

Dissipative axial inflation

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Abstract. We analyze in detail the background cosmological evolution of a scalar field coupled to a massless abelian gauge field through an axial term $\frac{\phi}{f_\gamma} F\tilde{F}$, such as in the case of an axion. Gauge fields in this case are known to experience tachyonic growth and therefore can backreact on the background as an effective dissipation into radiation energy density ρ_R , which can lead to inflation without the need of a flat potential. We analyze the system, for momenta k smaller than the cutoff f_γ , including the backreaction numerically. We consider the evolution from a given static initial condition and explicitly show that, if f_γ is smaller than the field excursion ϕ_0 by about a factor of at least $\mathcal{O}(20)$, there is a friction effect which turns on before the field can fall down and which can then lead to a very long stage of inflation with a generic potential. In addition we find superimposed oscillations, which would get imprinted on any kind of perturbations, scalars and tensors. Such oscillations have a period of 4–5 e-folds and an amplitude which is typically less than a few percent and decreases linearly with f_γ . We also stress that the curvature perturbation on uniform density slices should be sensitive to slow-roll parameters related to ρ_R rather than $\dot{\phi}^2/2$ and we discuss the existence of friction terms acting on the perturbations, although we postpone a calculation of the power spectrum and of non-gaussianity to future work and we simply define and compute suitable slow roll parameters. Finally we stress that this scenario may be realized in the axion case, if the coupling $1/f_\gamma$ to U(1) (photons) is much larger than the coupling $1/f_G$ to non-abelian gauge fields (gluons), since the latter sets the range of the potential and therefore the maximal allowed $\phi_0 \sim f_G$.

Keywords: axions, inflation, particle physics - cosmology connection

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1 Introduction

Inflation [1–3] is the best known candidate for providing initial conditions for our Universe, which can then evolve in the hot radiation dominated era. It explains, in fact, how the Universe can become nearly homogenous, isotropic and flat on very large scales. Moreover a great success of inflation is the possibility of generating an initial condition for metric perturbations which are consistent with the observed spectrum on large scales: adiabatic, gaussian and almost scale invariant. The simplest realization of inflation is arguably a single slowly rolling scalar field. This requires nonetheless a very flat potential, in such a way that the absolute value of the potential dominates over its derivatives, and so the Hubble friction dominates the evolution. It usually requires a large amount of fine-tunings to explain how such a potential can maintain its flatness during the required 60 e-folds of inflation. In the present paper we pursue the idea that inflation could actually be realized in a very different way, in which the slow-roll is not due to Hubble friction, but is dynamically generated by a non-trivial dissipation mechanism into other degrees of freedom. Such an idea has been invoked already in some scenarios, e.g. in [4]. In our view a simple and compelling concrete mechanism is the one proposed in [5],¹ in which a scalar ϕ is coupled to a CP-odd combination of a U(1) gauge field, with the term $\frac{\phi}{f_\gamma} F\tilde{F}$: in this case, in fact, a nonzero time derivative $\dot{\phi}$ induces a tachyonic instability for the gauge field, which can then grow until it backreacts on the scalar, slowing down its evolution. In the present paper we study such a system by fully taking into account of the backreaction in a numerical way, showing that indeed slow-roll can be realized even in the absence of Hubble friction and studying in detail the evolution of the system. As we will discuss, the analysis performed in [5] assumed that such a friction dominated regime could be realized and it was estimated within an analytical approximation. There are some issues which, however, cannot be addressed within such a

¹This was also called “natural steep” inflation, while we prefer to avoid the word natural, because it reminds necessarily of axion potentials, while the only ingredient is an axial coupling which allows dissipation into gauge fields, and so we prefer to call it dissipative axial inflation.

treatment. For instance such an estimate assumes that an inflationary solution with almost constant expansion rate H and almost constant field velocity $\dot{\phi}$ can be found, but does not address how such a solution could be reached from a given initial condition. In fact the question can be formulated as follows: starting with a field at rest at some initial value in a steep potential, can the backreaction term turn on before the field simply falls down its potential? The initial stage of evolution can be actually studied ignoring the expansion of the Universe since the field dynamics dictated by the potential is much faster than the Hubble expansion. The treatment of [5] cannot be used for this purpose, since it actually explodes exponentially in the limit $H \rightarrow 0$ and only makes sense after the backreaction has already turned on. Another reason why a numerical treatment is appropriate is to check whether the ansatz of constant $\dot{\phi}$ is actually realized and to which extent. For these reasons we study the system carefully in the present paper and show that it can be solved explicitly, therefore providing an inflationary background and finding new relevant features. Note that a similar numerical analysis has been performed in [6], although with the scalar starting in a Hubble friction regime and then only approaching the dissipative regime at the end of inflation. We will comment more on this study in the rest of the paper. Importantly, we also find that generically the energy density dissipated in radiation ρ_R is much bigger than the field kinetic energy and so it dominates the time derivative of the total energy density $\dot{\rho}$; we stress then that the amplitude of the primordial curvature perturbation ζ should be different from the usual expression for a single scalar field used in [5], although we do not perform here a full calculation of ζ . Finally we will also point out that this mechanism can work with generic potentials, but most naturally can be realized in the case of an axion or axion-like field coupled to a U(1) gauge field: in fact, using the well-known example of the QCD axion, we suggest that the scenario can work just by having the coupling between axion and abelian fields (the photon) much larger than than the coupling to non-abelian fields (the gluons).

The paper is organized as follows. In section 2 we review the setup. In section 3 we solve the system in flat spacetime, while in section 4 we solve it in an expanding background. In section 5 we show results for several potentials. Finally we discuss some features of the model in section 6 and we draw our conclusions in section 7.

2 Setup

We consider the action of a scalar field ϕ with a potential $V(\phi)$, coupled to a massless gauge field A_μ (the photon or another field), with field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. Such a coupling is naturally present for an axion field and it has been considered in [5] in the context of inflation. In principle one could also consider non-abelian fields, but this would introduce complications: (i) the presence of nonlinear self-interactions might change the production mechanism, (ii) producing friction mostly through such a coupling would not work, because it would also induce a contribution to $V(\phi)$ through instanton effects. We therefore only consider U(1) and discuss later the importance of the relative size of abelian vs. non-abelian couplings. Our action is therefore

$$S = - \int d^4x \sqrt{-g} \left(\frac{1}{2} \partial_\mu \phi \partial^\mu \phi + V(\phi) + \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{g(\phi)}{4} F_{\mu\nu} \tilde{F}^{\mu\nu} \right), \quad (2.1)$$

where we assume for simplicity the coupling function to be

$$g(\phi) = \frac{\phi}{f_\gamma}, \quad (2.2)$$

where f_γ is a mass scale and $\tilde{F}_{\mu\nu}$ is the antisymmetric dual field tensor. It is straightforward to generalize to other forms of $g(\phi)$. Note that such a coupling term has dimension five and therefore we can trust it only at energy scales smaller than f_γ . We will return to this point and to the possible origin of such a coupling from a more complete renormalizable theory at higher energy in section 6. We first analyze in section 3 the system in flat spacetime in order to have a clear picture of the dynamics and of the onset of the backreaction. In fact we are interested in cases in which the scalar field potential is steep, $\partial V/\partial\phi \equiv V_{,\phi} \gg 3H\dot{\phi}$, and therefore the Hubble friction is subdominant, at least in the initial stages of the ϕ evolution, until the backreaction term becomes important. Then we will include the expansion of the metric in section 4.

3 Flat spacetime analysis

Let us first analyze the equations in the simplest setup, namely in flat spacetime. Varying the action with respect to ϕ and only considering the spatially homogeneous mode $\phi(t)$ we get the equation of motion:

$$\ddot{\phi} + V_{,\phi}(\phi) + g_{,\phi}(\phi) \mathcal{B} = 0, \quad \mathcal{B} \equiv \langle F\tilde{F} \rangle/4. \quad (3.1)$$

A gauge field of three momentum \vec{k} evolves in the $\phi(t)$ background according to (see e.g. [7] and references therein):

$$\ddot{A}_\pm + (k^2 \mp k\dot{g})A_\pm = 0, \quad (3.2)$$

where $k \equiv |\vec{k}|$ and the index $+$ or $-$ represents the gauge field helicity, defined by a helicity vector $\vec{\epsilon}_\pm$, which satisfies $\vec{k} \cdot \vec{\epsilon}_\pm = 0$ and $\vec{k} \times \vec{\epsilon}_\pm = \mp ik\vec{\epsilon}_\pm$. The gauge field backreacts on the scalar through

$$\mathcal{B} = \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} k \frac{d[|A_+|^2 - |A_-|^2]}{dt}. \quad (3.3)$$

In absence of interactions the classical field evolution would be simply determined by the potential. The interacting system has instead a non-trivial evolution if the coupling $1/f_\gamma$ is large enough: there can be a very efficient dissipation of energy into gauge fields and this can slow down dramatically the scalar field evolution, being effectively a friction-like term. The physical essence is that a tachyonic instability signals that the initial background is not the correct one, and by solving for the full system we effectively find a new background. Note that the existence of such a friction is possible only because our interaction term involves a CP-odd combination $F\tilde{F}$, which does not respect Time reversal (T) and therefore it is sensitive to $\dot{\phi}$, contained in \dot{g} . The fact that the friction can be sustained for a long time is crucially due to the massless nature of the gauge field, so that, even if the field ϕ changes value, this does not induce a mass, which would otherwise prevent efficient dissipation.

Let us now place our field at rest at some initial time $t = 0$ with an initial value $\phi_0 > 0$ and for simplicity consider a constant force $V_{,\phi} \equiv -V_p < 0$, so that the solution without backreaction is

$$\phi(t) = \phi_0 - \frac{1}{2}V_p t^2, \quad (3.4)$$

and so ϕ would fall down its potential to a final value (introducing additional terms to the linear potential this could be a minimum of V , which we can also define to be $\phi = 0$ without

loss of generality), in a time given by $t_F \approx (\phi_0/V_p)^{1/2}$. In this case the gauge field behaves according to

$$\ddot{A}_\pm + (k^2 \pm \mu^2 kt)A_\pm = 0, \quad \mu^2 \equiv \frac{V_p}{f_\gamma}. \quad (3.5)$$

We can focus on the negative helicity modes, which for low k are unstable. We find the following solution in terms of the Airy functions of the first and second kind,

$$A_-(t) = c_1 \text{Ai} \left(\frac{k(\mu^2 t - k)}{(k\mu^2)^{2/3}} \right) + c_2 \text{Bi} \left(\frac{k(\mu^2 t - k)}{(k\mu^2)^{2/3}} \right), \quad (3.6)$$

where c_1 and c_2 are two constants related to the initial conditions. In the large t limit the second term of this equation is rapidly growing for low k ,

$$A_- \approx c_2 \frac{e^{\frac{2\sqrt{k}(\mu^2 t - k)^{3/2}}{3\mu^2}}}{\sqrt{\pi} \sqrt[12]{k} \sqrt[6]{\mu} \sqrt[4]{t}}. \quad (3.7)$$

More and more modes grow as time grows. At a given time all modes with $k < \mu^2 t$ grow explosively, with a time-scale given by $\bar{t} \approx (k\mu^2)^{-1/3}$. We should then integrate eq. (3.3) using eq. (3.7) for all the tachyonic k 's. This can be done analytically approximating $\mu^2 t - k \approx \mu^2 t$ in the exponents, leading to a \mathcal{B} which grows roughly as $e^{\frac{4}{3}(t\mu)^2}$. This means that we should compare the timescale μ^{-1} with t_F , in order to know whether the effect can become large before that the field falls down. This gives us the crucial condition that $f_\gamma \gg \phi_0$. Such an analysis is valid for a constant force, but a similar general lesson can be extracted from the more realistic (and complicated) case of a quadratic potential $V = \frac{1}{2}m^2\phi^2$. In this case the free field solution is periodic with some amplitude ϕ_0 , so that with suitable initial conditions we have:

$$\dot{\phi}(t) = m\phi_0 \cos(mt), \quad (3.8)$$

which leads to

$$\ddot{A}_\pm + \left(k^2 \mp \frac{km\phi_0}{f_\gamma} \cos(mt) \right) A_\pm = 0. \quad (3.9)$$

This equation has a well-known solution

$$A_\pm(t) = c_1 \text{MathieuC}[a, q, z] + c_2 \text{MathieuS}[a, q, z],$$

$$a \equiv \frac{4k^2}{m}, \quad q \equiv \pm \frac{2k\phi_0}{mf_\gamma}, \quad z \equiv \frac{mt}{2}, \quad (3.10)$$

in terms of even and odd Mathieu functions, whose behavior can be always written, according to Floquet's theorem, as $e^{irz} f(z)$, where $f(z)$ has period 2π and r is the so-called Mathieu characteristic exponent. It is possible to check that for $k < \frac{m\phi_0}{f_\gamma}$ such exponents have a negative imaginary part for one of the two helicities and so lead to an exponential growth of the modes. Such a growth can overcome the free field classical evolution if the $|\text{Im}(r)|$'s are much bigger than 1. Therefore one can plot $\text{Im}(r)$ and check that, fixing for instance units of $m = 1$, they grow linearly as $\kappa \frac{\phi_0}{f_\gamma}$, where κ is a number which depends on k and which has

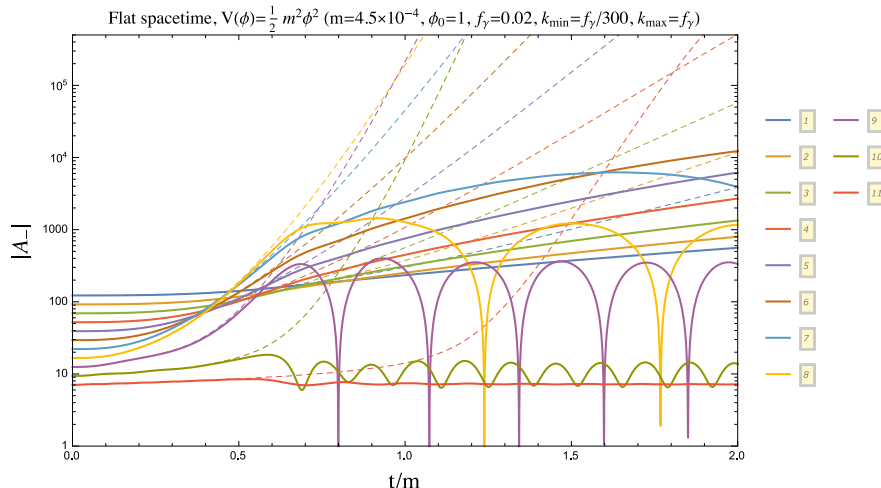


Figure 1. We show the time evolution, in a quadratic potential and in flat spacetime, of the growing gauge field A_- , for increasing values of k , from $k_{\min} = f_\gamma/300$ to $k_{\max} = f_\gamma$, equally spaced in logarithmic scale. In dashed lines we show the same modes, in absence of backreaction.

a maximum value of roughly 0.3. This shows again that one can get large backreaction as long as ϕ_0/f_γ is bigger than a few times κ^{-1} , which means $\mathcal{O}(10-20)$.

As a next step we solve numerically the coupled system eqs. (3.1)–(3.3), for a given potential $V(\phi)$. We solve eq. (3.2) for a relatively large number, $N_{\text{modes}} \approx \mathcal{O}(10-100)$, of different discrete values of k . We set the following initial conditions:

$$\begin{aligned} A(0) &= \frac{1}{\sqrt{2k}}, & \dot{A}(0) &= \frac{ik}{\sqrt{2k}}, \\ \phi(0) &= \phi_0, & \dot{\phi}(0) &= 0. \end{aligned} \quad (3.11)$$

The first line corresponds to vacuum fluctuations for the gauge field. Then, we include the backreaction term in the system of differential equations by discretizing the integral in N_{modes} logarithmic intervals. We check that the result is not sensitive to the number of modes, as long as we capture the relevant range. In order to find such a range we plot the backreaction spectrum, defined through

$$\mathcal{B} \equiv \int \frac{dk}{k} P_{\mathcal{B}}(k), \quad (3.12)$$

and we will check that we integrate over the relevant region.

We see in figure 1 that the modes evolve in time exponentially in the initial stage of the evolution until the backreaction term becomes important, and starting from this time the modes grow at a very low rate. However as time goes on there are more modes, with lower values of k , which follow the same behavior. This leads to a spectrum $P_{\mathcal{B}}(k)$ which is almost flat in a band, which becomes wider and wider as time goes on, as can be seen in figure 2.

The other important physical point, as we mentioned before, is that our coupling is an effective one, which we can trust only at $k < f_\gamma$ and so we always cut the integral off at $k = f_\gamma$. We did not attempt here at studying what happens in a more complete theory at higher energies, but we will comment on this in section 6. Note however that, if the range of tachyonic modes (i.e. the ones for which $k < k_p \equiv |\dot{\phi}/f_\gamma|$) is always smaller than the cutoff

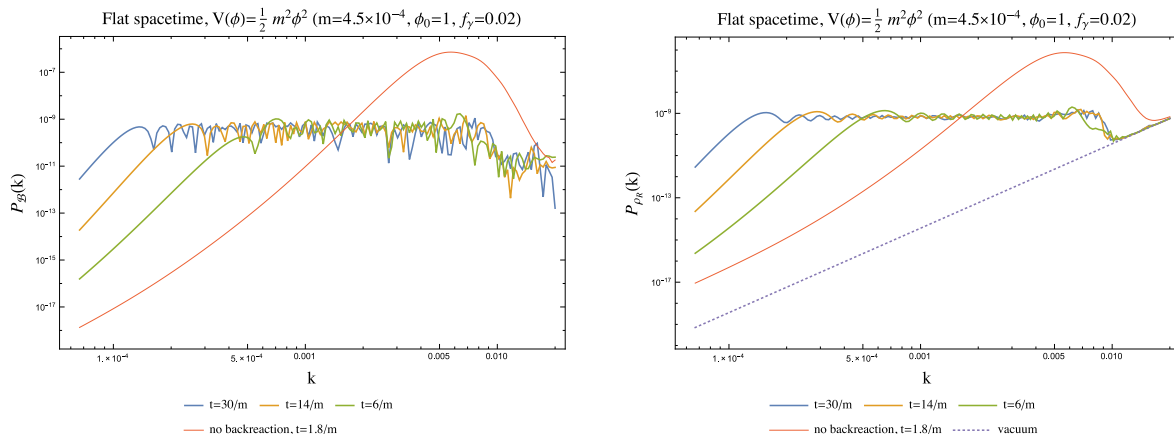


Figure 2. We show in the left panel the backreaction spectrum $P_B(k)$, defined in eq. (3.12), at three different times during the evolution, using a quadratic potential and in flat spacetime. In the right panel we show the spectrum of electromagnetic energy density $P_{\rho_R}(k)$, and also the vacuum energy density, which should be subtracted by renormalization. For both cases the flat region of the spectrum becomes wider as time goes on, since more infrared modes have enough time to get excited. The thin pink line represents the growth of modes in absence of backreaction, at a given time.

$k_c \equiv f_\gamma$, we do not need to consider such a high-energy completion of the theory. We will check in the following in which cases we this condition is met.

As anticipated we find that the evolution depends on the above discussed condition on f_γ/ϕ_0 . If we are in the regime $f_\gamma \gtrsim \phi_0$ the backreaction does not have enough time to become large, so nothing relevant happens and the fields falls down normally. If instead f_γ is at least about one order of magnitude smaller than ϕ_0 then the backreaction kicks in and we see a dramatic change of behavior: the field slows down and \mathcal{B} balances the potential for a very long time. Such a balance does not lead to a smooth behavior, but to rapid oscillations around an average slow evolution. We discuss a simple approximation in which is possible to verify the existence of oscillations in a simple way in appendix A. Such an average slow evolution leads eventually to an almost constant $\dot{\phi}$ regime, as we show in figure 3 for several values of f_γ . In figure 4 we show the size of the various terms (potential term, backreaction and $\ddot{\phi}$) in the equation of motion. The dependence of $\dot{\phi}$ (its average behavior, disregarding oscillations) on f_γ is found to be quadratic $\dot{\phi} = \alpha f_\gamma^2$, see figure 3, where $\alpha \approx -0.75$ is almost independent on the potential.

A similar dynamics with two stages of evolution must take place also in an expanding background, as long as the time scales of the potential and of the evolution of the gauge fields are much smaller than the Hubble time H^{-1} and as long as we follow the system for a time shorter than the Hubble scale (i.e. for much less than one efold of expansion of the metric). In fact with a steep potential and a rapidly growing gauge field, we should first have a free fall regime just because of the potential and then the backreaction should turn on exactly in the same way as in flat spacetime and start dominating the evolution. This should now lead to slow-roll and hence to inflationary expansion of the metric. Some differences will appear when following the evolution for more than one or two efolds: (1) we have to take into account of the redshift of the gauge fields modes, as inflation goes on for a long time, (2) there is a Hubble friction term also in the ϕ evolution. Such effects will lead to a third stage of the evolution, that we will address in the next section.

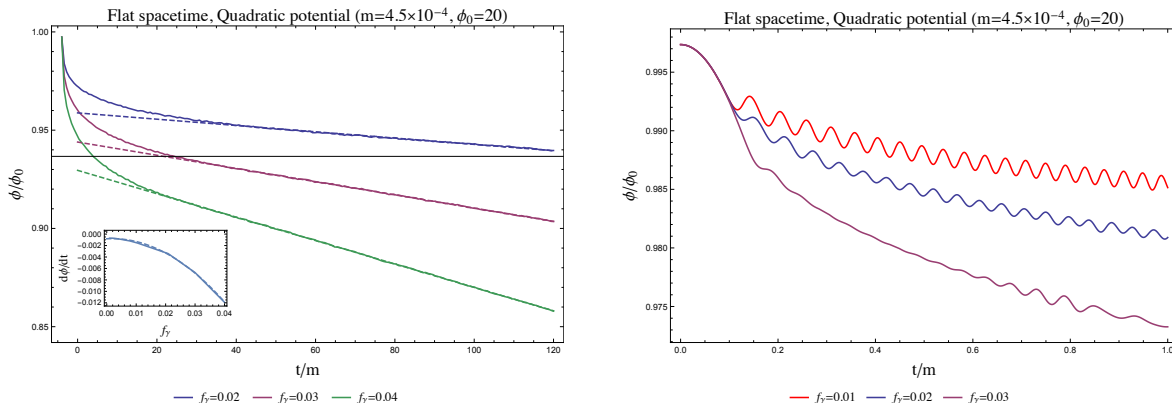


Figure 3. We show the time evolution of ϕ in the flat spacetime case, for a quadratic potential $V = \frac{1}{2}m^2\phi^2$, for several values of f_γ . The dashed lines represent linear interpolations of the average asymptotic evolution. We also show, in the small panel, the dependence of such asymptotic slope $d\phi/dt$ on f_γ , which turns out to be quadratic. In the right plot we show the ϕ evolution for a smaller range of time, in order to show that there are small oscillations at high frequency on top the average behaviors.

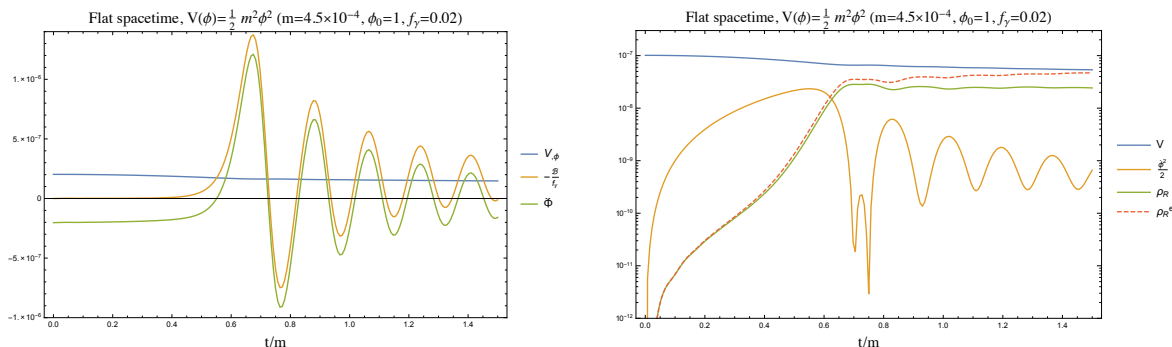


Figure 4. In the left panel we show the various terms (potential term, backreaction and $\ddot{\phi}$) in the equation of motion eq. (3.1), for a quadratic potential and in flat spacetime. In the right panel we show the size of the various energy densities: potential and kinetic energy of ϕ and the renormalized ρ_R , computed using eq. (3.13). The dashed line represents instead the result obtained using eq. (3.15), which agrees with the previous line at the beginning, but starts deviating at large time.

Finally we can also compute the energy density in the gauge field as,

$$\rho_R \equiv \left\langle \frac{B^2}{2} + \frac{E^2}{2} \right\rangle - \rho_R^{\text{vac}} = \frac{1}{2} \sum_{h=\pm} \int \frac{d^3k}{(2\pi)^3} \left[(|\dot{A}_h|^2 + k^2|A_h|^2) - k \right], \quad (3.13)$$

where E and B are electric and magnetic fields and where we have subtracted the vacuum contribution ρ_R^{vac} , using $A_k = e^{ikt}/\sqrt{2k}$, which takes care of the UV behavior. We also show its spectrum P_{ρ_R} , defined as $\rho_R = \int \frac{dk}{k} P_{\rho_R}$, in figure 2. Another way to estimate ρ_R is to rewrite eq. (3.1) as a conservation equation,

$$\dot{\rho}_\phi + \frac{\mathcal{B}\dot{\phi}}{f_\gamma} = 0, \quad \rho_\phi \equiv V + \frac{\dot{\phi}^2}{2}, \quad (3.14)$$

which shows that there is energy dissipation in the scalar field. Since the total energy density

is conserved, such an energy flow must go into a gauge field energy density ρ_R as follows:

$$\dot{\rho}_R - \frac{\mathcal{B}\dot{\phi}}{f_\gamma} = 0. \quad (3.15)$$

By integrating numerically this equation, the ρ_R computed in this “effective” way should coincide with the integral over the modes given in eq. (3.13). We find that indeed they coincide in the initial stages of evolution, see figure 4, but they start differing after some time, for unknown reasons. Note however that the two ways of computing ρ_R nicely coincide instead in the full case in an FLRW background, discussed in the next section. Another important remark is that we can see in the same plot that the kinetic energy of ϕ is subdominant with respect to ρ_R .

4 Inflation

We consider now our system in a FLRW metric with conformal time τ

$$ds^2 = a(\tau)(d\tau^2 - d\vec{x}^2). \quad (4.1)$$

The equations of motion are [5]:

$$\begin{aligned} \phi'' + 2aH\phi' + a^2V_{,\phi}(\phi) + a^2g_{,\phi}(\phi)\mathcal{B} &= 0, \\ A''_{\pm} + (k^2 \mp kg')A_{\pm} &= 0, \\ \mathcal{B} = \frac{\langle F\tilde{F} \rangle}{4} &= \frac{1}{2a^4} \int \frac{d^3k}{(2\pi)^3} k \frac{d[|A_+|^2 - |A_-|^2]}{d\tau}, \end{aligned} \quad (4.2)$$

where a $'$ means a derivative with respect to τ . Note that the gauge field equation is formally unchanged, while the scalar field has Hubble damping. Note also that $\langle F\tilde{F} \rangle$ has now a dilution factor a^{-4} . In an inflationary background this may seem to induce a large suppression, but it is actually not an issue. In fact such a dilution is counterbalanced by the explosive growth of the modes, keeping the value of $\langle F\tilde{F} \rangle$ roughly constant for a large number of e-folds. The equation of motion for the scale factor is simply the Friedmann equation,

$$H^2 \equiv \left(\frac{a'}{a^2}\right)^2 = \frac{V(\phi) + \frac{\phi'^2}{2a^2} + \rho_R}{3M_{\text{Pl}}^2}, \quad \rho_R \equiv \frac{1}{2a^4} \sum_{\pm} \int \frac{d^3k}{(2\pi)^3} [|A'_h|^2 + k^2|A_h|^2 - k], \quad (4.3)$$

where $M_{\text{Pl}} = 2.43 \times 10^{18}$ GeV is the reduced Planck mass and where the radiation energy density ρ_R has been renormalized. Both ρ_R and the field kinetic energy are subdominant compared to $V(\phi)$ if inflation is successfully generated.

Note that the k modes are getting redshifted and as inflation goes on the long wavelength modes go superhorizon and become less relevant in $\langle F\tilde{F} \rangle$ (because of the weight $k d^3k$), but at the same time new shorter wavelength growing modes are constantly generated. The expected outcome is an almost stationary process with an almost constant backreaction, which can be checked by integrating the full system of equations. Moreover, considering the full system also includes the intermediate case in which ϕ could have both electromagnetic and gravitational friction, in comparable amounts.

As we said, we expect that the initial behavior of ϕ must be very similar to the flat spacetime case, at least when following the system for less than about one e-fold. We are

interested in fact in a case in which the potential is steep and thus its typical timescale is much smaller than H^{-1} ; the same is true for the gauge fields, whose evolution is even faster. So the onset of backreaction happens under the same conditions that we have seen in the flat spacetime case, namely that $\phi_0 \gg f_\gamma$. However, once the backreaction sets in, it leads to a very slow evolution of ϕ and so the Hubble expansion may start to become important. This is indeed the case, as we discuss below.

Such a stage of evolution in an inflationary background has been studied in [5], where an analytical estimate of \mathcal{B} has been given. Assuming that backreaction can indeed become large the authors have worked under the hypothesis that a regime with constant $\dot{\phi}$ can be reached. In such a background they considered a de Sitter metric and computed analytically the evolution of the gauge fields setting an initial condition at infinite past time and looking for the asymptotic solution at future infinity. We have checked that their approximate solution

$$A_\pm^{an} = \frac{1}{\sqrt{2k}} \left(\frac{k}{2\xi aH} \right)^{1/4} e^{\pi\xi - 2\sqrt{2\xi k/(aH)}}, \quad (4.4)$$

gives the correct size for both the imaginary and the real part of the full numerical solution, as a late time behavior, assuming constant $\dot{\phi}$ and H , where $\xi \equiv \frac{\dot{\phi}_,N}{2f_\gamma}$. After an integration in k they have then derived the following equation for $\dot{\phi}$

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = \text{sign}(\xi) \frac{\mathcal{I}}{f_\gamma} \left(\frac{H}{\xi} \right)^4 e^{2\pi|\xi|}, \quad (4.5)$$

where $\mathcal{I} = 2.4 \times 10^{-4}$ (we have introduced the $\text{sign}(\xi)$ to include also the case $\xi < 0$). Given this equation, one could infer the value of ξ just equating approximately $V_{,\phi}$ to the backreaction term, as suggested in [5]. A caveat is that this would lead to two solutions: one at very small ξ and the other one at ξ larger than $\mathcal{O}(1)$. We checked that solving numerically eq. (4.5) leads rapidly to the first or the second solution, depending on the initial velocity of the field. Such an ambiguity is not present in our full numerical solutions, which we discuss in the next section, and the solution that is reached after 1–2 e-folds agrees qualitatively well with the second solution, as discussed in the next section. Such a second solution is approximately given by eq. (11) of [5]:

$$\xi \approx \frac{1}{2\pi} \log \left(\frac{9M_{\text{Pl}}^4 f_\gamma V_{,\phi}}{\mathcal{I}V^2} \right). \quad (4.6)$$

5 Potentials and results

In order to integrate the equations in a more efficient way we change variable from τ to the number of e-folds $N \equiv \log(a)$, choosing as initial value $N_0 = 0$ and using eqs. (3.11) as initial conditions (but replacing d/dt with $d/d\tau$), leading to

$$\begin{aligned} H^2 \phi_{,NN} + 3H^2 \phi_{,N} + HH_N \phi_N + V_{,\phi}(\phi) + g_{,\phi}(\phi)\mathcal{B} &= 0, \\ e^{2N} H^2 (A_{\pm,NN} + A_{\pm,N}) + e^{2N} HH_{,N} A_{\pm,N} \\ &+ (k^2 \mp ke^N Hg_{,N}) A_\pm = 0, \\ \mathcal{B} &= \frac{e^{-4N}}{2} \int \frac{d^3k}{(2\pi)^3} k H e^N \frac{d[|A_+|^2 - |A_-|^2]}{dN}, \\ H^2 &= \frac{V + \rho_R}{3M_{\text{Pl}}^2 - \phi_N^2/2}. \end{aligned} \quad (5.1)$$

We analyze potentials of the following types

$$(i) \quad V(\phi) = \frac{1}{2}m^2\phi^2, \quad (5.2)$$

$$(ii) \quad V(\phi) = \Lambda^4 \left[1 - \cos\left(\frac{\phi}{M}\right) \right]. \quad (5.3)$$

The first is a simple minimal choice, while the second is particularly interesting because it is an axion potential (plus a constant, such that the minimum at $\phi = 0$ has $V(0) = 0$). Let us in fact stress a crucial point here, taking as an illustration the QCD axion, though keeping in mind that this is generic for any axion-like field. In the QCD case the coupling which is needed to solve the strong CP problem is the one with gluons

$$\mathcal{L}_G = \frac{\phi}{f_G} G_{\mu\nu} \tilde{G}^{\mu\nu}. \quad (5.4)$$

Such a coupling to non-abelian fields breaks the shift symmetry $\phi \rightarrow \phi + c$, leaving only the symmetry $\phi \rightarrow \phi + 2\pi f_G$ and inducing a periodic potential of the above form with $\Lambda = \Lambda_{\text{QCD}}$ and $M = f_G$ (note that this is only an approximation for the QCD axion, which is good around minima [8]). The relevant point here is that we may assume that the field has a starting point in a generic position, displaced from the minimum by an amount of order $\phi_0 \approx f_G$. Therefore, as we have discussed, the condition we need for inflation is that $f_\gamma \ll f_G$, by about one or two orders of magnitude. Note that, although in the vast majority of analyses of the QCD axion these two couplings are considered to be comparable, they are actually two independent parameters. Moreover, even if the coupling to photons is much larger it does not break the ϕ shift symmetry and therefore it cannot induce large loop corrections to the gluon coupling. It is therefore allowed to have $1/f_\gamma \gg 1/f_G$.

Hence, a crucial observation is that an axion field coupled stronger to abelian than non-abelian gauge fields is a viable candidate for dissipative inflation. It is very interesting to check whether using the QCD axion itself for inflation with such couplings can work, which we discuss later.

It turns out to be possible to integrate eqs. (5.1), with a few complications. First, as seen also in the previous section, one needs to include a large enough density of modes so that the evolution converges to a smooth one. Second, one needs to take care of the fact that a huge range of k 's has to be used during inflation, since the relevant modes are constantly redshifted away. Moreover, as we already have discussed, we do not have control of the theory for physical momenta above the cutoff. However, unless f_γ is extremely small, typically the modes above the cutoff would just oscillate anyway. We deal therefore with this situation by freezing the modes in the numerical solution, as long as they satisfy $k/a > f_\gamma$ and then starting integrating only when such a condition is met. However, when f_γ is very small the modes with $k/a > f_\gamma$ could also be in the unstable region. In this case the effective theory is not adequate and the full answer should come from a UV complete theory.

We show the mode evolution in figure 5 and the relative spectra in figure 6. The field evolution is shown in figure 7, for a quadratic potential and for different values of f_γ . Some features are very striking. When the friction kicks in, it is very efficient and slows down dramatically the evolution of the field. Even if we integrate for only 10–20 e-folds it is obvious by visual extrapolation that the slow roll keeps going easily for much more than 60 e-folds, as long as f_γ is smaller than a threshold value. Our numerical results agree qualitatively with [6], although in that work the parameters have been chosen so that most

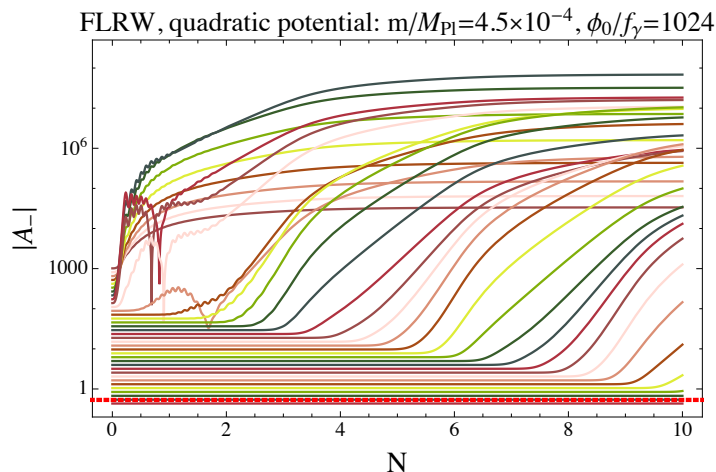


Figure 5. We show the evolution of the the gauge field A_- as a function of the number of e-folds, ranging from $N_i = 0$ up to $N_f = 11$, for different k 's in a FLRW inflating background. The last line in the bottom of the plot corresponds to a mode with maximal value of the momentum $k_{\max} = a_f f_\gamma$ while the other lines correspond to momenta which decrease with equal logarithmic spacing down to the minimal value of $k_{\min} = a_i f_\gamma / 50$, where $a_i = 1$ and $a_f = e^{N_f}$ are respectively the initial and final value of the scale factor. We used here units of $\phi_0 = M_{\text{Pl}} = 10$.

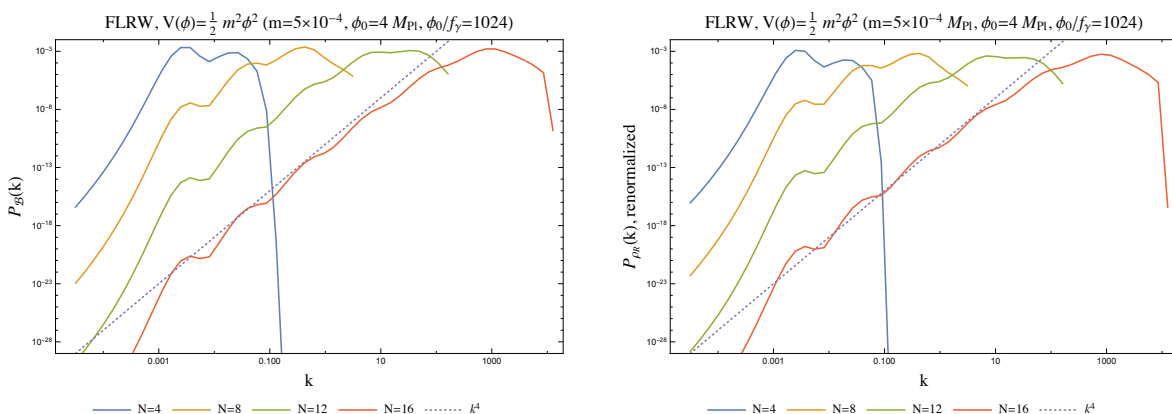


Figure 6. We show in the left panel the backreaction spectrum $P_{\mathcal{B}}(k)$, defined in eq. (3.12), at three different numbers of e-folds N during the evolution, using a quadratic potential and in a FLRW inflating background. In the right panel we also show the spectrum of the renormalized electromagnetic energy density $P_{\rho_{\text{R}}}(k)$. We show as a reference, in dotted line, a k^4 behavior. Both plots have a range of modes between $k_{\min} = a_i f_\gamma / 50$ and $k_{\max} = a_f f_\gamma$, where a_i and a_f are respectively the initial and final value of the scale factor.

of the inflationary trajectory is due to the usual Hubble friction dominated evolution and only a few more e-folds are added due to gauge field backreaction.² This has been motivated by the need of keeping the parameter ξ smaller than roughly 2, which supposedly is required to avoid large non-gaussianities. We do not apply such constraints, since as we argue here below, a reliable calculation of the curvature perturbations has not been performed yet in the backreaction dominated case, and hence it is even more the case for its three-point function.

²Similar results are found also in [9, 10], which uses also lattice simulations for the interacting scalar and gauge fields during and immediately before reheating.

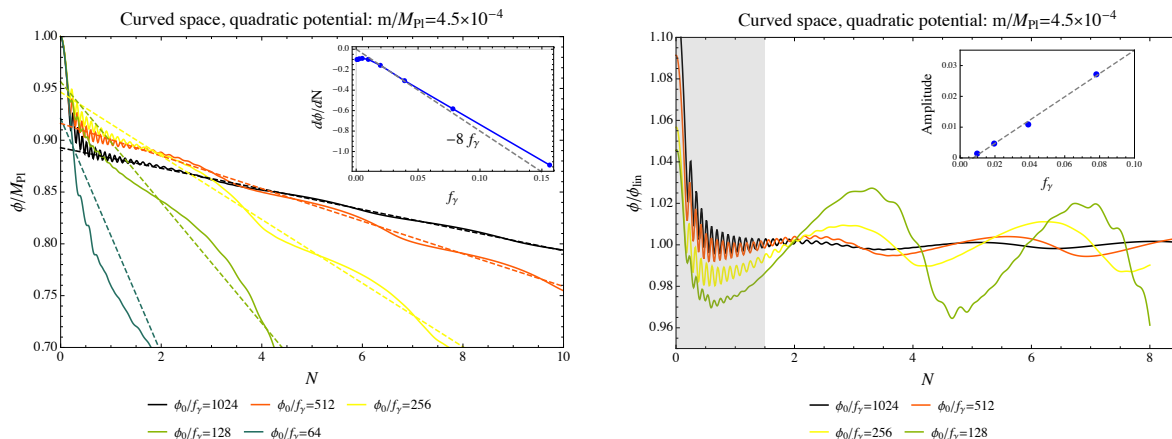


Figure 7. We show in the left panel the evolution of ϕ in a FLRW background, as a function of the number of e-folds N , for several values of f_γ . We show also linear interpolations in dashed lines and in the small panel we display the scaling of their slopes with f_γ : note that $d\phi/dN$ goes roughly linearly as $8f_\gamma/\phi_0$ (except at very small f_γ , where modes of $k > f_\gamma$ start being important, and are ignored in our effective treatment). This agrees quite well with the analytical scaling $d\phi/dN \approx 2\xi f_\gamma$ of [5], since $\xi \approx 3.5$ for these parameters. In the right panel we show the ratio of the full field evolution vs. the linear interpolation to highlight the presence of oscillations with a period of a few e-folds, after an initial transient. We show in the small panel the scaling of the amplitude of such oscillations with f_γ .

The main results of our analysis are as follows.

- If the field starts at rest it begins falling down rapidly but, if $\phi_0/f_\gamma \gtrsim \mathcal{O}(10)$, it is suddenly slowed down reaching a second stage of evolution similar to the one studied in the flat spacetime case, where its velocity $\dot{\phi}_1$ tends to a constant (plus some high frequency oscillations) and scales linearly as a function of f_γ as in figure 3. These two stages can be seen in the first efold of evolution, looking also at figure 8.
- If this friction-dominated regime lasts for at least about one efold, the field enters a third stage in which the Hubble friction and the redshift of modes become important: high frequency oscillations disappear and ϕ reaches another asymptotic velocity $\dot{\phi}_2$. This stage is quite well approximated by $d\phi/dN \approx 2\xi f_\gamma$, as in [5] (and so $\dot{\phi}_2 \approx 2\xi f_\gamma H$). Therefore the total number of e-folds is approximately $\phi_0/(2\xi f_\gamma)$.
- Another relevant feature is that the numerical solutions show a stable evolution, but with slow superimposed oscillations, as shown in the right panel of figure 7. Such oscillations do not seem to disappear for any of the analyzed values of f_γ and they do not vanish by increasing the precision of the numerical integration or the number of modes. Therefore we believe they are a physical feature of the evolution. The field starts falling down but then the backreaction stops it and as a consequence the gauge modes are afterwards less amplified and the field can move again until the same pattern repeats itself. Such a pattern is consistent with the spikes found also in [6]. The amplitude of such oscillations is found to depend linearly on f_γ and it is smaller than about $\mathcal{O}(0.1\%)$ for cases with a total number of e-folds of at least 60. The period instead seems almost independent of f_γ , and turns out to be around 4–5 e-folds. This phenomenon must lead to potentially observable consequences, at the level of the density and tensor perturbations, most likely as oscillations in the power spectrum and therefore as a

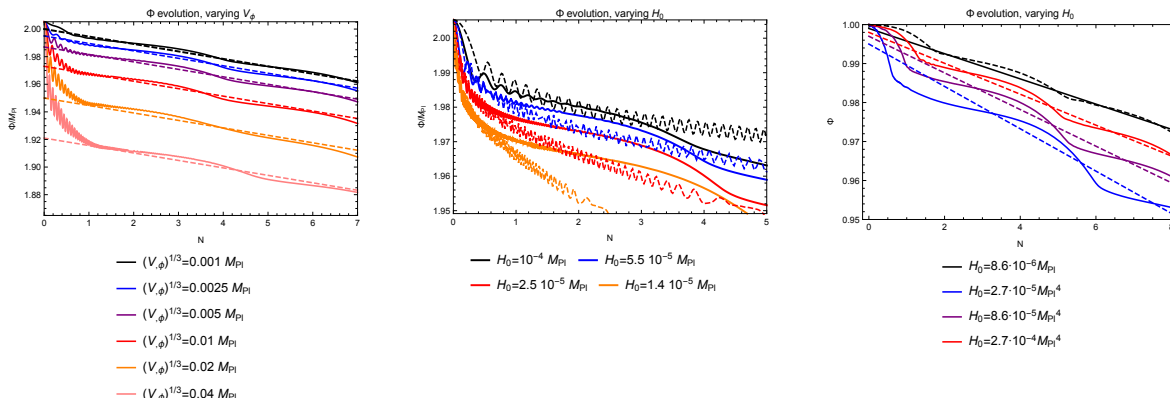


Figure 8. In the left panel we show the dependence of the field evolution on $V_{,\phi}$, keeping fixed all other parameters, including the height of the potential $V(\phi_0)$ (achieved by adding a suitable constant to it). Note that the initial transient stage at $N \lesssim 1$, as in the flat spacetime case depends on the size of $V_{,\phi}$. Instead in the subsequent stage, a dependence on $V_{,\phi}$ is not visible. The dotted lines are linear interpolations of the final stage, which all have the same slope of about 0.0027, corresponding roughly to 740 e-folds. We have chosen here a quadratic potential, with $H_0 = 5.5 \times 10^{-5} M_{\text{Pl}}$, $\phi_0 = 2M_{\text{Pl}}$, $f_\gamma = \phi_0/2500$. In the central panel we show in solid lines the dependence of the field evolution on H_0 , keeping fixed all other parameters, including $V_{,\phi}$. Note that after the initial transient stage at $N \lesssim 1$, there is a second stage, in which the slope $\phi_{,N}$ depends very weakly on H_0 . The dotted lines here would be the field evolutions in the flat spacetime case (using $N = H_0 t$ with $H_0^2 = V(\phi_0)/(3M_{\text{Pl}}^2)$). We have chosen here a quadratic potential, with $m = 6 \times 10^{-5} M_{\text{Pl}}$, $\phi_0 = 2M_{\text{Pl}}$, $f_\gamma = \phi_0/2500$. In the right panel we considered a wider range for H_0 , so that a weak dependence on it becomes visible. The dashed lines are the analytical expression given by eq. (4.6) (adjusted with a constant to account for the initial transient). Here $m = 6.6 \times 10^{-6} M_{\text{Pl}}$ and $\phi_0 = M_{\text{Pl}}$.

potentially unique signature of the model. It is tempting to mention that the presence of such oscillations might be associated to features of the CMB spectrum, which do not fit in the usual single field slow-roll scenario: for instance, the power suppression on large scales [11] or a possible oscillation in the power spectrum [11, 12]. We postpone anyway a detailed study to future work, also because in the present paper we do not use any constraints from perturbations around such a background solution, as we will explain later.

- If we decrease the energy scale of the potential keeping fixed all other parameters, we decrease H and so during the first friction-dominated stage the field falls down very quickly since $d\phi/dN \simeq \dot{\phi}_1/H \simeq \alpha f_\gamma^2/H$. If the evolution happens at very low energy H it can therefore happen that the field falls down to the minimum in much less than 1 efold, and so in this case the second stage of friction dominated inflation can never actually start. This is a concern in cases such as inflation with the QCD axion, where H is very small since the scale is Λ_{QCD} .

Then, we show the dependence of the asymptotic inflationary $\dot{\phi}_2$ on V' and V in figure 8, which confirm that the dependence is very weak, as expected from eq. (4.6). We also show in the left panel of figure 9 the size of the various terms in the ϕ equation of motion.

Note, moreover, that the background equations can be rewritten as:

$$3M_{\text{Pl}}^2 H^2 = \rho_\phi + \rho_{\text{R}}, \quad (5.5)$$

$$\dot{\rho}_\phi + 3H\dot{\phi}^2 + \frac{\mathcal{B}}{f_\gamma}\dot{\phi} = 0, \quad (5.6)$$

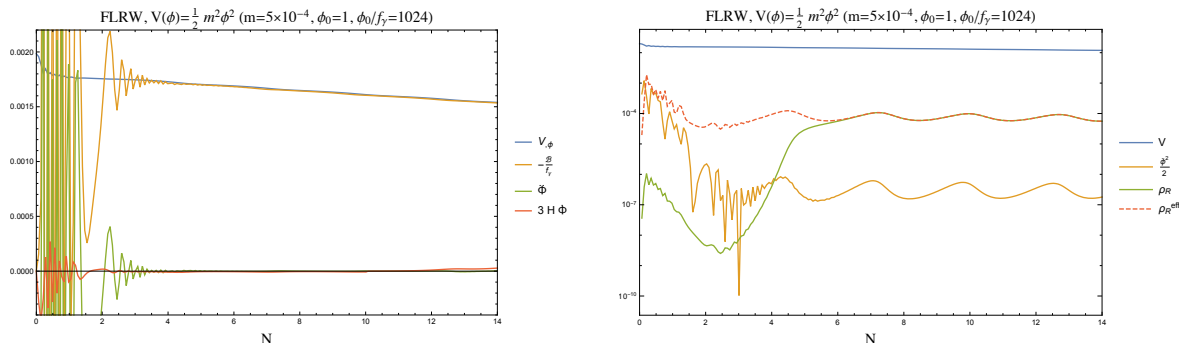


Figure 9. In the left panel we show the various terms (potential term, backreaction and $\ddot{\phi}$) in the full equation of motion eq. (4.2), for a quadratic potential and in FLRW spacetime. In the right panel we show the size of the various energy densities: potential and kinetic energy of ϕ and the renormalized ρ_R , computed using eq. (4.3). The dashed line represents instead the effective way of computing the energy density, as in eq. (5.7).

$$\dot{\rho}_R + 4H\rho_R - \frac{\mathcal{B}}{f_\gamma}\dot{\phi} = 0, \quad (5.7)$$

$$\rho_\phi = V(\phi) + \dot{\phi}^2/2. \quad (5.8)$$

As a response to the background ϕ evolution an electromagnetic field is produced, which on average is homogeneous and isotropic, and it turns out that $\rho_R \gg \dot{\phi}^2/2$, as can be seen from the right panel of figure 9. This fact is crucial for the treatment of perturbations in such a scenario, as discussed below.

It is also important to stress how the end of inflation and reheating happens here. Actually there is no clear distinction between inflation and reheating, since radiation is produced continuously and its energy density stays roughly constant in time. The crucial difference is only that during inflation the potential energy dominates. We can find the field value ϕ_{RH} and also the temperature T_{RH} at reheating just equating by definition the potential to the radiation energy density ρ_{RH} :

$$V(\phi_{RH}) = \rho_{RH}, \quad T_{RH} = \left(\frac{30\rho_{RH}}{\pi^2 g_*} \right)^{1/4}, \quad (5.9)$$

where, to define a temperature, we assumed that the radiation can actually reach quickly a thermal equilibrium (and g_* is the number of degrees of freedom in the plasma which is formed). Note that, remarkably, reheating here is fully fixed in this model and T_{RH} is already determined by the amount of radiation present during inflation (which is almost constant), in stark contrast with the usual slow-roll models with flat potentials, which have model-dependent reheating scenarios. We show one case, in figure 10, in which it is visible that the field is overdamped and just slows down approaching $\phi_{RH} \approx 0$, which is consistent also with [6], while ρ_R starts decreasing as a^{-4} after $\phi = \phi_{RH}$. In some cases, if the coupling $1/f_\gamma$ is not very large, the field may also perform a few damped oscillations before relaxing at zero. An estimate of ρ_{RH} , and so of the reheating temperature, is given by $\rho_{RH} \approx \mathcal{B} \approx V' f_\gamma$, as can be seen from figure 9.

It is easy to check that inflation with an axion-like potential, as in eq. (5.3), works practically in the same way as the quadratic case, as long as one starts in a generic point of the potential (we do not consider here fine-tuned situations with the initial ϕ very close to

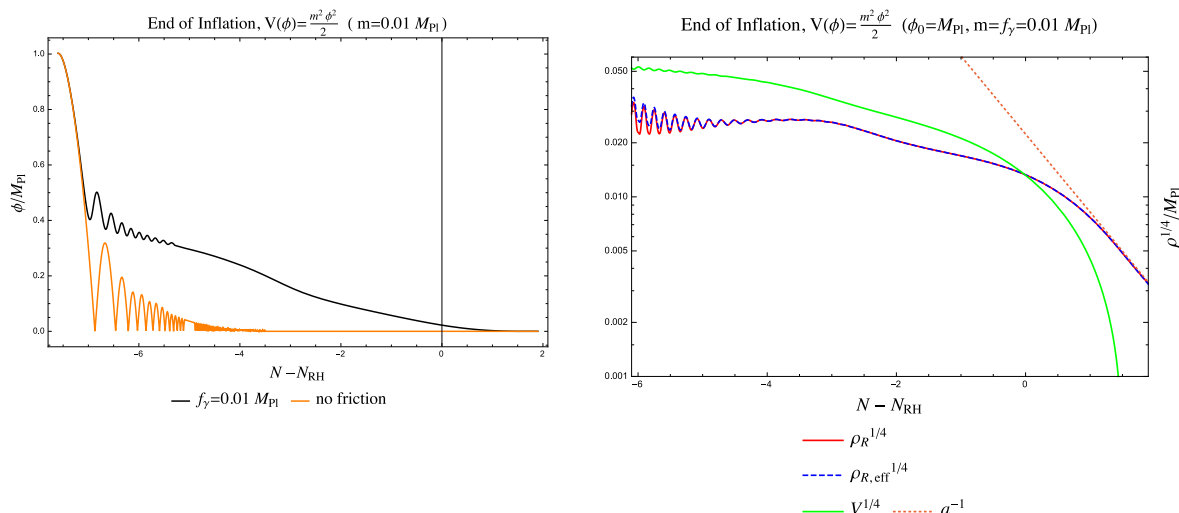


Figure 10. In the left panel we plot the field evolution close to the end of inflation: the field slows down and stops in zero, as opposed to the standard reheating picture with oscillations around the minimum. In the right panel we show the energy density stored in the potential V and in the gauge field. The red line corresponds to ρ_R found using the exact (renormalized) value eq. (4.3), while the blue dashed line corresponds to solving for eq. (5.7). Note that after that the potential becomes subdominant, at $N = N_{\text{RH}}$, radiation domination starts and ρ_R scales simply as a^{-4} .

the maximum). We show some solutions for the axion potential in figure 11. A particularly interesting case is the one of the QCD axion, with $\Lambda = \Lambda_{\text{QCD}}$ and so inflation happens at very low H . The reheating temperature would be at most of order $T_{\text{RH}} \lesssim \Lambda_{\text{QCD}} \approx \mathcal{O}(100 \text{ MeV})$. More precisely, according to the above estimate $T_{\text{RH}} \approx \rho_{\text{RH}}^{1/4} \approx (V' f_\gamma)^{1/4} \approx \Lambda_{\text{QCD}} \left(\frac{f_\gamma}{f_G}\right)^{1/4}$. This can satisfy bounds from primordial nucleosynthesis only if $T_{\text{RH}} \gtrsim 5 \text{ MeV}$. Unfortunately at such low values of H our numerical solutions becomes exceedingly slow, and so we could not directly check if the scenario can work. Nonetheless, as already mentioned above, a major concern is to make sure that during the transient phase, in the first efold of expansion, the field does not already fall down to zero. Using our flat space estimates this should be possible for very small f_γ , since $\dot{\phi} = \alpha f_\gamma^2$ and so we should impose that the field excursion $\Delta\phi \lesssim f_G$, where $\Delta\phi = \dot{\phi} \Delta t \approx \frac{\dot{\phi}}{H} \approx \frac{f_\gamma^2 M_{\text{Pl}}}{\Lambda_{\text{QCD}}^2}$, which means $f_\gamma \lesssim \left(\frac{f_G}{M_{\text{Pl}}}\right)^{1/2} \Lambda_{\text{QCD}}$. Such a condition can be satisfied only if f_G is several orders of magnitude larger than M_{Pl} . Otherwise f_γ turns out to be smaller than Λ_{QCD} and therefore well below the experimental bounds [13].

Finally, let us comment on the issue of density perturbations, which in such a scenario is non-trivial. First of all, one may wonder whether the gauge modes generated by the background evolution may have a spectrum which can be relevant on large scales. The answer to this question is negative, as could already be seen from figure 6. In fact a given k mode is rapidly produced, when $k \approx \dot{\phi}/f_\gamma$, but afterwards its momentum redshifts and the growth is not fast enough to overcome the a^{-4} dilution. For this reason the spectrum of energy density is suppressed on large scales, roughly as k^4 , and therefore its contribution to large scale density perturbations is negligible. For the same reason also magnetic fields on large scales would have a negligible amplitude today.³

³Note however that [14] and [10] invoked an *inverse cascade* scenario in which a helical magnetic field can transfer power from small to large scales, which should apply also in our case.

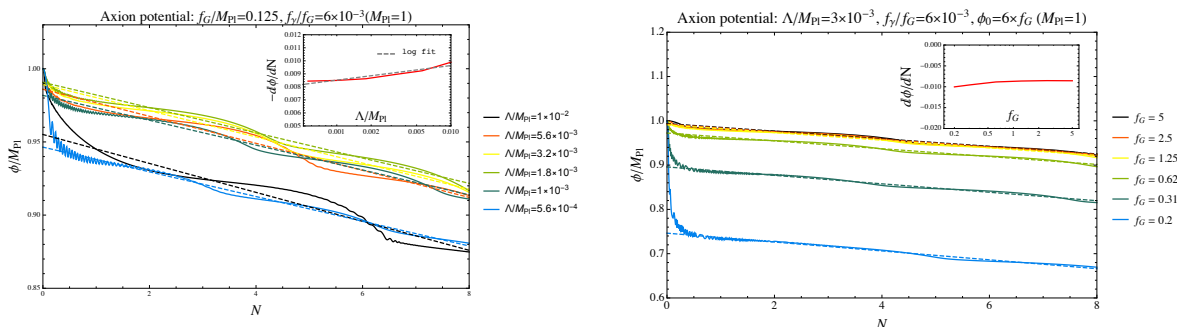


Figure 11. We show here the ϕ behavior on an Axion potential varying the overall scale Λ (left panel) and the scale f_G (right panel) in eq. (5.3). In both cases there is very weak dependence in the asymptotic late time behavior (in the small panels), while there is sensitivity only in the first efold of expansion.

Nonetheless there is another source of density perturbations, which is due to the usual vacuum quantum fluctuations of the field ϕ , coupled to a scalar part of the metric. To our knowledge, the study of such perturbations in the backreaction-dominated regime has been performed so far only in a heuristic way [5, 15–18] by looking at perturbations in ϕ and ignoring the metric components, while an equation for a gauge-invariant quantity, such as the comoving curvature perturbation has not been derived yet. Such a task is anyway highly non-trivial because of the presence of two components (the scalar and the photons), which exchange energy and because of the fact that the background itself has a contribution from the expectation value of the photon field, which is already an average over a collection of $k \neq 0$ modes, even in absence of fluctuations in ϕ and in the metric. We only mention here some qualitative features that we expect to be the most relevant.

One can think of estimating the amplitude of the primordial curvature perturbation A_ζ in the so-called spatially flat gauge, where $\zeta = H \frac{\delta\rho}{\rho}$ and where ρ is the total energy density and $\delta\rho$ its fluctuation; in usual single-field inflation this would give the well-known result

$$A_\zeta \simeq H \frac{V_\phi \delta\phi}{3H\dot{\phi}^2} \approx \frac{HV_\phi}{\dot{\phi}^2} \approx \frac{H^2}{\dot{\phi}}, \quad (5.10)$$

where we have used the fact that ϕ has an almost flat spectrum of fluctuations with amplitude $\delta\phi \approx \mathcal{O}(H)$. In our case instead such an expression would become

$$A_\zeta \simeq H \frac{\delta\rho}{\dot{\rho}} = H \frac{V_\phi \delta\phi}{4H\rho_R} \approx \frac{V_\phi \delta\phi}{\rho_R}, \quad (5.11)$$

where we have assumed that most of the energy density is stored in the potential of the ϕ field and so $\delta\rho \approx V_\phi \delta\phi$. The major challenge here is to know whether ϕ has a nearly flat spectrum and whether its amplitude is given by H or some other quantity. One would have to write down and solve the coupled system of gauge field, metric and ϕ fluctuations around our background and quantize it to find the vacuum fluctuations.

Other subtle points are the following. In [5] a term proportional to $\delta\dot{\phi}$ has been introduced by hand, on the basis that a $\delta\dot{\phi}$ should also dissipate its kinetic energy by exciting photons. However, we argue that the situation is more involved and a possible dissipative effect has to depend on the physical momentum $p_{\text{phys}} = p/a$ of the perturbation $\delta\dot{\phi}$, for two reasons. First, the equation of motion for the vector potential \mathbf{A} contains an extra term of

the form $\nabla(\delta\phi) \times \mathbf{A}'$ which becomes important for large p/a , so that the analogy with the photons excited by a background $\phi(t)$ breaks down and we cannot expect a coherent excitation of photons as a response to a $\delta\phi$. Second, even ignoring this, one would write down an equation of motion for A_{\pm} similar to eq. (5.1), assuming a time dependent $\delta\dot{\phi}$ as a source. For instance, a massless field at $p/a \gg H$ would just have vacuum fluctuations with $\delta\phi_p = \frac{1}{a} \frac{e^{ip\tau}}{\sqrt{2p}}$ behavior in Fourier space. A given region of physical volume L^3 would have field oscillations in real space roughly of amplitude $1/L$ due to all modes with $p_{\text{phys}} \lesssim 1/L$. Keeping the modes around $p_L/a \approx 1/L$ this translates in $\delta\phi \approx p_L/a \sin(p_L\tau)$, whose time derivative is (neglecting the expansion, in the regime $p_L/a \gg H$) roughly $\delta\phi' \approx p_L^2/a \cos(p_L\tau)$. This leads to an equation of motion of the form

$$A_{\pm}'' + \left(k^2 \mp \frac{kp_L^2}{af_{\gamma}} \cos(p_L\tau) \right) A_{\pm} = 0, \quad (5.12)$$

which is very similar to eq. (3.9). It is easy to check that only if $p_L/a \gg f_{\gamma}$ this equation can have large exponents for the Mathieu functions for some values of k . As a conclusion, when $p_L/a \lesssim f_{\gamma}$ we do not expect any backreaction to be possible. Finally another concern is that in [5] an ‘‘inverse decay’’ of the gauge modes created by the background was considered as a source for $\delta\phi$. However, as long as the $\delta\phi$ modes are not frozen completely there should be also a direct decay term $\delta\phi$ into gauge modes, and so the dynamics is likely to be much more complicated.

We have therefore given arguments to conclude that a treatment of perturbations is very involved in this scenario and so we postpone this to future work. We only stress here that as a consequence of eq. (5.11) the relevant slow-roll parameters are likely to be given by the ratio of ρ_R over the Hubble rate, and its time variation. In analogy with the usual slow-roll parameters ϵ_{ϕ} and η_{ϕ} defined from the derivatives of ϕ , we can then define two slow-roll parameters for radiation:

$$\begin{aligned} \epsilon_{\phi} &\equiv \frac{\dot{\phi}^2}{2M_{\text{Pl}}^2 H^2}, & \eta_{\phi} &\equiv 2\epsilon_{\phi} + \frac{1}{2} \frac{d \log \epsilon_{\phi}}{dN}, \\ \epsilon_R &\equiv \frac{2\rho_R}{3M_{\text{Pl}}^2 H^2}, & \eta_R &\equiv 2\epsilon_R + \frac{1}{2} \frac{d \log \epsilon_R}{dN}. \end{aligned} \quad (5.13)$$

One can also define and compute total slow-roll parameters as

$$\epsilon \equiv -\frac{\dot{H}}{H^2} = \epsilon_{\phi} + \epsilon_R \equiv \epsilon_R(1 + \delta_{\phi}), \quad \eta \equiv 2\epsilon + \frac{1}{2} \frac{d \log \epsilon}{dN} \simeq \eta_R + \frac{3}{2} \delta_{\phi}(\eta_R + \eta_{\phi} - \epsilon_{\phi}), \quad (5.14)$$

where we have expanded at first order in the small parameter $\delta_{\phi} \equiv \frac{\dot{\phi}^2}{2\rho_R}$, showing therefore that both total slow roll parameters are to a good approximation given by ϵ_R and η_R . Nonetheless, it may be useful to plot the four slow-roll parameters for some illustrative cases. As already mentioned we leave for future work the calculation of the spectrum of perturbations, but it is reasonable to assume that ϵ_R and η_R could be the relevant parameters. As it can be seen from figure 12 the ϵ_{ϕ} and ϵ_R parameters are small, with $\epsilon_{\phi} \ll \epsilon_R$; however they have some superimposed oscillations that translate in values of η_R and η_{ϕ} which are not very suppressed, $\mathcal{O}(0.1-1)$. Of course we know that decreasing f_{γ} the oscillations in ϕ have smaller amplitude, which would imply smaller η 's; however, as we have seen, when f_{γ} is tiny the modes beyond the cutoff of the effective theory become not completely negligible. And

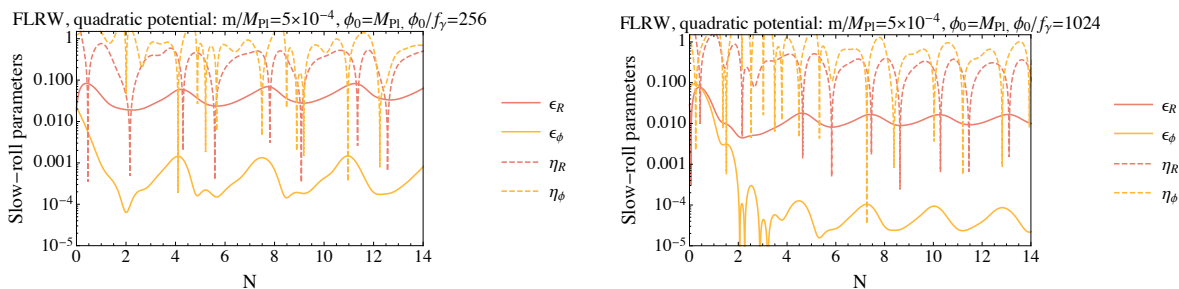


Figure 12. We plot the slow roll parameters defined in eqs. (5.13), for a quadratic potential $V = \frac{1}{2}m^2\phi^2$: ϵ_R is always much larger than ϵ_ϕ , and they are both always small. Instead η_R and η_ϕ are not extremely suppressed due to the oscillatory behavior of ϕ and in fact they are of $\mathcal{O}(0.1)$. Note also that all slow-roll parameters decrease for small f_γ .

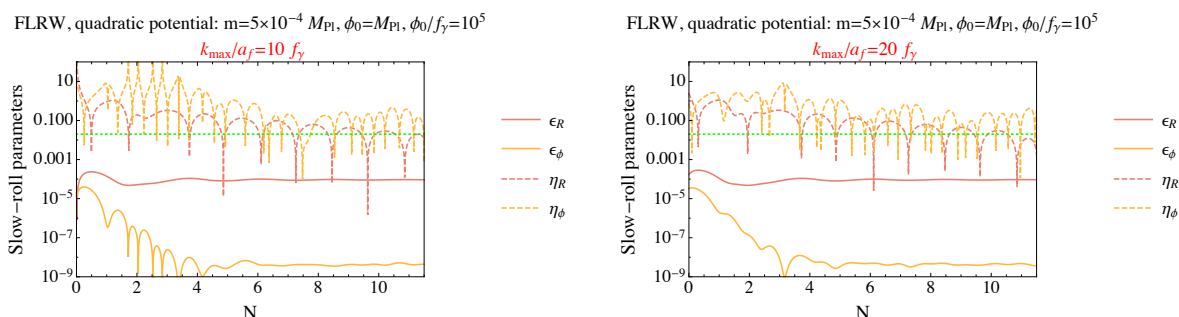


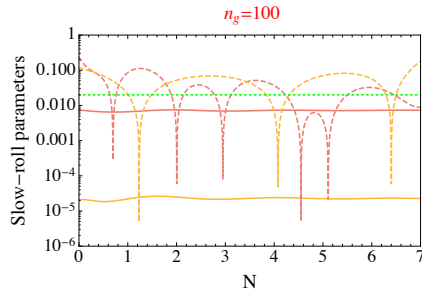
Figure 13. We plot the slow roll parameters defined in eqs. (5.13), for a quadratic potential $V = \frac{1}{2}m^2\phi^2$: ϵ_R is always much larger than ϵ_ϕ , and they are both always small. In this case we could obtain smaller η_R and η_ϕ than in figure 12 using two ingredients: a very small f_γ and also by including modes beyond the cutoff f_γ . The inclusion of such modes, while not justified in our effective treatment, turns out to provide a smoother evolution. In particular we have included modes up to $k_{\max}/a_f = 10f_\gamma$ and $k_{\max}/a_f = 20f_\gamma$ in the left and right panels, and also we have implemented in both a step function which freezes the modes only when $k/a > 80f_\gamma$. We show for visual reference the value 0.02 in green dashed.

in fact the only cases in which we could reach values of $\mathcal{O}(0.01)$ for η_R were obtained also by including modes beyond the cutoff f_γ . The inclusion of such modes, while not justified in our effective treatment, turns out indeed to provide a smoother evolution. For instance in figure 13 we have included modes up to $k_{\max}/a_f = 20f_\gamma$ and $k_{\max}/a_f = 10f_\gamma$, and also we have implemented a step function which freezes the modes only when $k/a > 80f_\gamma$. This shows that, in order to get a very smooth evolution, it seems to be necessary to be able to treat also the modes beyond the cutoff, which means dealing with a more fundamental renormalizable UV complete model. This is however beyond the scope of this paper.

Finally let us comment that another way to get a smoother evolution is to increase the multiplicity n_g of gauge fields (note this was also invoked in [5], but for the different purpose of suppressing the amplitude of perturbations). For instance in figure 14 if we consider a number of identical species $n_g \approx 10^2 - 10^3$ we get a smoother evolution and smaller and more stable values for η_ϕ and η_R .

Let us also stress that the background evolution implies that any additional light scalar field with mass $m \ll H$ would have an almost flat spectrum of fluctuations (but again with superimposed small oscillations) and therefore it may be used to induce a flat spectrum

FLRW, quadratic potential: $m=5\times 10^{-4} M_{\text{Pl}}$, $\phi_0=M_{\text{Pl}}$, $\phi_0/f_\gamma=1024$



FLRW, quadratic potential: $m=5\times 10^{-4} M_{\text{Pl}}$, $\phi_0=M_{\text{Pl}}$, $\phi_0/f_\gamma=10^5$

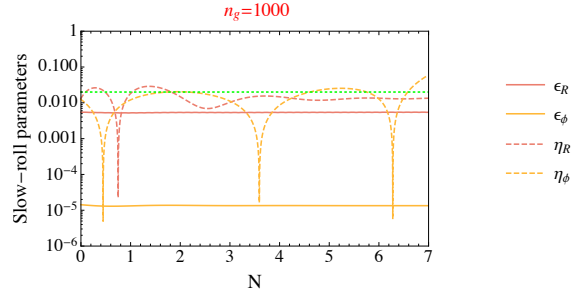


Figure 14. We plot the slow roll parameters defined in eqs. (5.13), for a quadratic potential $V = \frac{1}{2}m^2\phi^2$: ϵ_R is always much larger than ϵ_ϕ , and they are both always small. In this case we could obtain smaller η_R and η_ϕ than in figure 12 using a large number n_g of identical species for the gauge fields. Here the cutoff is at $k_{\text{max}}/a_f = f_\gamma$, as usual. We show for visual reference the value 0.02 in green dashed.

of density perturbations, if it could dominate at any time the total energy density of the Universe, i.e. through the so-called curvaton mechanism [19].

Another important fact is that the tensor modes from inflation should have two contributions: a vacuum fluctuation with an amplitude $A_T = \frac{8}{M_{\text{Pl}}^2} \left(\frac{H}{2\pi}\right)^2$, and another one sourced by the gauge field [17, 20–23]. Although we do not address such a calculation here, the generic prediction is again the presence of superimposed oscillations. In addition [17] there should also be parity violation, since the tensor modes produced by a source term should be chiral.

6 Discussion

We have found that a scalar with a generic potential can successfully achieve inflation at the background level through dissipation into gauge fields, as long as the coupling $1/f_\gamma$ is much larger than the inverse field excursion $1/\phi_0$, as in the case of an axion coupled to photons stronger than to gluons. This raises several questions and important issues.

First, it is interesting to consider whether such couplings might arise from a more complete theory. For instance, a way to generate such a coupling is to integrate out a fermion ψ , with U(1) charge g and with a $y\phi\bar{\psi}\gamma_5\psi$ term, where y is a coupling constant. This would mean that either ϕ is a pseudoscalar or alternatively, if ϕ is a scalar, that we have large CP violation. In any case this would generate an effective operator suppressed by the fermion mass m_f , of the form $g^2y\phi F\tilde{F}/m_f$, so that the new scale f_γ would be given by $f_\gamma \equiv m_f/(g^2y)$. However the very same coupling also induces a contribution to m_f of order $y\phi$. So in the end, barring cancellations with a tree level mass, we would typically have $f_\gamma = \phi/g^2$ and this could be much smaller than ϕ only if g is large, which would represent a strong coupling regime. In order to avoid such a strongly coupled regime one could invoke also the presence of several fermionic species. Let us comment also that in such UV completions of our model it should be possible also to treat cases in which the tachyonic physical momenta are larger than f_γ . However this is beyond the scope of the present paper.

It is also interesting to think about other implications. For instance a curious phenomenon may arise already during inflation. If the U(1) field is the actual electromagnetic field it would actually produce pairs of electrons and positrons and other particles if the en-

ergy scale is high enough, so this might induce an even more complex dissipative dynamics. We do not expect however this to change the overall qualitative picture. In fact if such pairs are created during inflation they would be rapidly diluted away.

Finally we comment again on the production of large scale magnetic fields: while we have seen that $P_{\mathcal{B}}$ due to the background evolution is suppressed on large scales, it is still possible that large scale perturbations on ϕ may source large scale magnetic fields or that some later mechanism can transfer power to larger scales (see [14] and [10]). Another interesting feature is the fact that we are producing a nonzero $\vec{E} \cdot \vec{B}$ and this might in principle be a source of parity violation in the early Universe [22], although again such an effect seems likely to be also suppressed at large scales. We postpone however such analysis to future work.

7 Conclusions

In this paper we have solved explicitly the dynamics for the background in a system in which inflation can take place without the need for a flat potential. This happens through an effective coupling of the form $\phi F\tilde{F}/f_\gamma$, as proposed already in [5], which produces an instability in the gauge fields that can be viewed as dissipation that slows down the ϕ field. We have studied the onset of such an instability starting from a static configuration of the scalar field by numerically solving the coupled system of scalar and gauge fields. We have shown that, even in flat spacetime, the mechanism is efficient as long as the instability is faster than the free field evolution, which can happen if the scale f_γ is smaller than the field excursion ϕ_0 . Then we have extended this setup to the FLRW case, showing that, if the mechanism can freeze the field value for at least about one efold, then the system enters the regime studied in [5] with the energy density staying nearly constant and the universe inflating. In particular $d\phi/dN \approx cf_\gamma$, where c is a numerical factor, which depends only logarithmically on the potential and on the Hubble scale and as long as the mass scale f_γ is much smaller than the field excursion ϕ_0 , the mechanism is highly efficient, providing easily more than the required 60 efolds of inflation. The mechanism is appealing, since it does not require any fine-tuned potential, nor superplanckian field excursions, and the basic requirements are: an equation of motion which violates CP (and therefore T), since it can contain a term proportional to $\dot{\phi}$, and the dissipation of energy into massless degrees of freedom, so that the friction is efficient for a very long time. We have studied the system cutting off modes with momentum $k > f_\gamma$. This is a good approximation except at extremely small f_γ (i.e. very large friction), in which case a full knowledge of the theory seems necessary beyond the effective theory. An important feature of the numerical solutions is the presence of oscillations in the background solution, with a typical period of about 4–5 efolds and an amplitude which is linear in f_γ and which is less than $\mathcal{O}(0.1)\%$ if a total number of efolds $N \gtrsim 60$ is required.

We have also argued that an axion field can have such properties if the coupling to photons $1/f_\gamma$ is much larger than the one to gluons $1/f_G$, since the latter scale determines the maximal field excursion, $f_G \sim \phi_0$.

We have left the calculation of the spectrum and the non-gaussianity of the perturbations for future work. This is a crucial point since according to [5] this model has a too large amplitude of the power spectrum A_ζ , unless a large number of species for the gauge fields is considered, and another concern is the size of the nonlinearity parameters f_{NL} , which was shown to grow large with ξ , at least in absence of backreaction [15, 21]. However we think that the dynamics of the perturbations in the backreaction dominated regime should be treated

more carefully. The action of a gauge-invariant variable (such as the comoving curvature perturbation ζ) should take into account of the non-trivial background in which an energy density ρ_R in radiation is present, so that $\dot{\rho} \approx 4H\rho_R$. So, an action to second and third order for ζ in the presence of ϕ , A_μ and other metric perturbations should be consistently written. Also, we have argued that the effect of gauge fields on the perturbations $\delta\phi$ cannot be treated on all scales as a dissipative term proportional to $\delta\dot{\phi}$, as in [5], due to the presence of gradients of $\delta\phi$ and due to the fact that such a dissipation should be efficient only on scales much larger than $1/f_\gamma$. Moreover we have pointed out that the interaction term between $\delta\phi$ and the gauge field should contain not only “inverse decays”, but also direct decays. Even if we did not solve the challenging task of solving for the perturbations, we have argued that new slow-roll parameters ϵ_R and η_R , proportional to ρ_R/H^2 and its time variations may be relevant for the evolution of scalar fluctuations. For this purpose we have shown that while ϵ_R can be rather small (though much bigger than the usual ϵ_ϕ parameter), it is more difficult to make η_R less than $\mathcal{O}(0.1)$. The latter may be achieved at tiny values of f_γ , but at the price of including modes of about a factor of 10 beyond the cutoff $k/a \lesssim \mathcal{O}(10)f_\gamma$. This is out of the regime of validity of our effective coupling, but nonetheless it may be an indication that in a UV complete theory a smooth evolution can be achieved. Alternatively we have shown that in the presence of a large number of gauge fields, $\mathcal{O}(10^2\text{--}10^3)$, η_R can be strongly reduced.

We have also argued that the addition of a curvaton field might imprint a flat spectrum of perturbations, in case the spectrum of ζ turns out to be negligible on large scales, since such a field would have an almost flat spectrum of amplitude H in this inflationary background. Finally we stress again that the background always has small superimposed oscillations in the evolution of H and thus, whatever is the mechanism of production of cosmological scalar and tensor perturbations, there should necessarily be an imprint on the observable density and tensor fluctuations at late times, which can constitute a distinctive generic feature of such a scenario.

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A Single mode approximation

We show here that it is possible, at least in the flat space case, to capture the physical effects by a single ordinary nonlinear differential equation which shows the existence of oscillations in the solution. This can be done under the assumption that a small range of k ’s, around a given k_0 , dominate \mathcal{B} in eq. (3.3), so we can approximate (keeping only one helicity)

$$\mathcal{B} \approx \frac{1}{4\pi^2} k_0^4 \frac{d}{dt} |A_-|^2. \quad (\text{A.1})$$

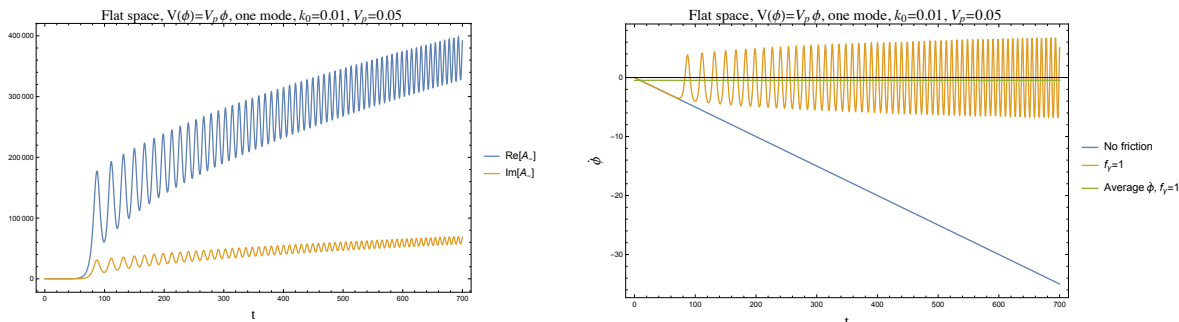


Figure 15. We show the time evolution, in a linear potential and in flat spacetime, of the gauge field A_- , for a single Fourier mode k_0 and we show that ϕ , taken from eq. (A.3), quickly goes to a friction dominated regime, in which it oscillates around a very small value.

We can also consider the special case of a linear potential $V(\phi) = V_p\phi$, so that we can rewrite eq. (3.1) as a conservation equation:

$$\frac{d}{dt} \left(\dot{\phi} + V_p t + \frac{1}{4\pi^2 f_\gamma} k_0^4 |A_-|^2 \right) = 0. \quad (\text{A.2})$$

This immediately points to an important fact, namely that the $\dot{\phi} = \text{constant}$ cannot be a solution. In fact, this could happen only if the third term canceled the second, which would mean $|A_-|^2 \propto t$. But this is impossible, since on a constant $\dot{\phi}$ background the solution is an Airy function (and at late time goes as an exponential), see eqs. (3.6)–(3.7). So the constant $\dot{\phi}$ solution can be reached only on average, which explains the presence of oscillations: if ϕ approaches a constant value then A_- grows very fast and this slows down ϕ , which in turn implies less A_- production and less friction, and so on.

The previous equation tells us that

$$\dot{\phi} = -V_p t - \frac{1}{4\pi^2 f_\gamma} k_0^4 |A_-|^2 + Q, \quad (\text{A.3})$$

where Q is given at an initial time $t_0 = 0$ by $Q = \frac{1}{4\pi^2 f_\gamma} k_0^4 |A(t_0)_-|^2 = \frac{1}{8\pi^2 f_\gamma} k_0^3$. We can now substitute $\dot{\phi}$ from the above equation in eq. (3.2) and get:

$$\ddot{A}_- + \left(k_0^2 + \frac{k_0}{f} V_p t \right) A_- + \frac{k_0^5}{4\pi^2 f_\gamma^2} A_- \left(|A_-|^2 - \frac{1}{2k_0} \right) = 0. \quad (\text{A.4})$$

So we have obtained a single differential equation, which however has a time-dependent mass and a nonlinear term. Such an equation does not seem to have an analytical solution. It is however very simple to solve this numerically and to show that it has a friction dominated regime with an oscillatory superimposed behavior. In particular we have checked that it has a good agreement with the full numerical solution and we show a solution in figure 15.

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