# SIMULATION OF HEAD-ON BEAM-BEAM LIMITATIONS IN FUTURE HIGH ENERGY COLLIDERS

X. Buffat, T. Pieloni, CERN, Geneva, Switzerland, J. Barranco, A. Florio, EPFL, Lausanne, Switzerland, C. Tambasco, CERN, Geneva, Switzerland and EPFL, Lausanne, Switzerland

*Abstract*

The Future Circular Hadron Collider (FCC-hh) project calls for studies in a new regime of beam-beam interactions. While the emittance damping due to synchrotron radiation is still slower than in past or existing lepton colliders, it is significantly larger than in other hadron colliders. The slow reduction of the emittance is profitable for higher luminosity in term of transverse beam size at the interaction points and also to mitigate long-range beam-beam effects, potentially allowing for a reduction of the crossing angle between the beams during the operation. In such conditions, the strength of head-on beam-beam interactions increases, potentially limiting the beam brightness. 4D weak-strong and strong-strong simulations are performed in order to assess these limitations.

## INTRODUCTION

In high energy lepton colliders, the strong emittance damping due to synchrotron radiations allows to reach small beam emittances. Yet the interplay between the non-linearities of the beam-beam forces and the quantum excitations results in an equilibrium emittance that is much larger than the lattice natural emittance, which is usually described as a maximum value of the beam-beam parameter [1] :

$$\xi = \frac{r_0 N_p}{4\pi\epsilon}, \qquad (1)$$

with $r_0$ the proton classical radius, $N_p$ the bunch intensity and $\epsilon$ the transverse normalised emittance assumed equal in both planes here. The design normalised emittance of the FCC-hh, $\epsilon = 2.2$ [$\mu$m] [2], is significantly larger than the lattice equilibrium emittance of $\epsilon_{\text{eq}} = 0.04$ [$\mu$m]. The effect of the quantum excitation is therefore negligible in the initial phase of the cycle. The emittance growth rate due to intrabeam scattering is also significantly longer than the emittance damping time due to the emission of synchrotron radiation ($\tau_{rad} = 1.1$ [h]), therefore it has a negligible impact with initial beam parameters. Consequently, the mechanisms limiting the beam brightness due to the non-linearities of beam-beam interactions and intrinsic sources of noise in lepton colliders are not applicable to the FCC-hh, at least in the initial phase of the cycle. Nevertheless the emittances may decrease and the beam brightness increase while producing luminosity, since the luminosity burn off is slower than the emittance damping mechanisms, reaching a point where the relative effect of intrabeam scattering and quantum excitations become dominant in the beam dynamics [3].

The understanding of the brightness limitations due to beam-beam interactions requires a proper modelling of the mechanisms in absence and in the presence of strong sources of intrinsic noise. This topic was covered, for example in [4], using the weak-strong approach. Here we recover some of these results and expose the issues when addressing the limitations of the weak-strong model with self-consistent macroparticle simulations.

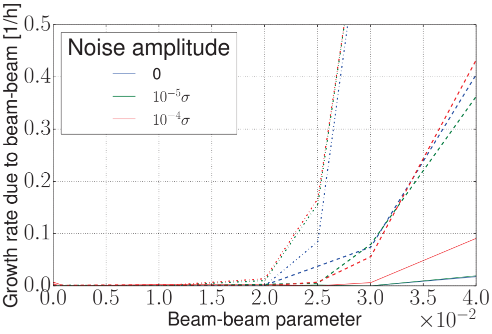

Figure 1: Simulated emittance growth due a single head-on beam-beam interaction in the weak-strong regime with different beam-beam parameters and in the presence of intrinsic noise. The solid lines corresponds to a configuration without chromaticity, the dashed to a chromaticity of 2 units and the dotted-dashed line to 10 units. The R.M.S. energy deviation is $10^{-4}$ and the horizontal, vertical and synchrotron tunes are respectively 0.31, 0.32 and $2 \cdot 10^{-3}$.

## WEAK-STRONG SIMULATIONS

We use the multiparticle tracking code COMBI [5] to evaluate the brightness limitations due to the non-linearity of the head-on beam-beam interactions in the presence of intrinsic sources of noise. A set of macroparticles initialised randomly in phase space with a Gaussian distribution are tracked through a lattice represented by a linear transfer map with the machine tune, including the first order chromaticity, and a non-linear beam-beam map. The latter assumes that the opposing beam has a Gaussian distribution with fixed parameters, the momentum of the particles are modified according to [6] :

$$\Delta x' = -\frac{2 r_0 N_p}{\gamma} \frac{x}{r^2} \left(1 - e^{-\frac{r^2}{2\sigma^2}}\right), \qquad (2)$$

with $\gamma$ the relativistic factor and $\sigma$ the transverse beam size at the interaction point. The intrinsic sources of noise are modelled as independent random kicks to the individual particles with a Gaussian distribution and a relative R.M.S

Copyright © 2016 CC-BY-3.0 and by the respective authors

ISBN 978-3-95450-147-2

amplitude of $\delta_i$. The emittance growth in absence of beam-beam interaction is given by :

$$\frac{1}{\epsilon}\frac{\partial \epsilon}{\partial t} = \frac{1}{2}\delta_i^2. \tag{3}$$

In order to characterise the effect of the non-linearities of the beam-beam interaction, we compare the evolution of the transverse emittances over $10^5$ turns in the presence of an intrinsic source of noise, with and without beam-beam interaction. The first $10^4$ turns are excluded from the analysis such as to avoid artefacts due to the strong non-linear re-matching. The difference in the fitted growth rates for different beam-beam parameters and noise amplitudes are shown in Fig. 1. In absence of chromaticity, the contribution of beam-beam interaction to the emittance growth remains negligible for beam-beam parameters up to 0.04. In the presence of strong intrinsic noise, non-linear diffusion mechanisms are enhanced and the impact on the emittance growth becomes significant for beam-beam parameters above 0.03. Similarly, the presence of chromaticity has a significant impact on the emittance growth which may be attributed to the modulation of the transverse tune of the individual particles with the synchrotron frequency. In these configurations, however, the presence of an intrinsic source of noise does not increase further the beam-beam contribution to the emittance growth.

Larger total beam-beam parameters were shown to have a low impact on the beam emittance in [4]. Two interaction points are considered with symmetric phase advances allowing for a mitigation of resonances between the two beam-beam interactions. While our implementation of the model is in qualitative agreement with former predictions, the configuration considered is pessimistic. It is used in the following to address the limitations of the weak-strong approach.

## STRONG-STRONG SIMULATIONS

The weak-strong model shows potential effects on the beam emittance for large beam-beam parameters. Since the non-linear diffusion mechanisms are not uniform in phase space, the beam distribution is distorted. This results in modifications of the beam-beam forces that are not taken into account in the weak-strong model. In order to go beyond the weak-strong approximation, the beam-beam forces have to be modelled in a self-consistent manner. The code COMBI implements such a strong-strong model, which differ from the weak-strong model as follows. Two sets of macroparticles are tracked synchronously through transfer maps representing the respective lattices, assumed identical for the two beams in our model. The non-linear beam-beam map is based on the other beam's charge distribution, derived from the macroparticle distribution. This approach requires the electromagnetic fields to be solved numerically at every beam-beam interaction, therefore the numerical solver needs to be both fast and accurate.

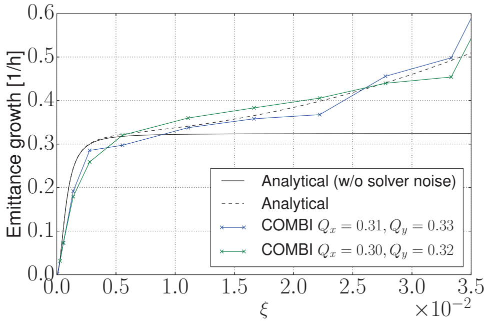

Figure 2: Transverse emittance growth in the presence of an external source of noise affecting identically all particles. The black lines show the analytical derivation [7] with and without the solver noise (Eq. 4) with $N_m = 10^6$. The self-consistent simulations with COMBI using the soft-Gaussian approximation follow the prediction of the analytical formula including the solver noise.

### *Noise Requirements*

To first order, the beam-beam force depend on the average position of the beam. When computed based on a set of $N_m$ macroparticles the relative error on the average is of order of $N_m^{-1/2}$. Considering the linear part of the beam-beam force, the resulting relative noise amplitude is given by :

$$\delta_{solver} = \frac{2\pi\xi}{\sqrt{N_m}} \tag{4}$$

The beam emittances reach an equilibrium value when the intrinsic sources of noise counter balance the effect of the synchrotron damping. Using Eq. 3, we find that at equilibrium the amplitude of the intrinsic noise is given by :

$$\delta_i = \sqrt{\frac{2}{\tau_{rad}}} \approx 4 \cdot 10^{-4} \tag{5}$$

In order to accurately simulate the beam-beam interactions in such regime, the solver noise must be significantly smaller, which translate into the following condition on the number of macroparticles :

$$N >> 2(\pi\xi)^2\tau_{rad} \tag{6}$$

For a beam-beam parameter of 0.03, we have $N >> 2 \cdot 10^5$. Strong-strong simulations of the emittance growth in the presence of external noise using the soft-Gaussian approximation show a good agreement with the theoretical expectation when the solver noise is properly included (Fig. 2). While the assumption taken to derive this first order estimate of the solver noise apply well to the soft-Gaussian approximation, they result in an significantly underestimation of the noise for other field solver.

### *Solver Noise*

In order to characterise the numerical noise of a given implementation of the field solver, we solve the field for a

Copyright © 2016 CC-BY-3.0 and by the respective authors

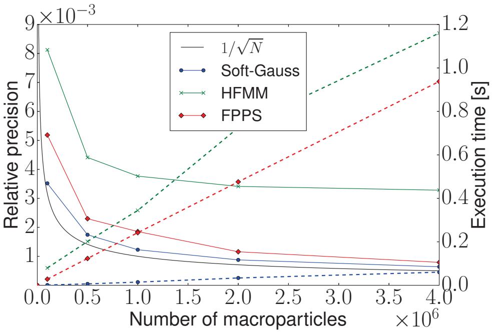

Figure 3: Noise and execution time on a 2.7 GHz Intel Core i7 processor with 8 Gb of RAM for three types of solver. For both the HFMM and the FPPS, the numerical parameters of the solver where chosen after a parametric study in order to minimise the numerical noise [8].

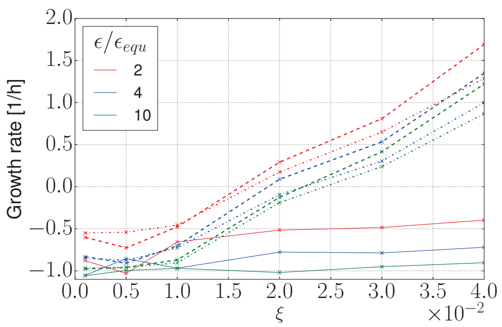

Figure 4: Comparison of the emittance variation in the presence of a head-on beam-beam interaction and synchrotron radiation. The beam-beam interaction is modelled in the the weak-strong (solid lines) and strong-strong regime, based on two different field solvers : the HFMM (dotted-dashed lines) or the FPPS (dashed lines).

set of random Gaussian distribution of charges and average the relative error compared to the theoretical value :

$$\sigma_p = \frac{\langle |K_i(x,y) - K_{\mathrm{th}}(x,y)| \rangle}{\langle K_{\mathrm{th}}(x,y) \rangle}, \tag{7}$$

where $K_{\mathrm{th}}(x,y)$ is the theoretical field at a position $(x,y)$, $K_i(x,y)$ is the computed field for the $i^{\mathrm{th}}$ macroparticle. The average is done over the macroparticles. The results for the field solvers implemented in COMBI is shown in Fig. 3. The soft-Gaussian solver [5] approaches the theoretical minimum of $N_m^{-\frac{1}{2}}$, yet it assumes that the beam distribution is Gaussian and therefore becomes inaccurate for other distributions. The Hybrid Fast Multipole Method (HFMM) [9] introduces significantly more noise, which may be attributed to the quad-tree algorithm used to improve the execution speed. The Fast Polar Poisson Solver (FPPS) was implemented in COMBI in order to overcome the noise limitations of the HFMM while keeping a reasonable execution time [8] profiting from the periodicity of the potential expressed in the polar coordinate and the FFT. While the improvement with respect to the HFMM is clear, the solver still introduces between 50 and 100% more noise than the first order estimate.

### Synchrotron Radiation

First simulations were performed introducing the effect of synchrotron radiations in the transverse plane. Since several photons are emitted each turn, we introduce the effect of the emission of synchrotron radiation as a damping of each particle emittance with $\tau_{rad}$ and an intrinsic noise source in the horizontal plane with a relative amplitude given by $\delta_i = \sqrt{\frac{2}{\tau_{rad}}\frac{\epsilon}{\epsilon_{equ}}}$ caused by the quantum nature of the synchrotron radiation [10]. The result of simulations using two types of field solvers are shown in Fig. 4, compared to the corresponding weak-strong simulation. As shown previously, the non-linear diffusion mechanisms are negligible in this model for beam-beam parameters below 0.03. Yet the strong-strong simulations show an important increase of the emittance growth due to beam-beam interaction, visible for beam-beam parameters larger than 0.01 and linearly growing with the beam-beam parameter. This suggest that the simulations are effectively dominated by the solver noise. The absence of improvement between the HFMM and the FPPS remains to be understood.

## CONCLUSION

The emission of synchrotron radiation in the FCC-hh result in a slow emittance damping time. The self-consistent modelling of the modification of the beam distribution in the presence of strong beam-beam interaction pushes significantly the computing requirements. Indeed, since the slow diffusion mechanisms take place over hundreds of thousands of turn, the beam-beam forces needs to be evaluated numerous times with a high accuracy. In particular it was shown that reliable simulations for large beam-beam parameters require more than $10^6$ macroparticles to model the beam distribution and the usage of state-of-the-art fast and accurate field solver. To achieve that purpose a new fast solver was implemented in the code COMBI, implementing a shared memory parallelization concept to further increase the execution speed.

## ACKNOWLEDGEMENTS

We would like to thank K. Ohmi and F. Zimmermann for fruitful discussions.

This research has used resources of the High Performance Computing centre of the École Polytechnique Fédérale de Lausanne, which was supported by Prof. L. Rivkin (Laboratory of Particle Accelerators Physics).

The European Circular Energy-Frontier Collider Study (EuroCirCol) project has received funding from the European Union's Horizon 2020 research and innovation programme under grant No 654305. The information herein only reflects the views of its authors and the European Commission is not responsible for any use that may be made of the information.

Copyright © 2016 CC-BY-3.0 and by the respective authors

## REFERENCES

[1] K. Ohmi et al. “Beam-Beam Limit in $e^+e^-$ Circular Colliders.” In: *Phys. Rev. Lett.* 92 (21 May 2004), p. 214801.

[2] A. Ball et al. *Future Circular Collider Study - Hadron Collider Parameters*. Tech. rep. FCC-ACC-SPC-0001. Geneva, Switzerland: CERN, Feb. 2014.

[3] X. Buffat and D. Schulte. “Evolution of the Beam Parameters During Luminosity Production in the Future Circular Hadron Collider.” In: *these proceedings*.

[4] Kazuhito Ohmi and Frank Zimmermann. “Fundamental beam-beam limit from head-on interaction in the Large Hadron Collider.” In: *Phys. Rev. ST Accel. Beams* 18 (12 Dec. 2015), p. 121003.

[5] T. Pieloni. “A Study of Beam-Beam Effects in Hadron Colliders with a Large Number of Bunches.” PhD thesis. EPFL, 2008.

[6] W. Herr. “Beam-beam interactions.” In: *CAS - CERN Accelerator School: Intermediate Course on Accelerator Physics*. Ed. by D. Brandt. Zeuthen, Germany: CERN, 15-26 September 2003 2006, pp. 379–410.

[7] Y. Alexahin. “A study of the coherent beam-beam effect in the framework of Vlasov perturbation theory.” In: *Nucl. Instrum. Methods Phys. Res. A* 480.2 (2002), pp. 253–288.

[8] Adrien Florio, Xavier Buffat, and Tatiana Pieloni. “Fast Poisson Solvers for Self-Consistent Beam-Beam and Space-Charge Field Computation in Multiparticle Tracking Simulations.” In: (Nov. 2015).

[9] F. W. Jones. “A hybrid fast-multipole technique for space-charge tracking with halos.” In: *AIP Conference Proceedings* 448.1 (1998), pp. 359–370.

[10] A. Wolski. *Beam Dynamics in High Energy Particle Accelerators*. London: Imperial College Press, 2014.

Copyright © 2016 CC-BY-3.0 and by the respective authors