### Small-radius jets to all orders

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With hadron colliders continuing to push the boundaries of precision, it is becoming increasingly important to have a detailed understanding of the subtleties appearing at smaller values of the jet radius R. We present a method to resum all leading logarithmic terms,  $\alpha_s^n \ln^n R$ , using a generating functional approach, as was recently discussed in Ref.<sup>1</sup>. We study a variety of observables, such as the inclusive jet spectrum and jet vetoes for Higgs physics, and show that small-R effects can be sizeable. Finally, we compare our calculations to existing ALICE data, and show good agreement.

# 1 Introduction

Jets are collimated bunches of particles produced by fragmentation of a quark or gluon.<sup>2</sup> They emerge from a variety of processes, such as scattering of partons in colliding protons, hadronic decays of massive particles (W, Z, H, t) and radiative gluon emissions. They are widely used as proxies for hard quarks and gluons. A precise understanding of jet processes at hadron colliders is critical in a wide variety of scenarios, such as background discrimination in Higgs production.

# 1.1 Jet algorithms

A jet definition includes a jet algorithm mapping final state particle momenta to jet momenta, the parameters required by the algorithm and a recombination scheme. Moreover, a jet definition should also be simple to implement in experimental analyses and theoretical calculations. It should yield finite cross sections at any order in perturbation theory and be relatively insensitive to hadronisation.

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Most jet definitions used at hadron colliders are based on sequential recombination algorithms, which cluster pairs that are closest in a metric defined by the divergence structure of the theory. This requires an external parameter, the jet radius R, specifying up to which point separate partons are recombined into a single jet.

# 1.2 Perturbative properties of jets

Jet properties will be affected by gluon radiation and  $g \rightarrow q\bar{q}$  splitting. In particular, emissions beyond the reach of the jet algorithm will reduce the jet energy.

We can calculate in the small-R limit the average energy difference between the hardest final state jet and an initial quark, considering gluon emissions outside of the jet. We find

$$\begin{split} \langle \Delta z \rangle_q^{\text{hardest}} &= \int^{\P(1)} \frac{d\theta^2}{\theta^2} \int dz (\max[z, 1-z] - 1) \times \frac{\alpha_s}{2\pi} p_{qq}(z) \Theta(\theta - R) \\ &= \frac{\alpha_s}{\pi} C_F \left( 2\ln 2 - \frac{3}{8} \right) \ln R + \mathcal{O}(\alpha_s), \end{split}$$
(1)

We notice the appearance of logarithmic terms of the form  $\alpha_s \ln R$ , which in the limit  $\alpha_s \ln R \sim \mathcal{O}(1)$  will spoil the convergence of the perturbative series, requiring resummation to all orders. This limit is of relevance for example in extreme environments, such as heavy ion collisions, where values down to R = 0.2 are used,  $^{8,9,10,11,12}$  and jet substructure tools such as filtering  $^{15}$  and trimming  $^{16}$  which resolve small-R subjets (with  $R_{\rm sub} = 0.2-0.3$ ) within moderate or large-R jets. Furthermore, even for the most common choice of the jet radius,  $R = 0.4 - 0.5^{17,18}$ , higher order corrections could be sizable, and small-R resummation could bring interesting insights into their effect.

We aim therefore to resum all leading logarithmic (LL) terms  $\alpha_s^n \ln^n R$  in the limit of small-R, for a wide variety of observables.

## 2 Method

We use generating functionals Q(x,t), G(x,t) to encode the parton content when resolving an initial quark or gluon with momentum fraction x on an angular scale defined through a variable t > 0, where

$$t = \int_{R^2}^{1} \frac{d\theta^2}{\theta^2} \frac{\alpha_s(p_t\theta)}{2\pi} \sim \frac{\alpha_s}{2\pi} \ln \frac{1}{R^2}.$$
 (2)

The evolution of a quark or gluon can then be described by two coupled differential equations. For an initiating quark, we have

$$\frac{dQ(x,t)}{dt} = \int dz p_{qq}(z) \left[ Q(zx,t) G((1-z)x,t) - Q(x,t) \right],\tag{3}$$

while for an initiating gluon the evolution is described by

$$\frac{dG(x,t)}{dt} = \int dz \, p_{gg}(z) \left[ G(zx,t)G((1-z)x,t) - G(x,t) \right] \\ + \int dz \, n_f \, p_{qg}(z) \left[ Q(zx,t)Q((1-z)x,t) - G(x,t) \right] \,. \tag{4}$$

These evolution equations allow us to resum observables to all orders numerically. They effectively exploit angular ordering.

# 3 Observables

We now present a few key observables of current interest where small-radius effects have been studied in detail.

### 3.1 Microjet vetoes in Higgs production

Jet veto resummation for Higgs production contain terms of the form

$$\alpha_s^m \ln^{2m} \frac{Q}{p_t} + \text{subleading} \tag{5}$$

Among the subleading terms, there are small-R enhanced terms such as

$$\alpha_s^{m+n} \ln^m \frac{Q}{p_t} \ln^n \frac{1}{R^2} + \dots \tag{6}$$

which have been suspected of playing an important role, and have been calculated to first order by several groups,  $^{19,20,21}$  as well as numerically to second order.  $^{22 \ b}$ 

These small-R terms can be accounted for by an overall factor  ${\cal U}$  that multiplies the jet veto efficiency, where  ${\cal U}$  has the form

$$\mathcal{U} \equiv P(\text{no microjet veto})/P(\text{no primary parton veto}) = \exp\left[-\frac{4\alpha_s(p_t)C}{\pi}\ln\frac{Q}{p_t}\int_0^1 dz \, f^{\text{hardest}}(z, t(R, p_t))\ln z\right],$$
(7)

where we defined  $f^{\text{hardest}}(z)$  the probability that the hardest microjet carries a momentum fraction z.

We extend therefore the calculation of small-R corrections in jet vetoes to all orders, and implement this result in JetVHeto<sup>19</sup>. The jet veto efficiency obtained is shown in figure 1.

The higher order small-R terms lead to a small shift in central value. But more noticeable is the change at low  $p_t$  of the bands calculated from scale variations, which grow larger. We attribute this to a non-trivial interplay between two classes of logarithms, of  $Q/p_t$  and R, where adding the small-R terms reveals uncertainties that are otherwise missed.



Figure 1 – Jet veto efficiency at 13 TeV with R = 0.2, in green (blue) without (with) small-R resummation

<sup>&</sup>lt;sup>b</sup>The values in the first version of Ref. <sup>22</sup> were in disagreement with our analytical calculations, but this was corrected in arXiv-v3.

### 3.2 Inclusive jet spectrum

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The jet spectrum can be obtained from the convolution of the inclusive microjet fragmentation function with the inclusive partonic spectrum from hard  $2 \rightarrow 2$  scattering

$$\frac{d\sigma_{\text{jet}}}{dp_t} = \sum_i \int_{p_t} \frac{dp'_t}{p'_t} \frac{d\sigma_i}{dp'_t} f_{\text{jet}/i}^{\text{incl}}(p_t/p'_t, t), \tag{8}$$

where  $f_{jet/i} \equiv \sum_j f_{j/i}$ , and we define, for an initial parton *i*,  $f_{j/i}^{incl}(z,t)$  the inclusive distribution of microjets of flavour j, at an angular scale *t*, carrying a momentum fraction *z* of the parton's momentum.

If we assume that the partonic spectrum is dominated by a single flavour *i* and that its  $p_t$  dependence is of the form  $d\sigma_i/dp_t \sim p_t^{-n}$ , then

$$\frac{d\sigma_{\text{jet}}}{dp_t} \simeq \frac{d\sigma_i}{dp_t} \int_0^1 dz \, z^{n-1} f_{\text{jet}/i}^{\text{incl}}(z,t) \equiv \frac{d\sigma_i}{dp_t} \langle z^{n-1} \rangle_i^{\text{incl}},\tag{9}$$

Small-R effects are therefore enhanced by a  $\ln n$  factor

$$\sim \alpha_s \ln \frac{1}{R^2} \ln n \,. \tag{10}$$

At the LHC, typical n values for the partonic spectrum range from about 4 at low  $p_t$ , to 7 or even larger at high  $p_t$ .

In figure 2, we show a comparison of the NLO prediction supplemented with small-*R* logarithms with existing ALICE data for the inclusive jet spectrum with R = 0.2. Here hadronisation is calculated using an analytic model <sup>23</sup>, and the theoretical error bands are obtained from the envelope of the  $0.5 < x_{\mu_R}, x_{\mu_F} < 2$  and  $1 < R_0 < 1.5$  variations, as well as an estimation of hadronisation uncertainties.

At small values of the jet radius, the small-R resummation improves agreement with the data, and reduces the scale dependence of the NLO prediction.



Figure 2 - Comparison of the matched resummation (blue) with ALICE data (red).

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