PROSPECTIVE PION-ELECTRON COLLIDING BEAM EXPERIMENTS USING K-SHELL OF HEAVY ATOM AS ELECTRON «STORAGE RING»

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(Presented by B. MAGLIĆ)

Direct experimental investigations of the electromagnetic form factor of the π-meson by means of e-π scattering are restricted at present to the low values of the momentum transfer obtainable in collisions of pions with atomic electrons. In order to measure pion charge distribution, with the purpose of obtaining r. m. s. radius of the distribution, it is essential to penetrate into pion to a distance shorter than its «physical radius» $r_\pi \simeq 1.4$ fermi but even with 25 GeV pions incident on electrons at rest, the maximum momentum transfer is $\Delta^2_{\max} = 0.014$ $(\mathrm{GeV})^2$, which corresponds to an interaction distance $d \sim hc/\Delta = 1.6$ fermi. With pions of 15 GeV — the energy at which intense beams are available — the corresponding figures are $\Delta^2_{\max} = 0.007$ $(\mathrm{GeV})^2$ or $d = 2.4$ fermi. In such distant $e - \pi$ collisions, only the total pion charge can be measured.

The situation is changed if the target electron, instead of being at rest, has a small amount of kinetic energy T_2. We shall refer to a device in which a beam of high-energy pions collides with a beam of low-energy electrons as«e —π-tron» or **epitron**. While with any storage ring technically conceivable at present an epitron would be unfeasible by an intensity factor of 10^9, it can be easily shown that «storage rings» with electrons of momenta up to ~ 1 MeV/c are readily available in the form of K-shell of heavy atoms such as uranium.

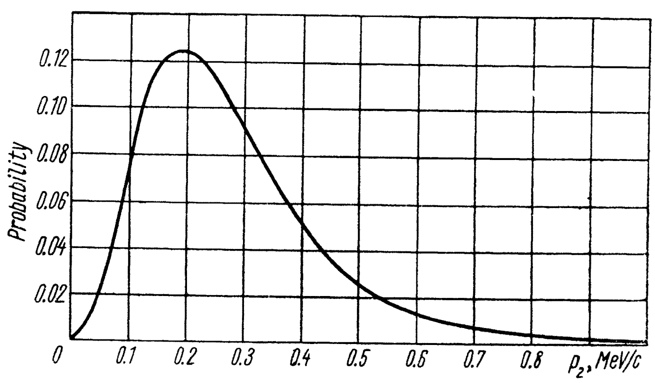
In this case, however, one has to take into account that the electron is bound. Suppose that the binding energy is $W = -116$ keV (see next Section) and the pion kinetic energy $T_1 = 25$ GeV; then, if electron of momentum $p_2 = 500$ keV$/c$ has a head-on collision, one gets $\Delta^2_{\max} = 0.032$ $(\mathrm{GeV})^2$, and the collision distance correspondingly shortens from 1.6 to 1.1 fermi. The equivalent pion kinetic energy needed to produce this Δ^2 with electron of zero momentum is $T_1 = 43$ GeV.

I. MOMENTUM DISTRIBUTION OF ELECTRONS IN K-SHELL

We consider the simple wave-function of the K-shell electron:

$$\Psi(r) = (\pi a^3)^{-1/2}\, e^{-r/a} \qquad (1)$$

where $a = a_0/z$, $a_0 =$ Bohr radius. The screening will be neglected in our considerations, because the K-shell energy obtained from eq. (1),



Momentum distribution of electrons in K-shell of uranium.

(1), $W_z = -(z^2e^2)/2a_0 = -115$ keV, is in sufficiently good agreement with the observed x-ray measurement * $W = -116$ keV.

Momentum distribution is obtained by Fourier transformation of eq. (1):

$$\Psi(k) = (2\pi)^{-3/2} \int \Psi(r) \exp(-i\mathbf{kr})\, d^3r, \qquad (2)$$

where $k = p_2/h$; after integration, we obtain (normalized to unity)

$$\Psi(k) = \frac{2\sqrt{2}}{\pi} \frac{a^{3/2}}{(1+a^2k^2)^2}. \qquad (3)$$

The probability that the magnitude of the momentum $[\mathbf{k}]$ has a value between k and

* Mack J. E. and Cock J. M. Phys. Rev., **30**, 741 (1927).

$k + dk$ in any direction is,

$$p(k)\,dk = 4\pi k^2 |\Psi(k)|^2\,dk \qquad (4)$$

and is plotted in Figure.

$p(k)$ has maximum at $k_{\max} = (1/3)^{\frac{1}{2}} a^{-1} = \sim 0.198$ MeV/c, while the average value is 0.292 MeV/c. One per cent of the electrons has $k = 0.650$ MeV/c, 0.5% has $k = 0.750$ MeV/c. The tail falls to 0.12% at 1 MeV/c.

II. DETERMINATION OF Δ^2 WITHOUT MEASURING EITHER THE CROSSING ANGLE OR ELECTRON MOMENTUM BEFORE SCATTERING

Let us label all pion quantities with 1, the electron ones with 2; the unprimed quantities before, and the primed ones after the scattering. Thus the laboratory momentum, kinetic energy and total energy are p_1, T_1, E_1 and $p_{1'}$, $T_{1'}$, $E_{1'}$ for π (m_1) and p_2, T_2, E_2 and $p_{2'}$, $T_{2'}$ and $F_{2'}$ for e (m_2). We have to take into account that $E_2 = m_2 + W$, where $W = -T_2$ is the binding energy of the electron in the K-shell. Angles are measured in respect to the pion direction, which is taken to be $\vartheta_1 = 0$; then, the electron angle before scattering is simply ϑ_2. For high-energy (15—25 GeV) pions and low-energy (100—600 keV) electrons, we can write down the c. m. energy, E_0, of the $e + \pi$ system:

$$E_0^2 \simeq m_1^2 + 2p_1(E_2 - p_2\cos\vartheta_2) \qquad (5)$$

where the electron mass is neglected. Product $p_2 \cos\vartheta_2$ is the effective electron momentum at the instant of collision.

On the other hand, E_0 can be directly determined by measuring outgoing momenta $p_{1'}$ and $p_{2'}$ and the π-e opening angle $\vartheta_{12'}$.

$$E_0^2 \simeq m_1^2 + 2E_{1'}E_{2'} - 2p_{1'}p_{2'}\cos\vartheta_{12'} = E_0^2(p_{1'},\ p_{2'},\ 1',\ 2') \qquad (6)$$

where $\vartheta_{12'} \simeq \vartheta_{1'} + \vartheta_{2'}$, and $\vartheta_{1'}$ and $\vartheta_{2'}$ are the laboratory angles of the pions and electrons after collision. The actual relation is: $\cos\vartheta_{12'} = \cos\vartheta_{1'}\cos\vartheta_{2'} + \sin\vartheta_{1'}\sin\vartheta_{2'}\cos\psi_{12'}$. but if $p_2 \ll p_1$ one can take $\psi_{12'} = \pi$

Momentum of either particle in c. m. is

$$q_0 = \frac{E_0^2 - m^2}{2E_0} = q_0(p_{1'},\ p_{2'},\ \vartheta_{1'},\ \vartheta_{2'}). \qquad (7)$$

The square of the momentum transfer is:

$$\Delta^2 = (p_{1'}^{(4)} - p_1^{(4)})^2 = 2q_0^2(1 - \cos\vartheta_0) = 2m_2^2 - 2E_1(E_{1'} - p_{1'})\cos\vartheta_{1'}) \qquad (8)$$

where ϑ_0 is the c. m. scattering angle. The momentum transfer in each event is then obtained by measuring only the momentum $p_{1'}$ and the angle $\vartheta_{1'}$. From these two quantities, we can infer the effective electron momentum before scattering, $p_2 \cos\vartheta_2$, but not separately p_2 and $\cos\vartheta_2$.

III. REJECTION OF THE EVENTS OTHER THAN π-e COLLISIONS WITH MOVING ELECTRONS, WHOSE CROSSING ANGLE $\vartheta_2 > 90°$

To the accuracy to which $p_2 \sin\vartheta_2 \ll p_1{}^*$, we can write down the electron energy $E_{2'}$ after scattering, as a function of the c. m. angle ϑ_0; or the laboratory angle $\vartheta_{2'}$.

$$E_{2'} = 2\gamma_c q_0 \cos^2\frac{\vartheta_0}{2} = \frac{p_1(E_0^2 - m_1^2)}{E_0^2} \times \frac{1}{(p_1/E_0)^2\tan^2\vartheta_{2'} + 1}. \qquad (9)$$

The maximum of $E_{2'}$ is obtained for $\vartheta_{2'} = 0$,

$$(E_{2'})_{\max} = 2\gamma_0 q_0. \qquad (9')$$

Let us call ε_0 the c. m. energy obtained when the target electron has zero momentum ($E_0 = \varepsilon_0$ for $p_2 = 0$ in eq. (5)); and the corresponding energy of the outgoing electron $\varepsilon_{2'}$ ($E_{2'} = \varepsilon_{2'}$ for $E_0 = \varepsilon_0$ in eq. (9)).

Dependence of $E_{2'}$ on E_0 means that at any angle $\vartheta_2{}'$

$$E_{2'}(\vartheta_2') > \varepsilon_{2'}(\vartheta_{2'}). \qquad (10)$$

By accepting only the electrons of energy $E_{2'}$ satisfying inequality (10), we reject the collisions with electrons of zero momentum or moving in the same direction as the pion. Ratio:

$$\delta = \frac{E_{2'}}{\varepsilon_{2'}} \qquad (11)$$

at 0° (where Δ^2 and $E_{2'}$ have maximum values) varies from 1.2 to 1.4 depending on p_1 and p_2. These large differences are easily detectable by means of magnetic spectrometers whose typical resolution is 1%.

IV. AVERAGE VALUES OF E_0, Δ^2 AND $E_{2'}$

It should be pointed out that eq. (4) gives the distribution of the scalar momentum in all directions, while the equations (5), (8) and (9) give the largest values of F_0, Δ^2 and $E_{2'}$ obtainable since they assume collisions with the

* The c. m. velocity is approximated by $\beta_c = p_1 + p_2/E_1 + E_2$, and $\gamma_c = p_1/E_0$.

most effective crossing angle $\vartheta_2 = 180°$; only 0.5% of the K-shell electrons may contribute to this configuration (180° ± 10°). It is more instructive to evaluate E_0, $(E_{2'})_{\max}$ and $\Delta^2_{\max}$ averaged over **all** crossing angles $\vartheta_2 > 90°$, which is equivalent to taking into account the effect of **half** of the electrons in the shell; the other half, having $\vartheta_2 < 90°$, will be rejected on the basis of too low $E_{2'}$ (see Section III, eq. (10)). The averages of eqs. (5), (8) and (9′) are:

$$\overline{E_0^2} = \int_{-1}^{0} E_0^2 d(\cos\vartheta_2) = m_1^2 + 2p_1\left(E_2 + \frac{p_2}{2}\right); \tag{5'}$$

$$\overline{\Delta^2_{\max}} = -\left(\overline{E_0^2} - 2m_1^2 + \frac{m^4}{2p_1p_2}\log\frac{\overline{E_0^2}+p_1p_2}{\overline{E_0^2}-p_1p_2}\right); \tag{8'}$$

$$\overline{(E_2')}_{\max} = 2\overline{\gamma_c q_0} =$$

$$= \frac{1}{2}(E_1+E_2)\left[1 - \frac{m_1^2}{2p_1p_2}\log\frac{\overline{E_0^2}+p_1p_2}{\overline{E_0^2}-p_1p_2}\right]. \tag{9''}$$

V. COINCIDENCE WITH K X-RAYS AS MEANS OF IDENTIFICATION OF EVENTS ORIGINATING IN K-SHELL

Every collision in K-shell will be followed by a K X-ray of 115 keV, while that in L-shell by 30 keV X-ray. The L-shell collisions can be rejected by requiring a 115 keV X-ray in coincidence with high-energy electron-pion pair. Similarly, by accepting only 30 keV X-rays, the observations can be restricted to L-shell collisions only.

The shape of the tail of the electron distribution, eq. (4), could be checked by (a) switching the measurement to L-shell only; (b) changing the target material and studying the shape of $\sigma(\Delta^2)$ for K- and L-shell. Another way to check the distribution would be by using muons, instead of pions, and comparing the two.

VI. CROSS SECTION FOR THE π-e SCATTERING ON FREE ELECTRONS

Cross section calculated by Bhabha, can be expressed as a function of the outgoing electron energy $E_{2'}$ only. After integration, from $((\varepsilon_{2'})_{\max} = \varepsilon_{2'}\ (\vartheta_{2'} = 0)$ to $(E_{2'})_{\max})$ we obtain:

$$\sigma(\varepsilon_{2'} \to E_{2'}) = 2\pi r_e^2 m_2 \times$$

$$\times \left[\frac{1}{\beta_1^2}\left(\frac{1}{\varepsilon_{2'm}} - \frac{1}{E_{2'm}}\right) + \frac{1}{E_{2'm}}\log\frac{\varepsilon'_{2m}}{E_{2'm}}\right] \tag{12}$$

or, using eq. (11),

$$\sigma\left(\frac{E_{2'}}{\delta} \to E_{2'}\right) + 2\pi r_e^2 m_2 \frac{1-\log\delta}{(E_{2'})_{\max}}. \tag{12'}$$

Taking $\beta_1 = 1$, $\delta = 1.3$, $E_{2'\ \max} = 13$ GeV, the cross section is $\sigma \approx 0.75 \times 10^{-29}$ cm^2.

The observed cross section in the K-shell epitron experiment, can be obtained as follows: one has to integrate first over all the values of p_2, ϑ_2 which are considered in the experiment, keeping Δ^2 constant:

$$\frac{d\sigma}{d\Delta^2} = \frac{1}{2}F(\Delta^2)\int_0^{\infty} dp_2 \int_1^{0} d\cos\vartheta_2 \cdot \sigma(\varepsilon_1, \Delta^2) P_{(p_2)}, \tag{13}$$

where $P_{(p_2)}$ is the momentum distribution given by eq. (4); $\sigma(\varepsilon_1, \Delta^2)$ is the cross section relative to the set of final configurations with $\Delta^2 =$ = const. (i. e. with constant values of $p_{1'}$, $\vartheta_{1'}$ as given from eq. (8)); ε_1 is the energy of the incident pion in the system in which $p_2 = 0$:

$$\varepsilon_1 = \frac{E_0^2 - m_1^2}{2E_2}.$$

Integration over the observed region of Δ^2 gives the required cross section.