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# On loop corrections to string effective field theories: field-dependent gauge couplings and sigma-model anomalies

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## Abstract

We show that certain one-loop corrections to superstring effective four-dimensional Lagrangians, involving a universal field-dependent renormalization of gauge couplings, can be consistently written in a standard $N = 1$ supergravity form. These loop corrections can be viewed as a renormalization of the gauge Yang-Mills kinetic term, or as a modification of the Kähler potential, depending whether the dilaton-axion multiplet is described by a linear or a chiral multiplet, the two formulations being related by a duality transformation. Because of the structure of the anomaly supermultiplets, these loop corrections can also be interpreted as supersymmetric anomaly-cancelling terms for mixed Yang-Mills–$\sigma$-model anomalies.

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# 1 Introduction

An important issue of superstring theory is to calculate quantum effects [1–7], which may remove the degeneracies of the classical superstring vacua, and lead to realistic mechanisms for gauge- and supersymmetry breaking.

In calculating these effects, it is important to take into account all the string symmetries, which severely constrain the superstring effective action describing the low-energy dynamics of the massless (or nearly massless) excitations. An example of these invariances is the duality symmetry [8] in the background moduli fields, which relates large and small radii as compared with the string size. These effects cannot be seen in Kaluza-Klein compactifications of the ten-dimensional point field theory limit, since the latter can only give information for large values of the moduli: in the sigma-model approach to two-dimensional world-sheet dynamics, such a situation corresponds to a weakly coupled theory. On the other hand, in the four-dimensional point field theory limit string loop corrections correspond to ordinary Feynman diagrams, with both massless and massive states circulating in loops. In four-dimensional superstrings, the loop effects on gauge couplings and other physical quantities are field-dependent.

For instance, qualitatively, the integration over the massless modes gives a correction to the gauge couplings [9] which is related to the standard field theory result

$$\left.\frac{1}{g^2(Q^2)}\right|_{1-loop} - \left.\frac{1}{g^2(R^{-2})}\right|_{1-loop} \propto b_0 \log(Q^2R^2). \tag{1}$$

This result is correct provided the scale $Q$ is much smaller than the inverse of the Kaluza-Klein radius $R$. However, for $1/R^2 \ll Q^2 \ll \Lambda_S^2$, where $\Lambda_S = O(\alpha'^{-1/2})$ is the string cutoff, the Kaluza-Klein string modes give a contribution which, for large $R$, goes as

$$\left.\frac{1}{g^2(Q^2)}\right|_{1-loop} = \left.\frac{1}{g^2(R^{-2})}\right|_{1-loop} \propto b_0\Lambda_S^2R^2 + \text{constant}, \tag{2}$$

where the constant does not depend on moduli. These effects get further corrections when $R^2$ is not large with respect to $\Lambda_S^{-2}$, due to the string winding modes which are responsible for the quantum symmetries. For toroidal compactifications, the latter are of the type

$$R \to \frac{\alpha'}{R}. \tag{3}$$

In supersymmetric compactifications, the radial variable $R$ always appears together with its pseudoscalar partner, the periodic variable $\varphi$, and quantum symmetries apply directly on

$$T = R^2 + i\varphi. \tag{4}$$

Renormalized couplings get then a dependence on the complex modulus $T$. In the simplest case, in which $\varphi$ periodicity and radius inversion symmetry are requested, the full invariance is the modular group $PSL(2, Z)$, and all quantities are expected to be modular functions of the $T$ variable.

In this paper, we would like to clarify the relation between these loop corrections and the one-loop effective action, which has to be consistent with supersymmetry, gauge invariance and the duality string symmetry. This problem has become particularly relevant in view of the recent calculation of some one-loop string corrections in heterotic strings on $(2,2)$ orbifolds [10].

In a generic supergravity theory, the one-loop $\beta$-function coefficient is related to a $U(1)$ chiral anomaly [11]. We will show that moduli-dependent renormalization effects on coupling constants can also be viewed as anomalies, mixing Yang-Mills fields and the holonomy connection of the Kähler manifold for the moduli fields. In string theory, there is a subtle cancellation mechanism for these anomalies, analogous to the ten-dimensional Green-Schwarz construction [12], but for a composite gauge connection of the $U(1)$ charge of a $N = 2$ superconformal algebra on the world-sheet, whose existence was implied in ref. [13]. In a supersymmetric framework, this mechanism consistently gives an effective action which for large radius reproduces the expected $R$-dependence,

$$\frac{1}{g^2(R,\varphi)} \sim a \log R^2 + bR^2 \quad (R \text{ large}), \tag{5}$$

of the one-loop corrected gauge coupling constant. The cancellation mechanism crucially relies upon the existence of an antisymmetric tensor $b_{\mu\nu}$, and is therefore naturally expressed using a linear multiplet coupled to supergravity [14]. It is well known that this formulation is equivalent, via a duality transformation, to the standard one, with the linear multiplet replaced by a chiral multiplet $S$: its real and imaginary parts are related to the tree-level gauge and axion couplings, respectively[1]

$$\left.\frac{1}{g^2}\right|_{tree} \propto \text{Re } S, \qquad \theta|_{tree} \propto \text{Im } S, \tag{6}$$

and the duality transformation relates Im $S$ and $b_{\mu\nu}$.

The structure of the present paper is the following. In sect. 2, we recall some supergravity results and we concentrate on the supersymmetrization of general field-dependent gauge couplings. We discuss in detail the introduction of loop corrections to the effective action, and the role played by the linear multiplet in ensuring gauge invariance. In sect. 3, we show how one can return to the standard formulation by performing a duality transformation, which replaces the linear multiplet by an equivalent chiral multiplet ($S$), and leads to a gauge kinetic function of the standard form, but to a modified Kähler potential. The duality transformation is modified order by order in the loop expansion, in such a way that gauge kinetic terms remain unchanged in terms of the (new) $S$-field. In this last formulation, the field-dependent corrections appear as a (wave function) renormalization of the $S$ multiplet. In sect. 4, we explain the connection between these renormalization effects and mixed gauge-holonomy anomalies. As an example, we discuss the case of (2,2) $Z_N$ orbifolds, and obtain the corrected form of the Kähler function for these theories. In sect. 5, we study the impact of the above corrections on the scalar potential. Finally, sect. 6 contains our conclusions.

# 2 Supersymmetry

The standard form of $N = 1$, $d = 4$ supergravity coupled to Yang-Mills and matter described by chiral superfields (up to two derivatives in the bosonic fields) [15,16] is defined in terms of two functions:

- A real and gauge invariant function $\mathcal{G}(\Sigma, \overline{\Sigma})$ of the chiral superfields $\Sigma^i \equiv (z^i, \chi^i, \mathcal{F}^i)$ and their conjugates $\overline{\Sigma}_i$, conventionally written as

$$\mathcal{G}(\Sigma, \overline{\Sigma}) = K(\Sigma, \overline{\Sigma}) + \log |w(\Sigma)|^2, \tag{7}$$

[1] We denote by $S$ both the chiral multiplet and its complex spin-0 component.

where the real function $K(\Sigma, \overline{\Sigma})$ and the analytic function $w(\Sigma)$ are called the Kähler potential and superpotential, respectively.

- An analytic function $f_{ab}(\Sigma)$ of the chiral superfields $\Sigma^i$, transforming as a symmetric product of adjoint representations of the gauge group G [2].

The Lagrangian is summarized by the superconformal action formula

$$\begin{aligned} e^{-1}\mathcal{L} &= -\frac{3}{2}\left[S_0\overline{S}_0 \exp\left\{-\frac{1}{3}\mathcal{G}(\Sigma, \overline{\Sigma}e^{2A})\right\}\right]_D \\ &+ \left(\left[S_0^3\right]_F + \text{h.c.}\right) - \frac{1}{4}\left(\left[f_{ab}(\Sigma)W^aW^b\right]_F + \text{h.c.}\right), \end{aligned} \tag{8}$$

where $A \equiv gA^aT^a$ is the Yang-Mills multiplet [in the Wess-Zumino gauge, $A^a \equiv (A^a_\mu, \lambda^a, D^a)$], $T^a$ are the gauge group generators, $g$ is the gauge coupling constant, $W^a$ is the chiral spinor superfield containing the Yang-Mills field strength $F^a_{\mu\nu}$, and the subscripts $D$ and $F$ indicate the action formula for a real vector density and for a chiral density, respectively. This action corresponds to conformal weights $\omega = 0$ for $\Sigma^i$, $\omega = 1$ for $S_0$ and $\omega = \frac{3}{2}$ for $W^a$. This choice of weights can be taken in full generality. The two functions $\mathcal{G}$ and $f_{ab}$ have then conformal weights zero. The chiral multiplet $S_0$ is used as a compensator, which gauge fixes the superconformal symmetry down to super-Poincaré. This is achieved by requiring that the Einstein term in the Lagrangian has the canonical form $-\frac{1}{2}eR$, which corresponds to the choice [15,17]

$$S_0 = \left(e^{\frac{1}{6}\mathcal{G}}, \frac{1}{3}e^{\frac{1}{6}\mathcal{G}}\mathcal{G}_i\chi^i, h_0\right). \tag{9}$$

In the above expression, $h_0$ is the supergravity auxiliary scalar and the usual conventions about derivatives ($\mathcal{G}_i \equiv \partial\mathcal{G}/\partial z^i$, ...) are used. It follows from eq. (8) that the kinetic terms for the gauge bosons,

$$-\frac{1}{4}\, Re\, f_{ab}(z^i)\, F^{a\,\mu\nu}F^b_{\mu\nu}, \tag{10a}$$

are always accompanied by the CP-odd term

$$+\frac{1}{4}\, Im\, f_{ab}(z^i)\, F^{a\,\mu\nu}\tilde{F}^b_{\mu\nu}, \tag{10b}$$

[2] To avoid too heavy a notation, we will take $G$ to be simple, but all our considerations can be trivially extended to a general $G$.

where $\tilde{F}^b_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{b\,\rho\sigma}$. One can then interpret $Re\, f_{ab}$ and $Im\, f_{ab}$ as field-dependent inverse gauge couplings and axionic couplings, respectively. The analyticity properties of the coefficients of $FF$ and $F\tilde{F}$ in expressions (10) follow from the fact that the corresponding chiral $F$-density in the superconformal action (8) has no explicit dependence on the compensator $S_0$ (the weight zero function $f_{ab}$ is analytic in the chiral multiplets $\Sigma^i$, which have conformal weight zero).

It was already observed long ago [15] that one could generalize eq. (8) by replacing it with the more general form

$$e^{-1}\mathcal{L} = -\frac{3}{2}\left[S_0\overline{S}_0 e^{-\frac{1}{3}\mathcal{G}}\right]_D - \frac{1}{4}\left(\left[S_0^3\mathcal{H}\left(WS_0^{-3/2},\Sigma\right)\right]_F + \text{h.c.}\right). \qquad (11)$$

This generalization of eq. (8) does not introduce couplings with more than two derivatives in the bosonic fields. The quadratic terms in the gauge boson field strengths are of the form (10), but with $f_{ab}$ replaced by

$$\frac{1}{2}\frac{\partial}{\partial t^a}\frac{\partial}{\partial t^b}\mathcal{H}(t,z)\bigg|_{t=\lambda e^{-\frac{1}{4}\mathcal{G}}}, \qquad (12)$$

which is no longer an analytic function of the chiral scalars $z^i$. This is the consequence of the dependence of the function $\mathcal{H}$ on the compensator $S_0$, and of the form of gauge fixing applied on $S_0$, eq. (9). This induced non-analyticity of $f_{ab}$ is however linked to fermionic (gaugino) terms, since $\mathcal{H}$ depends on the combination $W^a S_0^{-\frac{3}{2}}$. In the presence of gaugino condensates, this would naturally introduce a non-analytic gauge kinetic function [5]. In the absence of non-trivial fermionic backgrounds, the non-analyticity disappears and we recover expressions (10), provided that $\mathcal{H}$ is non-singular in the limit of vanishing fermionic background. The above discussion indicates however that, if one considers expression (10a) as a definition of the function $f_{ab}$, without referring to a specific superconformal action like eq. (8) or eq. (11), the analyticity of $f_{ab}$ is not a general consequence of local $N=1$ supersymmetry with up to two derivatives.

In the rest of this section, we will discuss the following question: given a kinetic term for the gauge bosons of the form [3]

$$-\frac{1}{8}h(z,\overline{z})\delta_{ab}F^{a\,\mu\nu}F^{b}_{\mu\nu}, \tag{13}$$

with $h$ a generic real and gauge invariant function of $z$ and $\overline{z}$ (not necessarily the real part of an analytic function of $z$), under which circumstances does this coupling admit an extension compatible with local $N=1$ supersymmetry and gauge invariance?

This problem turns out to be particularly relevant for the determination of the effective low-energy theories of $N=1$, $d=4$ heterotic string models. At the genus zero level, these are just standard $N=1$, $d=4$ supergravities, with [18]

$$\mathcal{G} = -\log(S+\overline{S}) + \hat{\mathcal{G}}(\hat{\Sigma},\hat{\overline{\Sigma}}e^{2A}) \tag{14}$$

and [19, 20]

$$f_{ab} = \delta_{ab}S, \tag{15}$$

where $S$ is a gauge singlet chiral superfield and $\hat{\Sigma}$ denotes collectively all the other chiral superfields of the effective low-energy theory, $\Sigma \equiv (S,\hat{\Sigma}^i)$. The real function $\hat{\mathcal{G}}$ is such that the F-term part of the scalar potential,

$$V_F = e^{\mathcal{G}}\left(\mathcal{G}^S\mathcal{G}_S^{-1\,S}\mathcal{G}_S + \hat{\mathcal{G}}^i\hat{\mathcal{G}}_i^{-1\,j}\hat{\mathcal{G}}_j - 3\right), \tag{16}$$

is positive semidefinite:

$$\mathcal{G}^S\mathcal{G}_S^{-1\,S}\mathcal{G}_S = 1, \qquad \hat{\mathcal{G}}^i\hat{\mathcal{G}}_i^{-1\,j}\hat{\mathcal{G}}_j - 3 \geq 0. \tag{17}$$

These last equations only apply to the specific case of string effective actions. Recently, one-loop string corrections to eq. (10a) were computed [10] in a class of orbifold models, and the result was found to be of the form (13), with a non-harmonic contribution

$$h(S,\overline{S},\hat{\Sigma},\hat{\overline{\Sigma}}) = S+\overline{S}+\Delta(\hat{\Sigma},\hat{\overline{\Sigma}}), \qquad \frac{\partial}{\partial z^i}\frac{\partial}{\partial \overline{z}_j}\Delta(z,\overline{z}) \neq 0. \tag{18}$$

[3]Compared to eq. (10a), we restrict ourselves to a coefficient function of the form $h\delta_{ab}$, instead of $h_{ab}$, only for simplicity: including an explicit dependence on the gauge indices is straightforward. Also, the most interesting application of our results is of the form (13).

If one tries to identify $h\delta_{ab}$ with $2Ref_{ab}$, this result seems incompatible with the standard form of the supergravity Lagrangian. Our first goal is to demonstrate that gauge kinetic terms of the form (13), (18) can be perfectly described by an effective supergravity action, provided that the problem is correctly formulated. Moreover, our results will single out a natural framework to include loop corrections into the effective supergravity action of superstrings. We will keep the argument at the general level: a detailed interpretation of the results of ref. [10] will be given in the following sections and elsewhere [21].

It is well known that the variation of the super-Yang-Mills Lagrangian

$$\mathcal{L}_{SYM} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu\,a} + \frac{i}{2}\overline{\lambda}^a\gamma^\mu D_\mu\lambda^a + \frac{1}{2}D^aD^a \tag{19}$$

under a global supersymmetry transformation is a total derivative, leading to an invariant action. The Lagrangian (19) can also be written as the real part of the $F$-component of the chiral superfield $-\frac{1}{4}W^aW^a$, which can be extended to local supersymmetry: this is the origin of the last term in the superconformal action (8), since a chiral superfield $W^aW^b$ multiplied by another chiral superfield $f_{ab}(\Sigma)$ is again a chiral superfield. Actually, $-\frac{1}{4}W^aW^a$ is the unique gauge invariant superfield containing the Lagrangian (19), up to two derivatives. It also possesses a term proportional to

$$i\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma} = 4i\epsilon^{\mu\nu\rho\sigma}\partial_\mu\Omega_{\nu\rho\sigma},$$

where $\Omega_{\nu\rho\sigma}$ is the Yang-Mills Chern-Simons tensor, whose gauge variation is a total derivative. To supersymmetrize the gauge kinetic terms (13), one has to look for an action containing the super-Yang-Mills terms (19) multiplied by the real function $h(z,\overline{z})$. However, if $\partial_z\partial_{\overline{z}}h \neq 0$, this action cannot be gauge invariant, since $W^aW^a$ cannot be used. Since the variation under supersymmetry of $h(z,\overline{z})\mathcal{L}_{SYM}$ is not a total derivative, further terms containing the Chern-Simons form will be required to restore supersymmetry. This argument could be carried on in component language, but it is much more convenient to use the superfield formalism, as we do in the following.

The natural way of supersymmetrizing eq. (13) is to embed it in a superconformal $D$-density. For the standard case $h(\Sigma,\overline{\Sigma}) = 2Re\,f(\Sigma)$, the mechanism leading to this result has been discussed in ref. [7]. The central point is to observe that the following identity holds (up to total derivatives)

$$[Tr\,f(\Sigma)WW]_F + \text{h.c.} \equiv \left[Tr\,(f(\Sigma)+\overline{f}(\overline{\Sigma}))\Omega\right]_D, \tag{20}$$

where the trace is taken over the gauge group indices and $\Omega$ is the non-Abelian Yang-Mills Chern-Simons supermultiplet associated with the gauge vector multiplet $A$:

$$\Omega(A) = 2\int_0^1 dt\, Tr\left([\nabla^\alpha(t), A]W_\alpha(t) + [\overline{\nabla}_{\dot\alpha}(t), A]\overline{W}^{\dot\alpha}(t) + A\{\nabla^\alpha(t), W_\alpha(t)\}\right). \tag{21}$$

In eq. (21), $W_\alpha(t) = (\overline{\mathcal{D}}\overline{\mathcal{D}} - 8R)\nabla_\alpha(t)$, $\nabla_\alpha(t) = e^{-tA}\mathcal{D}_\alpha e^{tA}$, and $\mathcal{D}_\alpha$ is the supersymmetric covariant derivative[4]. In the Abelian case, eq. (21) takes the simpler form

$$\Omega(A) = Tr[\mathcal{D}^\alpha A W_\alpha + \overline{\mathcal{D}}_{\dot\alpha} A \overline{W}^{\dot\alpha} + A\mathcal{D}^\alpha W_\alpha], \tag{22}$$

where now

$$W_\alpha = \left(\overline{\mathcal{D}}\overline{\mathcal{D}} - 8R\right)\mathcal{D}_\alpha A. \tag{23}$$

The left-hand side of eq. (20) is manifestly gauge-invariant. Gauge invariance of the right-hand side is ensured by the fact that, under gauge transformations, $\delta\Omega$ is a linear multiplet [7]. This can be seen directly from the constraint equation satisfied by the Chern-Simons multiplet:

$$\left(\overline{\mathcal{D}}\overline{\mathcal{D}} - 8R\right)\Omega = W^2, \tag{24}$$

which implies, since under gauge transformations $\delta W^2 = 0$,

$$\delta\Omega = \overline{\delta\Omega}, \tag{25}$$

$$\left(\overline{\mathcal{D}}\overline{\mathcal{D}} - 8R\right)\delta\Omega = 0. \tag{26}$$

These are precisely the two equations defining a real linear multiplet. It then follows that $[f(\Sigma)\delta\Omega]_D$ does not contribute to the action, since $f(\Sigma)$ is a chiral supermultiplet.

If we generalize the right-hand side of eq. (20) by replacing $f(\Sigma) + \overline{f}(\overline{\Sigma})$ with a general real and gauge-invariant function $h(\Sigma, \overline{\Sigma})$ (a vector multiplet), this would give a manifest supersymmetrization of eq. (13). At first sight however, since identity (20) does not apply, equivalence with the standard

[4]In these expressions, $\alpha$ and $\dot\alpha$ are the usual two-component spinor indices, not to be confused with gauge indices like $a$, which are omitted here for notational simplicity.

formulation of supergravity is lost. Also, for the same reason, gauge invariance is violated. In components, the new D-density

$$\mathcal{L}_\Omega = -\frac{1}{4}\left[h(\Sigma, \overline{\Sigma}^{2A})\Omega\right]_D \tag{27}$$

will give, besides eq. (13), a term of the form

$$-\frac{1}{8}e^{-1}\epsilon^{\mu\nu\rho\sigma}V_\mu\Omega_{\nu\rho\sigma}, \tag{28}$$

where

$$V_\mu = i(h^i\partial_\mu\bar{z}_i - h_i\partial_\mu z^i) \tag{29}$$

and

$$\Omega_{\mu\nu\rho} = \mathrm{Tr}\left(A_{[\mu}F_{\nu\rho]} - \frac{1}{3}A_{[\mu}A_\nu A_{\rho]}\right) \tag{30}$$

is the Yang-Mills Chern-Simons form, for which $2\epsilon^{\mu\nu\rho\sigma}\partial_\mu\Omega_{\nu\rho\sigma} = \mathrm{Tr}(F_{\mu\nu}\tilde{F}^{\mu\nu})$. When $h(\Sigma,\overline{\Sigma}) = f(\Sigma) + \overline{f}(\overline{\Sigma})$, as in the case of eq. (20), then $V_\mu = -i\partial_\mu(f - \overline{f})$, and by partial integration eq. (28) is equivalent to the standard axionic coupling of eq. (10b). Differently stated, $V_\mu$ can be seen as a composite $U(1)$ gauge field: in general, $V_\mu$ can be non-trivial, and only when $V_\mu$ is a pure gauge ($V_\mu = \partial_\mu\theta$) can one write down the standard axionic coupling. From the supersymmetry point of view, the axionic coupling is well defined provided identity (20), which also implies integrability of the term (28), holds. However, we will now show that, even in the case of a general $h(\Sigma,\overline{\Sigma})$, there is no obstruction to the construction of a supersymmetric action, with unambiguous couplings also for the axion.

Even though eq. (27) is manifestly supersymmetric, gauge invariance is lost, since $h$ is now a real vector supermultiplet. In components, the variation of eq. (27) under a gauge transformation with parameters $\Lambda^a$ takes the form

$$\delta\mathcal{L}_\Omega \propto \epsilon^{\mu\nu\rho\sigma}\hat{F}_{\mu\nu}(V)\Lambda^a\hat{F}^a_{\rho\sigma}(A) \quad + \quad \text{other terms}, \tag{31}$$

where $\hat{F}$ denotes the purely derivative part (curl) of the field strength $F$. This situation is strongly reminiscent of the way anomalies manifest themselves in $N = 1$ , $d = 10$ supergravity coupled to Yang-Mills. The expression

(31) is clearly of the form of a mixed anomaly [5] for gauge fields and the vector field $V_\mu$. To recover gauge invariance, we can pursue the analogy with the Green-Schwarz anomaly cancellation mechanism in ten dimensions [12]. The crucial ingredient of its four-dimensional version is the existence of a physical linear supermultiplet $L$ containing, as physical degrees of freedom, the dilaton, the antisymmetric tensor $b_{\mu\nu}$ (the tree-level dualized $\mathrm{Im}\, S$–field) and their Majorana fermionic partner. In order to restore gauge invariance, the action which reproduces gauge kinetic terms of the form (13) has to be modified as follows:

$$[h\Omega]_D \longrightarrow [h\,(\Omega - L)]_D \qquad (32)$$

and gauge invariance is enforced by requiring that under gauge transformations

$$\delta L = \delta\Omega. \qquad (33)$$

In components, the antisymmetric tensor $b_{\mu\nu}$ contained in $L$ transforms according to

$$\delta b_{\mu\nu} \propto -\epsilon_{\mu\nu\rho\sigma}\Lambda^a \hat{F}^{a\,\rho\sigma}(A). \qquad (34)$$

We need finally to complete our action by terms describing propagation and interactions of the linear multiplet. Notice that this is absolutely necessary, because, with modification (32) only, the equation of motion of $L$ would simply tell us that $h$ is the real part of an analytic function. Taken alone, the modification (32) would in fact give an action completely equivalent to standard supergravity, rewritten after use of identity (20). This would bring us back to our starting point, eq. (8). This is no longer the case when propagation terms for $L$ are introduced, in which case one has the superconformal action formula

$$e^{-1}\mathcal{L} = -\frac{3}{2}\left[S_0\overline{S}_0\Phi\left(\frac{L-\Omega}{S_0\overline{S}_0}, \hat{\Sigma}, \hat{\overline{\Sigma}}e^{2A}\right)\right]_D + \left(\left[S_0^3\right]_F + \text{h.c.}\right). \qquad (35)$$

[5] When considering superstring effective actions, such an anomalous term would be expected to arise from one-loop corrections. In fact, we have just shown that this term is intrinsically related to the supersymmetrization of eq. (13), and the occurrence of the latter was explicitly demonstrated at one loop for the class of models considered in [10]. A more complete discussion of the anomaly structure of superstring effective actions will be given in ref. [21].

The arbitrary real, gauge invariant function $\Phi$ defines entirely the theory. We have to take [7]

$$\Phi = \frac{2\sqrt{2}}{3} e^{-\frac{1}{2}\hat{g}} \left( \frac{L - \Omega}{S_0 \bar{S}_0} \right)^{-1/2}, \qquad (36)$$

in order to recover the tree-level superstring effective action defined by eqs. (14), (15), but expressed equivalently in terms of the linear supermultiplet $L$ instead of the chiral supermultiplet $S$: a supersymmetric duality transformation leads from eqs. (35) and (36) to eqs.( 14) and (15). Adding a term of the form

$$-\frac{1}{4} \left[ \Delta \left( L - \Omega \right) \right]_D \qquad (37)$$

to the superconformal action formula (35), which would correspond to adding the term $\frac{1}{6}\Delta \left( \frac{L-\Omega}{S_0 \bar{S}_0} \right)$ to $\Phi$, will include the one-loop (and higher order as well) correction to the gauge kinetic terms.

We close this section by two remarks. Firstly, the essential role of the linear multiplet is to restore gauge invariance: it does not appear in the gauge kinetic terms (13) which we have supersymmetrized. Secondly, if one considers only the couplings of gauge fields to scalars, the Lagrangian in the standard formulation with vector and chiral multiplets, eq. (8), and the Lagrangian with linear and Chern-Simons multiplets, eq. (35), cannot be distinguished only with the help of the first correction to the kinetic terms, the 3-point couplings $z - A^a_\mu - A^b_\nu$ with two derivatives. In both formulations, the $zF^2$ terms are identical. The $zF\tilde{F}$ part of Lagrangian (8) would give a term of the form

$$\epsilon^{\mu\nu\rho\sigma} z (\partial_\mu A^a_\nu)(\partial_\rho A^a_\sigma),$$

while eq. (35) would lead to

$$\epsilon^{\mu\nu\rho\sigma} (\partial_\mu z) A^a_\nu (\partial_\rho A^a_\sigma),$$

but both terms are clearly equivalent after partial integration. The 3-point amplitudes for one scalar and two gauge fields are then completely identical for both Lagrangians[6]. This is to be contrasted with, for instance, the 4-point amplitudes with two scalars and two gauge fields, which differ in the two cases.

[6] This 3-point function was actually considered in sect. 3 of ref. [10].

# 3 S–field formulation of the loop-corrected action

Up to here, we have shown how to write down a gauge-invariant supergravity Lagrangian able to accommodate gauge boson kinetic terms of the form (13), (18). This was obtained at the price of introducing a linear multiplet $L$, whose transformation properties are such as to cancel a potential anomaly in the gauge transformations. It is however well known that a linear multiplet $L$ can always be transformed into a chiral multiplet $S$ by a (supersymmetric) duality transformation. After the duality transformation, the action (35) will be in the standard supergravity form, with matter described by chiral multiplets only. We now want to show explicitly this relationship, and discuss its implications. The first step is just to write down a superconformal action equivalent to eqs. (35), (36):

$$-\frac{3}{2}\left[S_0\overline{S}_0\Phi\left(\frac{U}{S_0\overline{S}_0},\hat{\Sigma},\hat{\overline{\Sigma}}e^{2A}\right)+\frac{1}{6}(S+\overline{S})(U+\Omega)\right]_D+\left(\left[S_0^3\right]_F+\text{h.c.}\right), \quad (38)$$

where we have introduced a real, unconstrained superfield $U$ instead of the combination $L-\Omega$. This last action is equivalent to eq. (35), since the equation of motion of $(S+\overline{S})$ implies that $L\equiv U+\Omega$ is a linear multiplet. Integrating over $S$ gives then eq. (35). But we could as well integrate over the unconstrained superfield $U$, to obtain an equivalent form of the action in terms of $S$, $\hat{\Sigma}^i$ and $A$. Explicitly, this amounts to solving the superfield equation of motion for $U$, which reads

$$6\,\Phi'\left(\frac{U}{S_0\overline{S}_0},\hat{\Sigma},\hat{\overline{\Sigma}}e^{2A}\right)=-(S+\overline{S}), \quad (39)$$

where $\Phi'=\frac{\partial}{\partial x}\Phi(x,\hat{\Sigma},\hat{\overline{\Sigma}}e^{2A})$, and to substituting back the solution into eq. (38). The resulting action is always characterized, in the standard formulation of refs. [15,16], by a gauge kinetic function of the form (15), and, with eq. (36), by the $\mathcal{G}$ function (14). If we now add to the Lagrangian (36),(38) the term (37), with $(L-\Omega)\to U$, eq. (39) becomes

$$6\,\Phi'=-(S+\overline{S}+\Delta),$$

and we clearly obtain a theory of the standard type, but characterized by

$$\mathcal{G}_{corr.}=-\log(S+\overline{S}+\Delta)+\hat{\mathcal{G}}(\hat{\Sigma},\hat{\overline{\Sigma}}e^{2A}), \quad (40)$$

$$f_{ab}^{corr.} = \delta_{ab} S, \tag{41}$$

since this addition to the action (38) only modifies the equation of motion of $U$, eq. (39), by the replacement of $S + \overline{S}$ by $S + \overline{S} + \Delta$. Notice that, even though, in the formulation with the linear multiplet, the modification introduced by the $\Delta$ term has to do with gauge kinetic terms, in the formalism of refs. [15,16], the gauge kinetic function $f_{ab}$ does remain unchanged: it is entirely determined by the couplings of $S$ in eq. (38). The duality transformation relating $S$ to $U = L - \Omega$ [eq. (39)] is however modified by the correction term (37).

Since one might want to consider the addition of the term (37) as a one-loop correction to a tree-level superstring effective action, it could be appropriate to rewrite

$$\mathcal{G}_{corr.} = -\log(S + \overline{S}) - \log\left(1 + \frac{\Delta}{S + \overline{S}}\right) + \hat{\mathcal{G}}. \tag{42}$$

This suggests clearly that the one-loop contribution to the gauge kinetic terms should be viewed as a wave function renormalization of the $S$ field, when interpreted in the standard formulation. It should not be regarded as a renormalization of $f_{ab}$, which remains unchanged. Notice however that this distinction only makes sense as long as the corrections introduced by $\Delta$ are not the real part of an analytic function. If $\Delta = 2Re\,\delta(\Sigma)$, there is clearly an analytic field redefinition of the $S$ field

$$S' = S + \delta \tag{43}$$

for which

$$\mathcal{G}_{corr.} = -\log(S' + \overline{S'}) + \hat{\mathcal{G}} \tag{44}$$

$$f_{ab}^{corr.} = \delta_{ab}(S' - \delta) \tag{45}$$

After this field redefinition, which is acceptable in the standard formulation since it is analytic, the one-loop effect appears now in the gauge coupling constant, in $f_{ab}$. If however $\Delta$ is not the real part of an analytic function, there is clearly no analytic field redefinition of $S$ able to move $\Delta$ into $f_{ab}$. Insisting on a change of variable of the form

$$S_{new} + \overline{S}_{new} = S + \overline{S} + \Delta \tag{46}$$

is inconsistent with the standard formulation of supergravity. We have actually shown above how to implement such a non-analytic field redefinition, which is what is required by the renormalization group equations (1), in a consistent, supersymmetric way.

If one insists on manifest target-space duality invariance [8] of the effective Lagrangian, in which case the tree-level Lagrangian and $\Delta$ [10] are separately duality invariant, eqs. (40) and (41) are the correct definitions for the loop-corrected effective action. Using (44) and (45) would spoil duality symmetry. In fact, the duality invariant form of $\Delta$ is (up to a multiplicative constant) $\Delta = \hat{K} + 2\,\mathrm{Re}\,\phi$, where $\hat{K}$ is the (tree-level) Kähler potential for all chiral multiplets but $S$,

$$\hat{\mathcal{G}} = \hat{K} + \log|w|^2, \tag{47}$$

and $\phi$ is a holomorphic modular function [8, 10].

# 4 Gauge-holonomy anomalies

In $N = 1$, $d = 4$ supergravity, the axial 'auxiliary field' $V_\mu$ has the geometrical interpretation of a gauge connection, whose field strength locally coincides with the Kähler two-form $J$. Local supersymmetry actually implies a global property, namely that the first Chern class of the line bundle $L$ (with gauge connection $V_\mu$) coincides with the cohomology class of $J$ [22]. This is the definition of restricted Kähler space, which can be conveniently coupled to supergravity. The gauge transformations of $V$ and the Kähler transformations of $K$ ($J = \partial\overline{\partial}K$) are related to the imaginary and the real parts of an analytic function $\phi(z)$

$$\begin{array}{lll} V & \rightarrow & V + 2\,\mathrm{d}(\mathrm{Im}\ \phi) \\ K & \rightarrow & K + 2\,(\mathrm{Re}\ \phi) \end{array}, \tag{48}$$

which gives the gauge transformation of holomorphic sections of $L$ (such as the superpotential, determining the Yukawa couplings). The invariance $K \rightarrow K + \phi + \overline{\phi}$ corresponds to an exact invariance of the tree-level Lagrangian. The composite holomorphic connection $\partial_i K$ is similar to the spin connection of Lorentz transformations in General Relativity. The transformations (48) may be anomalous, in the same way as Lorentz transformations may. Let us consider now a possible loop-correction to the tree-level string effective action in which the $S$ dependence is changed as follows

$$\mathcal{G}(S+\overline{S}) \rightarrow \mathcal{G}(S+\overline{S}+\lambda\hat{K}), \tag{49}$$

where $\hat{K}$ is the tree-level Kähler potential for the chiral superfields different from $S$, and, as will be discussed below, $\lambda$ is proportional to the coefficient of the $N=1$ beta-function. It is clear that this action is still Kähler invariant provided we demand

$$S \rightarrow S - \lambda\phi \quad \text{when} \quad \hat{K} \rightarrow \hat{K} + \phi + \overline{\phi}. \tag{50}$$

However, under a Kähler transformation one has also

$$SW^2 \rightarrow SW^2 - \lambda\phi W^2. \tag{51}$$

In components, this means that the overall change in the Lagrangian includes

$$\frac{1}{4}\lambda\,\mathrm{Re}\,\phi(z)F^{a\,\mu\nu}F^a_{\mu\nu} - \frac{1}{4}\lambda\,\mathrm{Im}\,\phi(z)F^{a\,\mu\nu}\tilde{F}^a_{\mu\nu} \tag{52}$$

among many other terms. The second term is precisely an anomaly term for chiral fermions charged under the local $U(1)$ gauge symmetry associated to $V$, and the first term is a $\beta$-function contribution due to the gauge coupling constant renormalization ($\mathrm{Re}\,S \rightarrow \mathrm{Re}\,S + \frac{1}{2}\lambda\hat{K}$). This indeed shows that the $\beta$-function is related by supergravity to the axial anomaly, which is the holonomy anomaly of the Kähler manifold of the matter scalar fields.

As an example, we consider now the case of $(2,2)$ $Z_N$ orbifolds [23]. In these heterotic vacua, the six internal coordinates are compactified on the quotient of an appropriate 6–torus by a point group $Z_N$, with elements $g^k$, $k=0,\ldots,N-1$. In a suitable basis, the generator $g$ acts on the complexified internal coordinates $z_i$ $(i=1,2,3)$, with phases $e^{2i\pi\eta_i}$. Twisted sectors $k\neq 0$ have phases $\eta_i^k = k\eta_i$ (modulo 1). In $N=1$ orbifolds, $\sum_{i=1}^3 \eta_i^k =$ integer. For discussing physical states and vertex operators, the most convenient picture is such that $0\leq\eta_i^k<1$ and left-handed (right-handed) sectors have $\sum_{i=1}^3\eta_i^k=1$ ($\sum_{i=1}^3\eta_i^k=2$). For these $(2,2)$ orbifolds, the gauge group is $E_6\times H\times E_8'$, where $H^{(1)}=U(1)^3$ if the three phases $\eta_i$ of the point group generator $g$ are all different, $H^{(2)}=SU(2)\times U(1)$ if two phases are equal, and $H^{(3)}=SU(3)$ if the three phases are $1/3$. The representations of massless chiral multiplets in the untwisted sector will be denoted by $R_i^u$ $(i=1,2,3)$. The index $i$

indicates that the GSO phase of $R_i^u$ is $e^{2i\pi\eta_i}$. Chiral multiplets are neutral under $E_8'$, and, for (2,2) orbifolds,

$$\begin{array}{lcl} H^{(1)}: & R_i^u = 27 & \text{of } E_6 \\ H^{(2)}: & \left\{ \begin{array}{l} R_i^u = (27,2) \\ R_3^u = (27,1) \end{array} \right. & \text{of } E_6 \times SU(2) \\ H^{(3)}: & R_i^u = (27,3) & \text{of } E_6 \times SU(3) \end{array} \tag{53}$$

(for case $H^{(2)}$, we have chosen $\eta_1 = \eta_2$). If a phase $\eta_i = \frac{1}{2}$, each representation 27 is accompanied by a multiplet $\overline{27}$. In the untwisted sector, the number of (1,1) and (2,1) moduli are equal to the number of massless 27 and $\overline{27}$ representations, respectively. In the twisted sectors $k \neq 0$, the representation of the massless chiral multiplets will be denoted by $R_k^t$.

Our goal is to compute the anomaly generated by the triangle diagram with two external gauge fields $A$ and one composite holonomy connection $V$, related to isometries of the sigma-model describing (untwisted) moduli. Such sigma-model anomalies have been studied in ref. [24]. The connection $V$ can be written as a linear combination of the three internal orbifold currents $J_m = i\partial z_m$ ($m = 1,2,3$). The anomaly diagram gives a contribution quadratic in the gauge coupling constant $g^a$ of $E_6$, $H$ or $E_8'$, and linear in the coupling constants of matter and gauge multiplets to $V$. The latter can be simply obtained from the corresponding vertex operators [25]. The anomaly for the current $J_i$ is characterized by the formal 6-form [26]

$$I_6^{i,a} = c\,(g^a)^2 \delta^{i,a} J^i \,\mathrm{Tr}(F^a)^2, \tag{54}$$

where $J = dV$ and $F^a$ are the holonomy and gauge curvature two-forms, and $c$ is a normalization constant. The coefficients $\delta^{i,a}$ ($i = 1,2,3$ and $a$ refers to $G^a = E_6$, $H$ or $E_8'$) can be computed for arbitrary orbifolds and read (see also [21]):

$$\begin{array}{lll} J_1: & \delta^{1,a} = & C(G^a) + T(R_1^u) - T(R_2^u) - T(R_3^u) + \sum_k (2\eta_1^k - 1) T(R_k^t), \\ J_2: & \delta^{2,a} = & C(G^a) - T(R_1^u) + T(R_2^u) - T(R_3^u) + \sum_k (2\eta_2^k - 1) T(R_k^t), \\ J_3: & \delta^{3,a} = & C(G^a) - T(R_1^u) - T(R_2^u) + T(R_3^u) + \sum_k (2\eta_2^k - 1) T(R_k^t). \end{array} \tag{55}$$

In these expressions, $C(G^a)$ is the quadratic Casimir of each factor $G^a$ of the gauge group, and $T(R) = \mathrm{Tr}(T_R)^2$ for an arbitrary generator $T_R$ of representation $R$ of $G^a$. It is apparent that the anomaly for the universal, internal current $J \equiv J_1 + J_2 + J_3$ of the $N = 2$ superconformal algebra is proportional to the coefficients $b_0^a$ of the $N = 1$ one-loop $\beta$-function:

$$\sum_{i=1}^{3} \delta^{i,a} = 3C(G) - \sum_{i=1}^{3} T(R_i^u) - \sum_k T(R_k^t) = b_0^a \tag{56}$$

and is characterized by $I_6 = c(g^a)^2 b_0^a J Tr(F^a)^2$. Following the standard procedure of ref. [26], the consistent anomaly reads

$$\begin{aligned} I^a &= \int d^4x I_4^a, \\ I_4^a &= c(g^a)^2 b_0^a [\alpha J Tr(\Lambda^a dA^a) - (1-\alpha)\sigma Tr(F^a)^2], \end{aligned} \tag{57}$$

where $\alpha$ is an arbitrary real constant and $\Lambda^a$ and $\sigma$ are the zero-form parameters for gauge and internal (holonomy) local transformations. The ambiguity in the regularisation of the triangle diagram is reflected by the parameter $\alpha$: choosing for instance $\alpha = 0$ corresponds to a regularization preserving gauge invariance, with only the internal symmetries remaining anomalous. Since we can use the antisymmetric tensor $b_{\mu\nu}$, there exists a counterterm able to cancel the anomaly $I^a$: this is the four-dimensional analogue of the Green-Schwarz mechanism [12]. The counterterm is

$$\delta S^a = c\,(g^a)^2 b_0^a \int d^4x [dB - (1-\alpha)\Omega_3^a] V, \tag{58}$$

where $\Omega_3^a$ is the Chern-Simons form for the gauge group $G^a$ and $B = b_{\mu\nu} dx^\mu \wedge dx^\nu/2$. It is clear that for $\alpha = 0$, the cancellation of the holonomy anomaly for the current $J$ leads then to a gauge invariant contribution to the one-loop effective action:

$$\sum_a \delta S^a = c \sum_a (g^a)^2 b_0^a \int d^4x [dB - \Omega_3^a] V, \tag{59}$$

assuming that, under gauge transformations [see also eqs. (34), (35)],

$$\delta dB = \frac{\sum_a (g^a)^2 b_0^a \delta\Omega_3^a}{\sum_b (g^b)^2 b_0^b}. \tag{60}$$

When supersymmetrized, eq. (59) is precisely of the form (37) discussed previously, with

$$\Delta = c \sum_a (g^a)^2 b_0^a V \tag{61}$$

and $\Omega$ replaced by

$$\tilde{\Omega} = \frac{\sum_a (g^a)^2 b_0^a \Omega_3^a}{\sum_b (g^b)^2 b_0^b}. \tag{62}$$

The vector superfield $V$ is the supersymmetrization of the composite (moduli dependent) holonomy connection.

This example shows again how one-loop corrections to gauge kinetic terms, as in expression (37), can be viewed as the counterterm for cancelling holonomy anomalies. This analysis has been performed at the field theory level only. Limiting ourselves to its moduli dependence, $\Delta$ can still be modified by the real part of a holomorphic function of the moduli $f(T)$ which, however, in order to respect the continuous Peccei-Quinn symmetry, is fixed to be linear in $T$, so that $\mathrm{Re}\, f(T) = a(T + \overline{T})$. String corrections, due to winding modes, break this symmetry in such a way that $\Delta = c \sum_a (g^a)^2 b_0^a [V + \mathrm{Re}\, f(T)]$ is a modular invariant function, as in ref. [10].

We have only considered the universal current $J \equiv J_1 + J_2 + J_3$. This is indeed sufficient for the $Z_3$ orbifold: the manifold of the nine $(1,1)$ moduli is $SU(3,3)/SU(3) \times SU(3) \times U(1)$ and only the Abelian current, which coincides with $J$, can possibly generate a mixed anomaly. This is also sufficient for $Z_7$ orbifolds, for which

$$\delta^{1,a} = \delta^{2,a} = \delta^{3,a}.$$

Other $Z_N$ orbifolds, with $N$ non-prime, will generate several anomalies [21].

We close this section by considering the part of the function $\mathcal{G}$ describing moduli and generation kinetic and superpotential terms (this is the function denoted by $\hat{\mathcal{G}}$ in eq. (40)). It is clear from eq. (47) that the superpotential $w$ transforms under (50) according to

$$w \longrightarrow e^{-\phi} w. \tag{63}$$

The effect of gaugino condensates [27, 19, 28] can be viewed as an additional term to the superpotential $w$, which follows from the following argument. The scale at which an asymptotically-free gauge interaction, with running coupling constant $g^a(Q)$, becomes strong is $M_C \exp(\frac{8\pi^2}{b_0^a (g^a)^2})$, where $g^a$ is the value of $g^a(Q)$ at the Kaluza-Klein scale $M_C \sim R^{-1}$. At tree-level [7],

$$(g^a)^{-2} = \mathrm{Re}\, S,$$

[7] We omit the Kac-Moody level, for simplicity.

but this is no longer the case in the loop-corrected effective theory. In the generalization of Lagrangian (38) to the case of a semisimple gauge group $G = \prod_a G^a$, gauge kinetic terms arise from $-\frac{1}{4}[(S+\overline{S})\tilde{\Omega}]_D$, $\tilde{\Omega}$ being defined in eq. (62). The relation between the gauge coupling constants (at the Kaluza-Klein scale) and the $S$-field becomes then

$$\frac{1}{(g^a)^2} = \frac{(g^a)^2 b_0^a}{\sum_b (g^b)^2 b_0^b} \,\mathrm{Re}\, S = \frac{b_0^a}{\sum_b b_0^b} \,\mathrm{Re}\, S,$$

using the previous relation for the tree-level coupling constants. The complete superpotential, including the contributions from gaugino condensates, takes then the form

$$w = \sum_a \kappa_a e^{24\pi^2 S/\sum_a b_0^a} + w_{ijk} C^i C^j C^k, \tag{64}$$

where $C^i$ stands for the generation superfields and $\kappa_a$ replaces gaugino condensates. Under the Kähler transformation of the moduli space (50), $w_{ijk} \to e^{-2\phi} w_{abc}$, $C^i \to e^{\phi/3} C^i$ and, in order to fit the transformation of $S$,

$$\lambda = -\sum_a \frac{1}{24\pi^2} b_0^a, \tag{65}$$

which in turns determines

$$c = -\frac{1}{24\pi^2} \tag{66}$$

in the one-loop contribution (61), since we have demonstrated in sect. 2 and 3 how loop corrections can be represented as a correction to $S$ [as in eq. (40)] or as a correction to gauge kinetic terms. Eq. (64) also shows the self-consistency of our anomaly considerations.

# 5 Scalar potential

In view of possible applications, for example the discussion of supersymmetry breaking via gaugino condensation [27,19,28] and/or other mechanisms [29,30], it is useful to write down explicitly the scalar potential associated to the supergravity theory specified by eqs. (40), (41). Defining for convenience

$$Y \equiv S + \overline{S} + \Delta, \tag{67}$$

[recall that $Y$ can be viewed as the inverse bare coupling constant, eq. (46)] the Kähler metric mixes now the $S$ field with the other fields $\Sigma^i$:

$$\mathcal{G}_S{}^S = \frac{1}{Y^2}, \quad \mathcal{G}_S{}^j = \frac{\Delta^j}{Y^2}, \quad \mathcal{G}_i{}^S = \frac{\Delta_i}{Y^2},$$

$$\mathcal{G}_i{}^j = \frac{\Delta_i \Delta^j}{Y^2} + \hat{\mathcal{G}}_i{}^j - \frac{\Delta_i{}^j}{Y}. \tag{68}$$

We will make now the ansatz, suggested by the previous discussion

$$\Delta_i{}^j = k \hat{\mathcal{G}}_i{}^j. \tag{69}$$

This allows us to write

$$\mathcal{G}_i{}^j = \frac{1}{Y^2}\left(\Delta_i \Delta^j + Y(Y-k)\hat{\mathcal{G}}_i{}^j\right), \tag{70}$$

with the positivity of the Kähler metric implying $Y > k$. The inverse Kähler metric is easily found to be

$$\mathcal{G}^{-1} = \frac{Y}{Y-k}\begin{pmatrix} Y(Y-k) + \Delta^k \hat{\mathcal{G}}_k^{-1\,l}\Delta_l & -\Delta^k \hat{\mathcal{G}}_k^{-1\,j} \\ -\hat{\mathcal{G}}_i^{-1\,l}\Delta_l & \hat{\mathcal{G}}_i^{-1\,j} \end{pmatrix}, \tag{71}$$

and from this the corresponding F-term part of the scalar potential reads

$$\frac{V_F'}{e^{\mathcal{G}'}} = \frac{V_F}{e^{\mathcal{G}}} + \frac{k}{Y-k}\hat{\mathcal{G}}^i \hat{\mathcal{G}}_i^{-1\,j}\hat{\mathcal{G}}_j, \tag{72}$$

which is certainly positive semidefinite for $k \geq 0$.

In the case of superstring low-energy effective actions, however, the gauge group $G$ is the product of different simple or $U(1)$ factors $G^a$, and the structure of the scalar potential is going to be more complicated: a detailed discussion of this case goes beyond the aim of the present paper and will be given elsewhere [21].

# 6 Conclusions

To conclude, we have shown in this paper that, contrary to common wisdom, the occurrence of non-harmonic functions in the gauge boson kinetic terms can be perfectly consistent with (local or global) supersymmetry. The supersymmetrization procedure relies on the use of the Yang-Mills Chern-Simons superfield $\Omega$ instead of the usual superfield $W^2$. However, gauge invariance requires the introduction of a linear multiplet, which implements an anomaly cancellation mechanism closely analogous to the Green-Schwarz mechanism. Indeed, the use of the linear multiplet does not provide any constraint on the form of the gauge kinetic terms: even if each theory formulated with a linear multiplet can be dualized into a supergravity of the standard form [15,16], the non-analytic terms are transported into the Kähler function by the duality transformation. The function $f_{ab}$, which defines gauge kinetic terms in the standard formulation, is completely insensitive to the non-harmonic terms.

By a superfield duality transformation, this theory can always be transformed to a standard supergravity Lagrangian with $f_{ab} = S\delta_{ab}$, $L$ being replaced by the chiral multiplet $S$. The non-harmonic contributions to the gauge kinetic terms of the original, 'linear' theory are moved to the Kähler potential for $S$, in the transformed standard theory. The axionic coupling to $F\tilde{F}$ cannot be obtained in the $L$ version of the theory: such a term does not appear in the Lagrangian, and it cannot be obtained by partial integration. Only the duality transformation will generate it. Remember that the duality transformation can be viewed as a non-analytic field redefinition: such a field redefinition is not compatible with the standard formulation of supergravity as given in ref. [15,16].

The loop effects considered in this paper are not directly related to the threshold corrections computed in ref. [10], which are due to extra isometries of the moduli space. However, the interpretation of these effects as $\sigma$–model anomalies should also apply to that case.

Finally, we would like to mention that the full supersymmetric anomaly term should also contain $F$–terms coupling the $S$–field to the $\sigma$–model and gravitational curvatures, of the form [7,13]

$$cSW_\alpha(K)^2 + dSW^2_{\alpha\beta\gamma}. \tag{73}$$

These terms represent however higher derivative corrections to the effective action [31], and have been neglected in the context of our discussion.

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