises measurements of absolute and relative branching fractions of charmed-hadrons used throughout the LHCb experimental programme. While LHCb is a dedicated flavour physics experiment, there are instances where the production process or detector capabilities prevent precision measurements of these inputs. This document aims to detail some of the measurements that are possible with data from the the BESIII experiment and would be particularly beneficial to the LHCb physics programme.

3 Datasets, timescales and sensitivity projections

For the purposes of this document it is useful to set out some initial assumptions on which further discussions will be based. These are given in Table 1 which details the past, current, and future LHCb run periods and estimates by what factor the cumulative yields of hadronic B decays will increase in comparison to those obtained within the Run 1 dataset. These increases take account of the growth in integrated luminosity, the change in the $b\bar{b}$ cross-section with energy, and the higher trigger efficiency that will be available at the upgrade.

Table 1 includes an entry for a phase-2, high-luminosity upgrade, which is a project currently at the conceptual level. Table 1 also provides the timescale by which these yields will be collected.

Table 1: Projected increases of hadronic B decay yields for Run 2, the phase-1 LHCb upgrade and potential phase-2 upgrade along with the timescales on which those yields could be attained.

Run Period [E_{CM}]	Collected / Projected luminosity per run	Cumulative yield factor compared to Run 1	Year attained
Run 1 [7,8 TeV]	3 fb^{-1}	1	2012
Run 2 [13 TeV]	5 fb^{-1}	4	2018
LHCb phase-1 upgrade [14 TeV]	50 fb^{-1}	60	2030
LHCb phase-2 upgrade [14 TeV]	300 fb^{-1}	$\sim$400	2035(?)

The general assumptions are that in 2019, when the Run 2 data have been analysed the LHCb statistical uncertainties will halve compared to Run 1 and that on the order of decade they will reduce by a factor 7 [3]. Naturally there are some differences in the precision of external inputs required for these two periods. For the phase-2 upgrade it is possible that for several measurements the systematic uncertainties will become dominant, and hence the best possible precision on external inputs is required.

The inputs that would be useful for studies at LHCb are discussed in light of the datasets already accumulated at BESIII. This includes the dataset at the $\psi(3770)$ ($2.9\,\mathrm{fb}^{-1}$) which is almost 4 times as large as that recorded by CLEO-c ($0.8\,\mathrm{fb}^{-1}$). It is understood that the plans for BESIII include a further larger 10-20 fb^{-1} dataset at the $\psi(3770)$ and an upgrade in collision energy which would allow a significantly larger Λ_c dataset to be recorded.

4 Importance of strong-phase related measurements at BESIII

The $\psi(3770)$ dataset collected at BESIII allows for direct measurement of strong-phase related parameters in a variety of D meson decays. There are two uses of D meson strong-phase related parameters at LHCb: the measurement of the CKM angle γ and the search for indirect CP violation in charm mixing. For states that are not self-conjugate, the coherence factor, R, and average strong-phase difference, δ, can be measured [4]. For self-conjugate states there are two choices: either a measurement of the CP-even fraction, F_+ [5], or a measurement of the amplitude weighted average cosine and sine of the strong-phase difference, c_i and s_i, where the index i refers to the region of phase space of the given multibody decay [6]. It is also possible to measure R, δ, or F_+ in regions of phase space. This can be advantageous for measurements of γ if the region has high coherence, or is strongly CP even or CP odd. The choice between either F_+ or c_i and s_i depends on the CP content, and on the sensitivity to s_i in the data. For LHCb it may well be advantageous to have both, as decay channels with lower yields may prefer to use the simpler F_+.

4.1 CKM angle γ

The CKM angle γ is the least well known of the angles of the unitarity triangle and the only one that is accessible with tree-level decays in a theoretically clean way. It provides a Standard Model benchmark against which other measurements can be compared. The precise measurement of this parameter is a top priority for LHCb. One of the key methods to measure γ that the experiment exploits is through the interference between $B^+ \to \bar{D}^0 K^+$ and $B^+ \to D^0 K^+$ decays which occurs if the final state of the charm-meson decay is accessible to both the D^0 and $\bar{D}^0$ mesons (hereafter designated by D). While any hadronic final state is accessible to both flavours, in practice some are more attractive options than others. Hence there are a number of D modes that could be used, and furthermore, the analyses can be extended to other similar B modes such as $B^0 \to DK^{*0}$, $B \to DK\pi\pi$, and $B \to DK^{*+}$. It should also be noted that other important methods exist that are independent of D decays, for example a time dependent analysis of the family of modes $B_s, \bar{B}_s \to D_s^{\pm} K^{\mp}$. To relate the experimental observables of CP violation with the CKM angle γ, knowledge of the D meson decay, and in particular on the strong-phase related observables, is required. The use of direct measurements of the strong-phases (as opposed to those derived from amplitude models) is essential for the measurement of γ to remain free of the uncertainties associated with amplitude models, which are very difficult to assign reliably. To achieve the best sensitivity to γ, a large variety of D and B decay modes will be required. Strong-phase information in the following D decay modes is available from the CLEO-c data and has been used in the most recent combination of γ:

- measurement of c_i and s_i in $D \to K^0_S\pi^+\pi^-$ and $D \to K^0_S K^+K^-$ [7];
- measurement of the coherence factor and average strong-phase difference in $D \to K^\pm\pi^\mp\pi^+\pi^-$ and $D \to K^\pm\pi^\mp\pi^0$ [8];
- measurement of the coherence factor and average strong-phase difference in $D \to K^0_S K^\pm\pi^\mp$ [9];
- measurement of the CP even content of $D \to \pi^+\pi^-\pi^+\pi^-$, $D \to \pi^+\pi^-\pi^0$, and $D \to K^+K^-\pi^0$ [10];
- measurement of the strong-phase difference in $D \to K^\pm\pi^\mp$ [11].

In the case of the $D \to K^\pm\pi^\mp\pi^+\pi^-$ decay, complementary strong-phase information comes from LHCb measurements of charm-mixing [12], and in the case of the $D \to K^\pm\pi^\mp$ decay the strong-phase information is dominated by charm mixing measurements from LHCb, CDF, and the B factories which are averaged by HFAG [13].

The precision of the existing measurements of the $D \to K^\pm\pi^\mp$ strong-phase performed by the BESIII collaboration [14], and that of the preliminary measurements of the strong-phases in $D \to K^0_S\pi^+\pi^-$ [15], demonstrate how powerful the full suite of strong-phase measurements from BESIII could be.

4.1.1 Impact of current strong-phase knowledge on the uncertainty of γ

It is useful to assess the contribution of the uncertainties of the CLEO-c inputs on the current LHCb uncertainty on γ. Studies in Ref. [7] conclude that the limiting uncertainties would be of order 2-4° for measurements that use the $D \to K^0_S\pi^+\pi^-$ final state, where the range is dependent on the binning scheme used. However, this decay mode, although very important, is only one of many used in the global γ combination. Limiting uncertainties due to other CLEO-c measurements have not been previously discussed.

The determination of γ is made by combining the observables from the different D and B modes. The uncertainty due to the CLEO-c inputs on the parameter γ is estimated for the LHCb Run-1 data with a simple study and is found to be 1.9°. For Run-2 data this uncertainty is expected to be between 1.7 and 2.2° and for the LHCb phase-1 upgrade this is expected to be between 1.8 and 2.5°. The range in values, both within and between different datasets, is due to differing assumptions on the evolution of the LHCb experimental systematic uncertainties of these measurements, which in turn change the relative weighting of different D modes in their contribution to a combined γ measurement. For the subsequent discussion, it is assumed that the CLEO-c inputs contribute 2° uncertainty to the overall error on γ.

- The Run-1 precision is approximately 8° and is driven by the LHCb statistical uncertainties. In this case, the impact of a 2° uncertainty from the strong phases only has a very small impact on the total uncertainty on γ.

- For Run-2 the anticipated statistical uncertainty is less than 4° due to the four-times larger data set and to additional B and D decay modes that will be analysed. The impact of a 2° uncertainty from the strong-phase measurements will start to have a noticeable impact.

- One of the aims with the LHCb phase-1 upgrade dataset is to reach a degree-level precision. This matches the precision of lattice-QCD uncertainties in performing an indirect measurement of γ using CKM parameters dominated by loop decays. However, the current knowledge on the strong-phase measurements from CLEO-c data is about twice the desired overall precision. Hence, improved strong-phase and coherence factor information is essential to allow this goal to be met.

- Assuming that the strong-phase and coherence-factor uncertainties can be reduced to a sub-dominant level for the LHCb phase-1 upgrade analyses, the aim of a phase-2 upgrade would be to improve further the precision on γ, and be able to cross-check different decay modes against each other. To gain the most from this, the external input of the strong-phase information must be as precise as possible.

The improvements in the precision of the strong-phase and coherence-factor information can be made in three ways. They are

- Use the B data to improve the knowledge on these parameters at the same time as determining γ. This is possible since the system of LHCb observables is over-constrained, as there are more observables than physics parameters and constrained inputs.

- Use LHCb charm-mixing measurements to determine better the strong-phase and coherence-factor parameters.

- Improve the knowledge of strong-phase and coherence-factor parameters via direct measurement performed with the larger $\psi(3770)$ samples that are available now and/or in the future at BESIII.

Of these, the first option is furthest from ideal, since there will be a negative impact on the precision on γ if the B data is simultaneously improving the charm strong-phase information. The second remains a possibility and in certain cases can provide orthogonal and valuable information [12] to the $\psi(3770)$ data. While these opportunities will be explored, preliminary studies show that to improve upon CLEO-c uncertainties, a much larger dataset than that currently available will be required [16, 17]. Furthermore the use of the LHCb charm data to determine the strong-phase related and charm CP violation parameters simultaneously could have a negative impact on the latter [16], which are also a priority of the LHCb collaboration. Therefore the third option is by far the most attractive. The current $\psi(3770)$ dataset is approximately four times as large as that of CLEO and hence would contribute an uncertainty on γ of the order of 1°. This precision will be sufficient for the LHCb Run-2 measurements. However to maintain a sub-leading contribution to the uncertainty from the strong-phases in the LHCb upgrade era, a larger $\psi(3770)$ dataset would be essential, ideally as large as 20 fb^{-1}, which would lead to a strong-phase uncertainty of approximately 0.4°. With data from both LHCb and BESIII, degree level precision on γ can be achieved.

4.1.2 Desired strong-phase inputs from BESIII

In Table 2 the decay modes of interest are listed along with the desired measurements. Some comments are made on binning schemes on the Dalitz plot that could be employed. The modes are listed in the general order of importance for LHCb measurements.

4.2 Inputs for indirect CP violation in charm mixing

Another goal of the LHCb experiment is the measurement of the indirect CP violation in charm mixing. Methods by which these parameters can be accessed are described in the literature [16, 19, 20]. These methods all make use of the charm strong-phase parameters. Although detailed studies in all decay modes have not been performed, the information available in Refs. [16, 19, 20] suggests that precision requirements on strong-phase knowledge are not as demanding as for the γ measurements. Therefore having the strong-phase measurements listed in Table 2 performed with the current BESIII dataset and eventually the future BESIII dataset will keep any strong-phase related uncertainties sub-dominant for these measurements.

5 Measurements of absolute and relative branching fractions

There are a number of areas where improvement in the precision in relative or absolute charm-hadron branching fractions would have a significant impact on the LHCb physics programme. It is difficult to measure these branching fractions in the pp environment of the LHC. Inclusive measurements or measurement of decay modes containing multiple neutral particles are also difficult due to the high multiplicity of LHC collisions.

5.1 Motivation for absolute branching fraction measurements

Absolute branching fraction measurements of charm meson and baryon decays to Cabibbo-favoured charged final states are highly beneficial for improving key measurements at LHCb.

The magnitude of the CKM-element ratio, $|V_{ub}|/|V_{cb}|$, is another key parameter of the unitarity triangle. The ability of LHCb to make precise measurements of $|V_{ub}|/|V_{cb}|$ in the B_s and Λ_b sector is a vital input in this area, as this provides measurements with complementary uncertainties to the traditional methods performed by the B factories. The largest experimental uncertainty in the recent measurement performed with $\Lambda_b \to p\mu\nu$ and $\Lambda_b \to \Lambda_c\mu\nu$ [2] decays is due to the knowledge of the branching fraction of the Cabibbo favoured decay $\Lambda_c \to pK\pi$. The forthcoming analysis involving B_s decays is similarly expected to have a leading experimental uncertainty due to the limited knowledge of the branching fraction of the decay $D_s^+ \to K^+K^-\pi^+$. For the Λ_c case, the recent BESIII measurement [21] demonstrates the future improvement on the knowledge of this quantity that is possible with a larger data set. Further improvements to these uncertainties would be highly beneficial on the timescale of the LHCb upgrade analyses, as on this timescale improvements in lattice calculations will lead to lower theoretical uncertainties.

Table 2: A priority-ordered list of strong-phase related measurements that are important for precision measurements of γ and indirect CPV in charm-mixing.

Decay mode	Quantity of interest	Comments
$D \to K^0_S\pi^+\pi^-$	c_i and s_i	Binning schemes as those used in the CLEO-c analysis. With future, very large $\psi(3770)$data sets, it might be worthwhile to explore alternative binning.
$D \to K^0_S K^+K^-$	c_i and s_i	Binning schemes as those used in the CLEO-c analysis. With future, very large $\psi(3770)$data sets, it might be worthwhile to explore alternative binning.
$D \to K^\pm\pi^\mp\pi^+\pi^-$	R, δ	In bins guided by amplitude models, currently under development by LHCb.
$D\to K^+K^-\pi^+\pi^-$	c_i and s_i	Binning scheme can be guided by the CLEO model [18] or potentially an improved model from LHCb in the future.
$D\to \pi^+\pi^-\pi^+\pi^-$	F_+ or c_i and s_i	Unbinned measurement of F_+. Measurements of F_+ in bins or c_i and s_i in bins could be explored.
$D\to K^\pm\pi^\mp\pi^0$	R, δ	Simple 2-3 bin scheme could be considered.
$D\to K^0_S K^\pm\pi^\mp$	R, δ	Simple 2 bin scheme where one bin encloses the K^* resonance.
$D\to \pi^+\pi^-\pi^0$	F_+	No binning required as $F_+ \sim 1$.
$D\to K^0_S\pi^+\pi^-\pi^0$	F_+ and c_i and s_i	Unbinned measurement of F_+ required. Additional measurements of F_+ or c_i and s_i in bins could be explored.
$D\to K^+K^-\pi^0$	F_+	Unbinned measurement required. Extensions to binned measurements of either F_+ or c_i and s_i possible.
$D\to K^\pm\pi^\mp$	δ	Of low priority due to good precision available through charm-mixing analyses.

LHCb has the ability to measure a large number of charm meson and baryon branching-fraction ratios due to the high yields given by the large charm production cross section. The conversion from the branching-fraction ratio to the absolute branching fraction incurs uncertainty due to the precision of the branching fraction of the normalisation mode. Commonly used normalisation channels are $D^0 \to K^-\pi^+$, $D^0 \to K^-\pi^+\pi^-\pi^+$, $D_s^+ \to K^+K^-\pi^-$ and $\Lambda_c \to pK\pi$. Improvements in the absolute branching fractions of these decay modes would reduce the uncertainties across the physics programme, as it is expected that the uncertainty on the normalisation modes will become the dominant uncertainty in a number of measurements. Absolute branching fraction measurements of the other charmed baryons, Ξ_c and Ω_c are also strongly desired. Although it is acknowledged that these would be very hard to measure with the current energy of the collisions at BESIII, the importance of these measurements is stressed here in case future developments could make them possible.

5.2 Inputs required for the study of hadronic tau decays at LHCb

There are also some specific relative branching fractions where improvements in knowledge or indeed a first measurement would have an impact on LHCb physics. A prime example of this can be found when considering the recently demonstrated ability of LHCb to measure semi-tauonic decays of B hadrons and use them to test lepton flavour universality at the 10% level [22]. In extending this programme, modes where the τ lepton decays hadronically in three prongs are particularly promising, and the projected statistical uncertainty for Run-1 and Run-2 data is at the level of a few percent. In order to keep the systematic uncertainty at the same level, the knowledge of backgrounds where the B hadron decays into two charmed hadrons, one of which decays to final states involving at least three charged pions, must be improved. Therefore, precise measurements of branching fractions for D^0, D_s and D^+ inclusive decays to three charged pions and other neutral particles, and exclusive decays to final states with neutral kaons and pions (*e.g.* $D_s^+ \to \eta'\pi^+\pi^0$, $D^+ \to K^0\pi^+\pi^-\pi^+\pi^0$ and decay modes contributing to $D^{0,+} \to \eta X$), are highly desirable.

These measurements are ranked here according to relative importance. The most important measurement is the inclusive measurement of the branching fraction of $D_s^+ \to \pi^+\pi^-\pi^+X$ relative to the exclusive decay $D_s^+ \to \pi^+\pi^-\pi^+$. While lepton contamination arising from semileptonic decays should be removed, the contribution from pions that decay in flight to muons should be treated with care, since the different kinematic spectra of pions at LHCb and BESIII will lead to differing contributions. If decay of pions in flight are included in the measurement, either in the analysis directly or through corrections from simulations, their contribution should be quoted separately. This is to insure that the contribution of decays of pions in flight can subsequently be adjusted through simulation for the different kinematics found in LHCb. Furthermore, in order to allow correction to our simulation distributions it would be useful to publish the efficiency corrected three pion invariant mass spectrum and the mass spectrum distributions of each of the pion pairs from the inclusive sample.

The branching fractions useful in D_s^+ decays are listed in Table 3.The branching fractions useful for D^+ and D^0 decays are listed in Tables 4 and 5, respectively.

Table 3: A list of useful branching fractions of the D_s^+ meson to aid understanding of backgrounds to hadronic τ decays. The notation X refers to a collection of one or more neutral particles.

Decay mode	Priority	Comments
$D_s^+ \to \pi^+\pi^-\pi^+X$ inclusive	very high	See text.
$D_s^+ \to N3\pi$	high	N is any neutral meson
$D_s^+ \to \eta\pi^+X$	medium	
$D_s^+ \to \eta'\pi^+X$	medium	
$D_s^+ \to \phi\pi^+X$	medium	
$D_s^+ \to \omega\pi^+X$	medium	
$D_s^+ \to \pi^+\pi^-\pi^+\pi^-\pi^+X$ inclusive	medium	

Table 4: A list of useful branching fractions of the D^+ meson to aid understanding of backgrounds to hadronic τ decays. The notation X refers to a collection of one or more neutral particles.

Decay mode	Priority	Comments
$D^+ \to \pi^+\pi^-\pi^+X$ inclusive	high	
$D^+ \to K^0\pi^+\pi^-\pi^+\pi^0$	high	
$\frac{D^+\to K^0\pi^+\pi^-\pi^+X}{D^+\to\pi^+\pi^-\pi^+X}$	medium	A missing mass technique can be used
$D^+ \to \pi^+\pi^-\pi^+\pi^0$	low	

Table 5: A list of useful branching fractions of the D^0 meson to aid understanding of backgrounds to hadronic τ decays. The notation X refers to a collection of one or more neutral particles.

Decay mode	Priority	Comments
$\frac{D^0\to K^-\pi^+\pi^-\pi^+X}{D^0\to\pi^+\pi^-\pi^+X}$	medium	A missing mass technique can be used
$D^0 \to K^-\pi^+\pi^-\pi^+\pi^0$	low	
$D^0 \to K^-\pi^+\pi^-\pi^+\pi^0\pi^0$	low	

6 Summary

The LHCb physics programme can benefit greatly from measurements performed at BESIII. The ability to measure charm strong-phases, and associated parameters, directly is of significant benefit to LHCb. Without input from the BESIII open-charm data set it will take significantly longer for LHCb to meet its physics goals, and the ultimate precision of its measurements will likely be limited. There are a large number of very useful studies that can already be performed with the existing BESIII data sets. However there is particular need for additional data to allow for larger samples of all charm hadrons for further improvements in the knowledge of charm hadron branching fractions. Finally, significantly more data at the $\psi(3770)$ are needed to improve the knowledge of the charm strong-phase and coherence factor measurements to the level required for the b-hadron samples that will be accumulated during the LHCb upgrade eras.

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