# AN ALTERNATE APPROACH TO UNSTABLE PARTICLE SPECTROSCOPY: MAXIMUM ANGLE MISSING-MASS SPECTROMETER WITH ON-LINE COMPUTER

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(Presented by B. MAGLIĆ)

We describe here the theoretical, experimental and instrumental basis of, as well as the operational experience with the «missing-mass spectrometer», an instrument built with the specific purpose of searching for unstable particles. The motivation for this work was not the lust for instrument development but a feeling shared by some physicists that only systematic investigations of the mass spectrum of unstable particles can lead to finding a certain order and perhaps an underlying law common to all «elementary particles». While our method is general, we chose to apply it to the reaction

$$\pi^- + p \rightarrow p + N\pi \qquad (1)$$

where $N = 1,2 \ldots 10 \ldots$ ; the pions can be mass-correlated, in which case we refer to them as particle $X^-$, so that we search for the two-body reaction of the type

$$\pi + p \rightarrow p + X^- \qquad (1')$$

in the background of Reaction (1). The method can be applied to other reactions, provided the kinematic condition

$$\beta_c > \beta_3^0 \qquad (2)$$

is satisfied. [$\beta_c$-velocity of the c. m. system in the laboratory and $\beta_3^0$-velocity of the recoil particle in the c. m. system].

## METHOD

**Basic Method.** With the fixed incident pion momentum, $p_1$, in reaction (1) a simultaneous measurment of two quantities, the outgoing proton momentum $p_3$ and the proton angle $\theta_3$ gives the missing mass of the proton, $MM$, which is equal to the effective mass of the $N$ pions, $M$,

$$(MM)^2 = M^2 = (E_1 + m_2 - E_3)^2 - p_1^2 - p_3^2 + 2p_1p_3\cos\theta_3 \qquad (3)$$

For uncorrelated pions, the distribution in $M$ is a smooth function with a broad maximum; however, if $N$ pions «resonate», the process becomes a two-body one, reaction (1), and a peak in the distribution will occur at a set of combinations of $p_3$ and $\theta_3$ (a mass $M$). From equation (3) one obtains

$$\cos\theta_3 = \frac{M^2 - m_1^2 + 2E_0T_3}{2p_1p_3}. \qquad (4)$$

In Fig. 1, *a*, *b* some graphs for $\cos\theta_3$ are drawn as function of $p_3$, for different values of $M$, and for $p_1 = 6$ GeV/c and 8 GeV/c. With each value of $M$, we also consider the value $M + \Gamma$ ($\Gamma = 30$ MeV).

One sees that all the curves have a minimum which corresponds for given $p$ and $M$, to the maximum angle $\theta_3^{max}$ which is allowed in the laboratory system. The maximum for $\theta_3$ occurs at the energy of the recoil proton:

$$(E_3)_{\theta\,max} = m_3\left[\frac{m_3}{m_2} - \frac{N^2 + 2m_2^2}{2m_3E_0}\right]^{-1} = m_3\left[1 - \frac{M^2 - m_1^2}{2m_3E_0}\right]^{-1} \qquad (5)$$

and is given by:

$$\cos\theta_3^{max} = \left(\frac{2m_2E_1 - N^2}{2m_3p_1}\right)\sqrt{K^2 - 1} = \frac{1}{2m_3p_1}[(M^2 - m_1^2)(4m_3E_0 - M^2 + m_1^2]^{1/2}. \qquad (6)$$

In equations (5) and (6), first expressions are for the general case $m_1 + m_2 \rightarrow m_3 + m_4$, the second ones for our case, $m_3 = m_4$.

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The symbols are:

$N^2 = m_4^2 - m_1^2 - m_2^2 - m_3^2$; $K = (1/m_3)(E_3)\,\theta^{\max}$.

It was shown by Puppi * that at the maximum angle the following relation holds (in the approximation $\beta_1 \approx i$):

$$p_1 (p_3 \cos\theta)_{\theta\,\max} \cong (M^2 - m_1^2) \times \times \left(1 + \frac{M^2 - m_1^2}{4mE_0}\right) \cong M^2 - m_1^2. \quad (7)$$

One sees that both $(p_3)$ at $\theta^{\max}$ and $\cos\theta^{\max}$, for a fixed value of $p_1$, increase with $M$.

A glance at Eq. (6) shows that the maximum laboratory angle of particle 23, $\theta_3^{\max}$, does not depend on its momentum, $p$ (or energy $E_3$). The missing mass of the recoil nucleon (or the effective mass of $N$ pions) is related only to $\theta_3^{\max}$.

The question arises: how to establish the presence of $\theta_3^{\max}$ and measure it, when Reaction of type (1′) is in the continuous background of effective masses from Reaction (1), to which corresponds a continuum of values of $\theta_3^{\max}$?

Let us derive an important property of the Jacobian of c. m. → lab transformation at $\theta_3^{\max}$

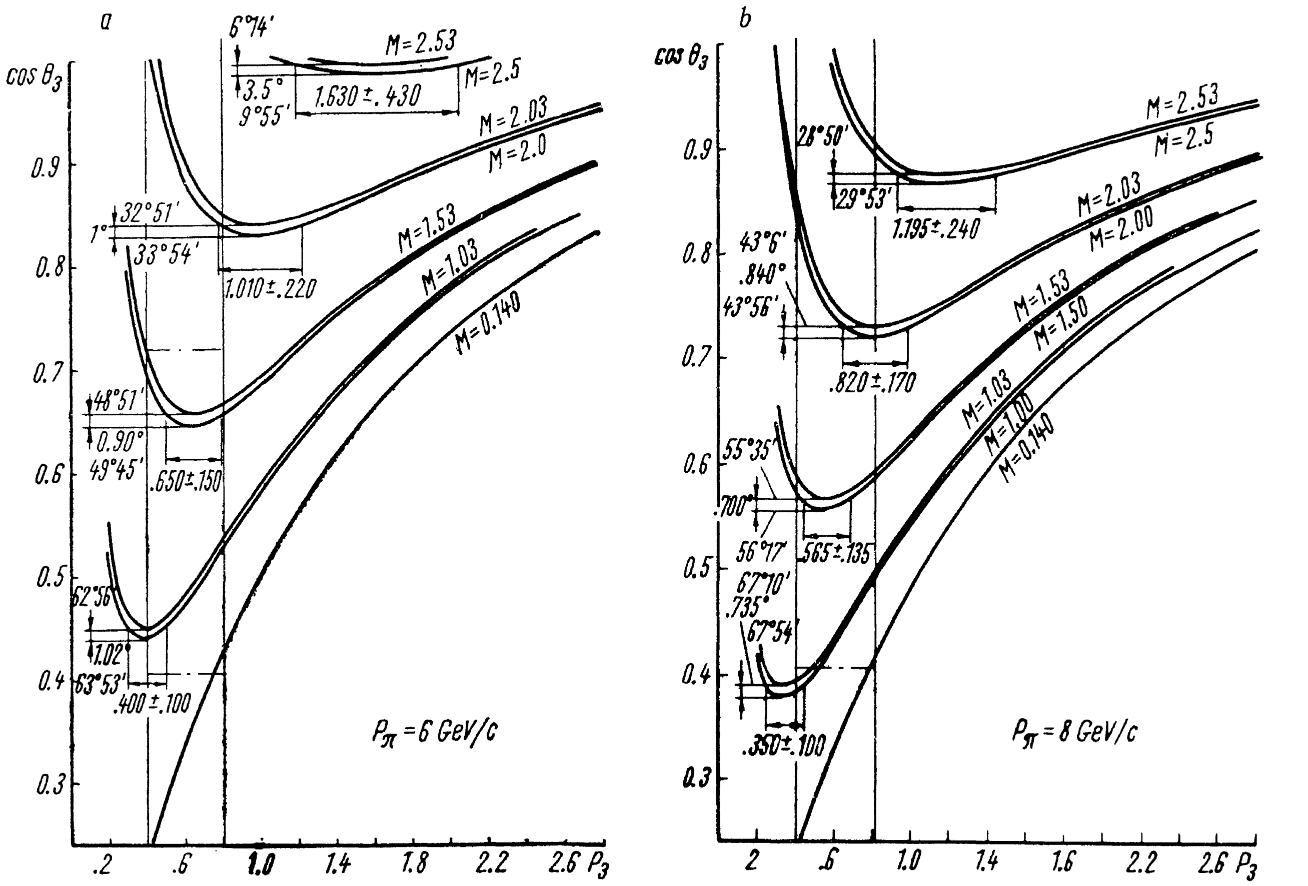


Fig. 1 Mass lines in the cos $\theta_3$ versus $p_3$ diagram for incident pion momentum (*a*) $p_1 = 6$ GeV/c, (*b*) $p_1 = 8$ GeV/c.

**Novelties in our application of the method.** We can show that with any fixed $p_1$ and properly chosen kinematic conditions, the existence of Reaction (1′) can be experimentally determined by measuring only one quantity: the angle $\theta_3$. The proton momentum $p_3$ does not have to be measured; it is sufficient to set (experimentally) the upper and the lower limits to the proton momenta. The existence of a discrete mass will manifest itself as a peak in the angular distribution of protons of all momenta in a given momentum band.

* Puppi G. Private Communication.

for cases $\beta_c > \beta^0$, which has been exploited in low-energy nuclear physics in the past.

## SPIKES IN THE ANGULAR DISTRIBUTION AT $\theta_3^{\max}$

To transfer a given angular distribution of the recoil proton from the c. m. system to the laboratory system, we need simple the Jacobian of the transformation, which can be written, by use of the Lorentz transformations, in the following way:

$$J(p_3, p_3^0) = \frac{(p_3)^3 \sqrt{E_0^2 - p_1^2}}{m_3 p_3^0 E_0} \times \times \left[E_3\left(1 - \frac{M^2 - m_1^2}{2m_3E_0}\right) - m_3\right]^{-1}. \quad (8)$$

We see at once that $J$ goes to infinity at the value of $E_3$ given by (5), which corresponds to the maximum angle $\theta_{\max}$.

The occurrence of this pole can be understood by observing that $J$ represents the angular distribution in the laboratory system corresponding to an isotropic distribution in the c. m. We remark that a pole in the physical region, corresponding to a maximum angle $\theta^{\max} < \pi$, can be present only for $M > m_1$. For $M < m_1$ the expression in brackets (8) never vanishes and all the angles are permitted in the laboratory system. ($M = m_1$ represents the limiting case, in which $J \to \infty$ at $\theta = \pi/2$).

Equation (8) gives the laboratory angular distribution when the c. m. distribution is isotropic. However, the infinity at $\theta_{\max}$ exists in all shapes of the c. m. distribution. Of course, the integral over a finite range of $\theta$ close to $\theta^{\max}$ is finite.

## ENHANCEMENT OF THE JACOBIAN PEAKS DUE TO PERIPHERAL MECHANISM

We have considered the exchange of a pseudoscalar particle ($\pi$ and $\eta$), and of a vector particle ($\varrho$, $\omega$) in the production process. Going from the pion mass to the $\varrho$ mass, we can cover different degrees of «peripherism».

The laboratory angular distributions for $\pi$ and $\eta$ exchange are given in Fig. 2.

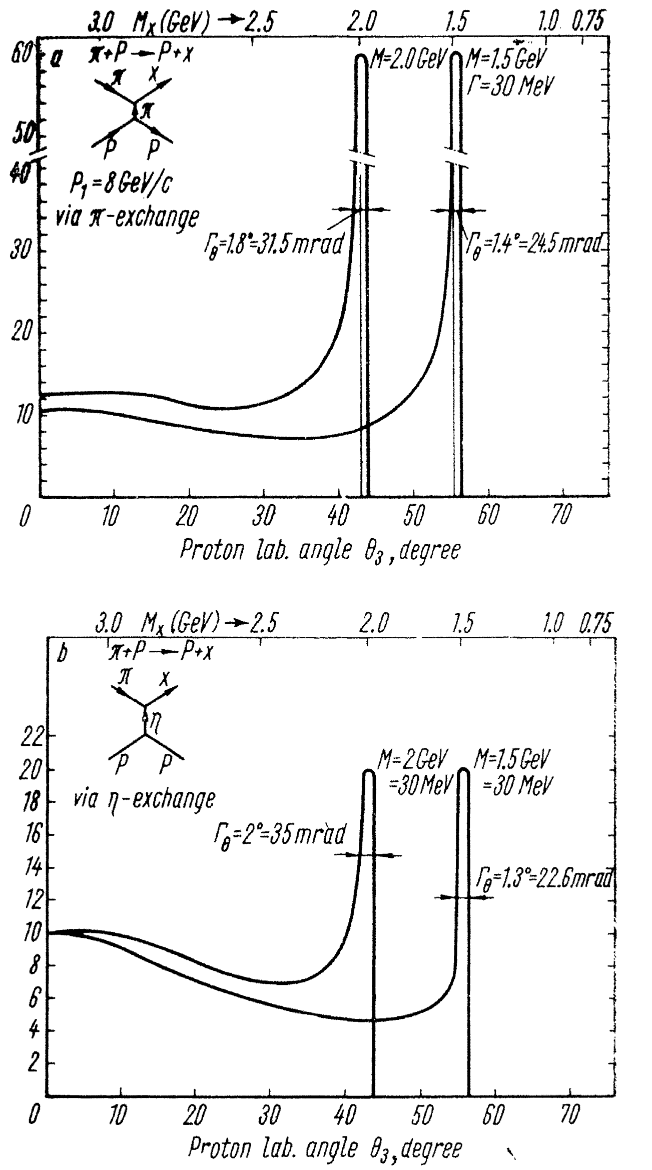


Fig. 2. Jacobian peaks for (*a*) $\pi$ exchange, and (*b*) $\eta$ exchange for $p_1 = 8$ GeV/c.

## OPERATION OF THE MISSING-MASS SPECTROMETER

The CERN missing-mass spectrometer is a system consisting of sonic spark chambers, time-of-flight, pulse-height and hodoscope counters (see Fig. 3).

According to its function, the instrument can be divided into two parts:

Part i): «Pion-line». It contains:
— hodoscopes $H_1 - H_2$
— «vertex» counter, $V$

Part ii): «Proton line». It contains:
— sonic chambers $S_1 - S_2$
— hodoscopes for recoil proton $R_1 - R_2 - R_3$
— aluminium degraders $D - D - D$ of variable thickness.

**Measured physical quantities.** Equation (3) can be rewritten *:

$$M^2 = m_1^2 - 2\,[(p_1^2 + m_1^2)^{1/2} + m_2] \times \times [(p_3^2 + m_3^2)^{1/2} - m_2] + m_3^2 - m_2^2 + 2p_1p_3\cos\theta_3. \quad (3')$$

All quantities in Eq. (3') are in the laboratory system. Knowledge of $M$ requires simultaneous measurement of six quantities; they are listed in Table, together with the description of the measuring technique used.

All measured quantities listed in Table are first digitized and then registered in scalers. The existing CERN scaler read-out logics is used to transfer the information from the scalers, either onto magnetic tape or the Mercury computer.

In the following text we describe only the essential facts about the operation of our system. Over-all description of the inter-connections and parts of the instrument will not be given.

**Data storage.** All raw data without any preselection or pre-computation, are stored from the scalers onto the magnetic tape. We use the

* We give general form: $m_1 \neq m_3$.

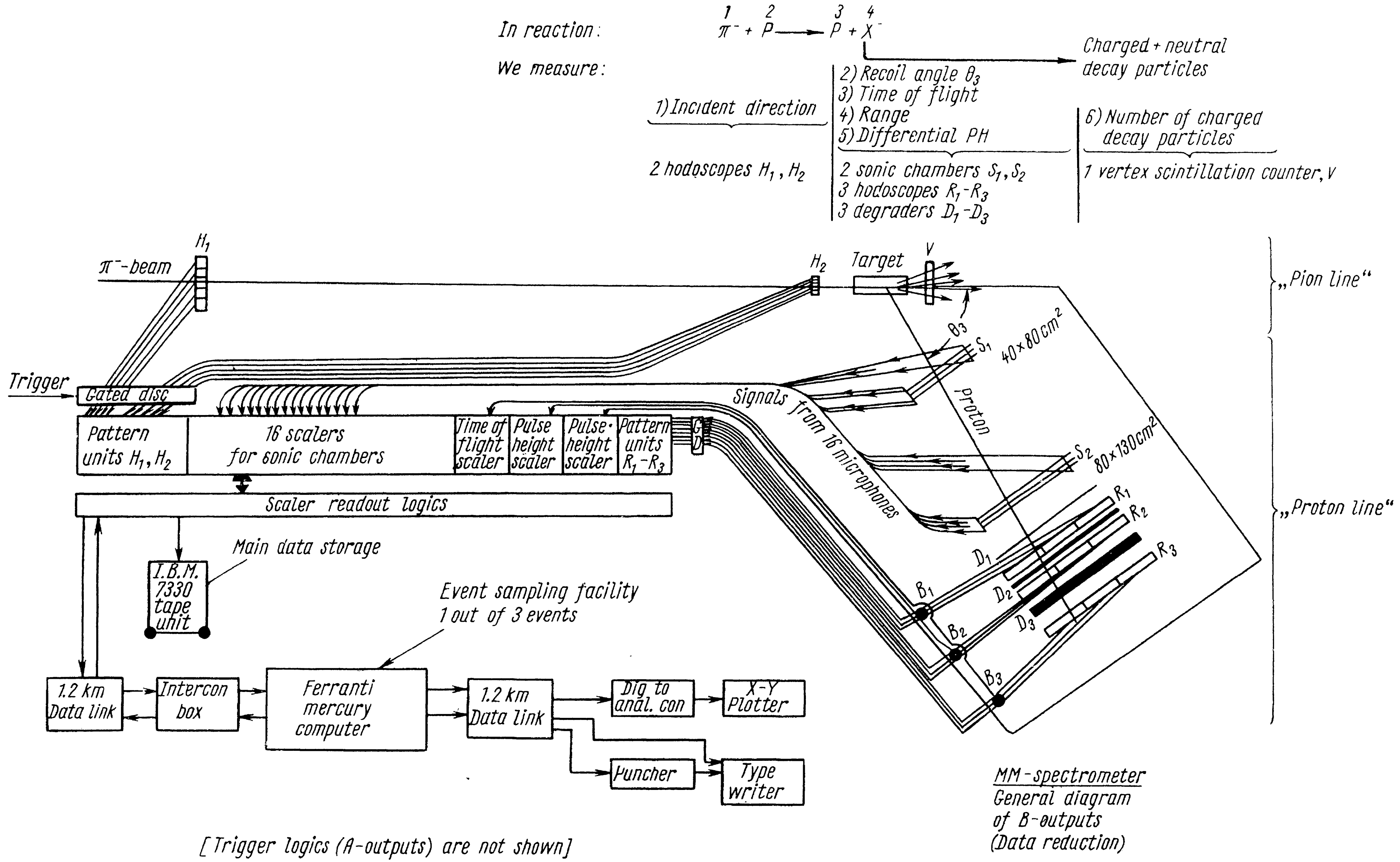


Fig. 3. Diagram of the CERN missing-mass spectrometer.

**Physical quantities involved in measuring «missing-mass spectrum»**

| | Observable | Method of measuring | Accuracy |
|---|---|---|---|
| 1 | Mass of incident pion, $m_1$ | Not measured. Beam is presumed to contain 99.3% prons | |
| 2 | Momentum of incident pion, $p_1$ | Double magnetic spectrometer (120 mrad total deflection) | $\pm 1\%$ |
| 3 | Direction of incident pion, $p_1$ | 2 counter hodoscopes $H_1 - H_2$ | $\pm 1$ mrad |
| 4 | Mass of recoil proton, $m_3$ | Rande (3 hodoscpes $R_1 - R_2 - R_3$) and time-of-flight | $\pm 20\%$ |
| 5 | Momentum of recoil proton, $p_3$ | Time-of-flight ($3m$) | $\pm 1.0$ nsec |
| 6 | Direction of recoil proton, $p_3$ (measurements 3 and 6 give angle $\theta_3$) | 2 two-gap sonic spark chambers | $\pm 1$ mrad |
| 7 | Number of charged particles $N$, into which $X$-boson decays (not directly related to the measurement of $m_3$, but essential to our method of eliminating background | Pulse height counter | In 5% cases one more particle will be counted due to Landau tail |

IBM unit, placed in an air-conditioned trailer. The tape speed is 36″/sec with a density of 200 characters per inch. A 2,400 ft reel can store $3 \times 10^4$ events (from 30 scalers).

Assuming a 100% running efficiency and 100 msec for the recording of an event, one expects, on the average, 2 events per 300 msec/burst of the proton synchrotron, or 3,600 events/hour. Our estimate is a 30% efficiency; thus, the expected data-getting rate is 1,000 events/hour or $2.4 \times 10^4$ events/day.

**Role of the on-line computer.** Due to the slowness of the Ferranti «Mercury» computer used, only one out of three to four events can be processed to the point of computing $M$, Eq. (3'). This makes the computer a sampling facility. The importance of sampling is two-fold:

i) It makes it possible to test the goodness of the physics result (missing-mass distributions) by applying certain prescribed checks on the $\theta$ versus $P$ dot density distributions. The propesed method of «particle hunting» depends on these tests (described in the next paragraph). The $\theta$ versus $P$ display is done by the mechanical $x - y$ plotter.

II) It constantly checks the functioning of the whole instrument: whenever an event cannot be «fitted», the reason for the failure (type of error) is displayed on the typewriter on-line.

**On-line checks against false peaks.** In the previous bubble chamber work peaks in missing-mass distribution, frequently turned out to be false ones. All unstable bosons were found in the effective mass, rather than the missing-mass distributions.

The proposed method prescribes definite tests to be made in the course of the experiment, which should be capable of revealing if the peak is false or not.

**$\theta^3$ versus $P_3$ scatter diagram.** In this technique the result is not displayed in the usual $N$ versus $MM$ histogram. Since, at fixed $P_1$, $MM$ is a function of only two variables, $MM = f(\theta_{,3}\ P_3)$, each event can be represented by a dot in a $\theta_3 - P_3$ plane. This gives more information than $N$ versus $MM$ histogram. First, it shows simultaneously, the cross section as a function of the momentum transfer $\Delta^2$ ($\approx P_3^2$). Secondly, the scatter diagram provides two independent tests:

Test 1: If there is a discrete mass in $X$ the dots should lie along one mass-line whose shape is well known at any incident $P_1$-fixed. (See, for example, $\theta_3 - P_3$ mass lines for $P_1 = 6$ GeV in Fig. 1). An increase of dot-density, if the effect is real, is expected along a given line. If this condition appears to be satisfied in the course of the run, the physicist proceeds to:

Test 2: The mass-lines shift in a known manner with the change of the incident momentum $P_1$: the increase (decrease) of $P_1$ by 1 GeV/c, typically increases (decreases) the maximum angle by about 3°. At any beam setting, the variation of the beam momentum of $\pm$ 1 GeV/c is possible without changes in the beam design.

The second test therefore, consists of repeating the run at $P_1 + 1$ GeV/c and $P_1 - 1$ GeV/c and seeing if the density distribution follows the scale prescribed by kinematics.

**Use of typewriter on-line.** The typewriter is in the experimental area, on-line with the computer output: it gives the evidence on the functioning of every instrument of the system at any time.

I) Whenever an event cannot be «fitted» by the missing-mass programme, the reason for the failure is typed. This can be illustrated by the following examples:

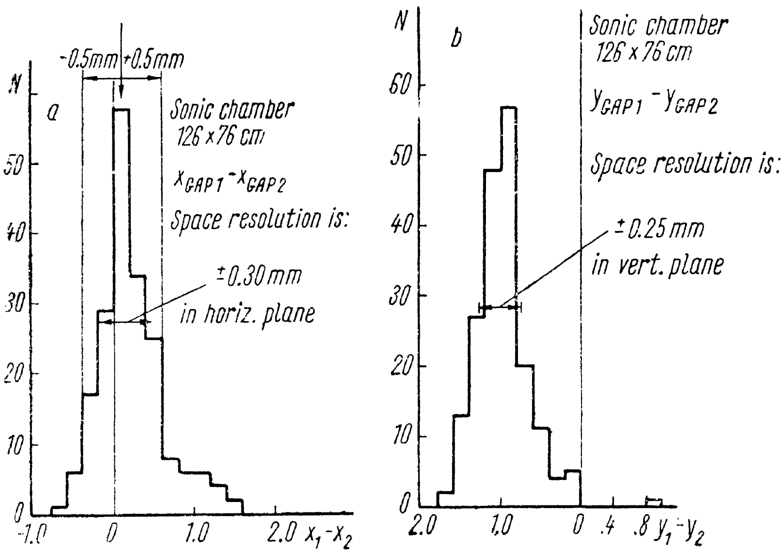


Fig. 4. Distribution of the differences of the spark positions in two gaps in the sonic chamber of active area of 126 $\times$ 76 cm, for the $x$ and $y$ coordinates, respectively. The systematic shift of 1 mm in the $y$ coordinate helped us in finding out that the chamber was inclined 1° to the vertical. This space resolution of $\pm$ 0.25 mm implies an angular resolution of $\pm$ 0.05° with two chambers 150 cm apart.

— If the pion line operates properly only one counter in each of the hodoscopes $H_1 - H_2$ should go off. If, in one hodoscope, two counters give signals, the event will not be fitted.

— One spark-chamber gap became inefficient. This would result in the absence of the stop signals in any of the four microphones in the gap.

An event does not satisfy the pulse-height criterion for proton of given time-of-flight.

II) The repetition of any of these errors draws the experimentalist's attention to the specific part of the apparatus. He requests the computer operator to output the quantities related to the part of the system which is malfunctioning. He can plot the distribution of each of the quantities.

An example of this is given in Fig. 4, 5. Distribution has revealed that the spark chamber $S_1$ was inclined by 1° to the vertical.

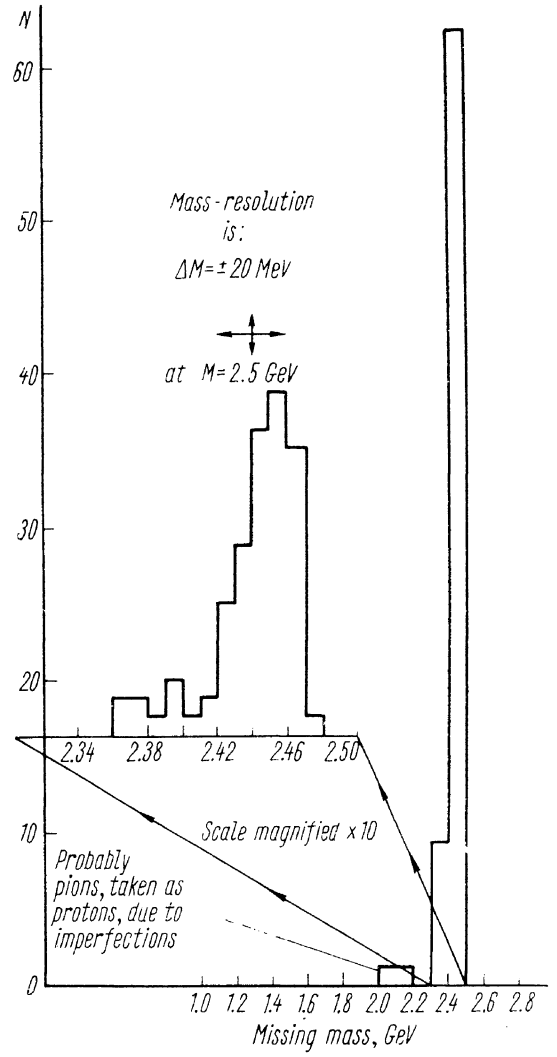


Fig. 5. Missing-mass resolution obtained with the monoergic proton beam from CERN Proton Synchrotron (used for calibration).

## DISCUSSION

J. Perez Y. Jorba

What is the increase in mass resolution of your spectrometer due to the energy width of the incoming $\pi$ beam because when the $\frac{\Delta E_\pi}{E_\pi}$ is large, you have to put in your Jacobian a quantity which is something like $\frac{dE_\pi}{dP_p}$ and which goes to 0 at maximum angle, destroying the peak in the Jacobian?

B. Maglić

Pion-beam momentum spread is $\pm 1.5\%$ at worst. This produces the Jacobian peak so that it gives error in missing-mase of 7 MeV.