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HADRONIC TAU-DECAYS AND QCD

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ABSTRACT

The total τ -hadronic width can be accurately predicted using standard QCD-methods. The theoretical analysis of this observable is updated and it is shown how the τ -decay width can be used to infer a value of the QCD-scale $\Lambda_{\overline{MS}}$.

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1. INTRODUCTION

Besides its intrinsic interest as a sequential lepton in the standard model of electroweak interactions, the τ is the only presently known lepton heavy enough to decay into hadrons. Therefore, its semileptonic decays appear to be an ideal laboratory for studying the hadronic weak currents in very clean conditions. Moreover, contrary to the well-known $e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons}$ process, which only tests the electromagnetic vector current, the semileptonic τ -decay modes offer the possibility to study the properties of both vector and axial currents.

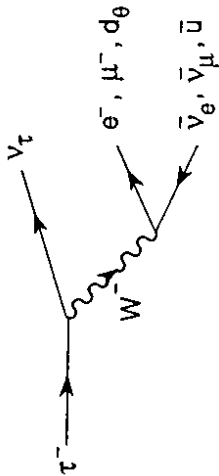


Fig. 1. Feynman diagram for the decay of the τ -lepton.

Within the standard model, the τ lepton decays via the W -emission diagram shown in figure 1. Since the W -coupling to the charged current is of universal strength,

$$L_{cc} = \frac{g}{2\sqrt{2}} W_{\mu}^{+} \left\{ \sum_l \bar{\nu}_l \gamma^{\mu} (1 - \gamma_5) l + \bar{u} \gamma^{\mu} (1 - \gamma_5) d \right\} + h.c., \quad (1.1)$$

there are five equal contributions (if final masses and gluonic corrections are neglected) to the τ^- -decay width. Two of them correspond to the decay modes $\tau^- \rightarrow \nu_{\tau} e^{-} \bar{\nu}_e$ and $\tau^- \rightarrow \nu_{\tau} \mu^{-} \bar{\nu}_{\mu}$, while the other three are associated with the three possible colours of the quark-antiquark pair in the $\tau^- \rightarrow \nu_{\tau} d_{\theta} \bar{u}$ decay mode ($d_{\theta} \equiv \cos \theta_c d + \sin \theta_c s$). Hence, the branching ratios for the different channels are expected to be approximately:

$$Br(\tau^- \rightarrow \nu_{\tau} l^{-} \bar{\nu}_l) \simeq \frac{1}{5} = 20\%, \quad (l = e, \mu), \quad (1.2a)$$

$$R_H \equiv \frac{\Gamma(\tau^- \rightarrow \nu_{\tau} + \text{hadrons})}{\Gamma(\tau^- \rightarrow \nu_{\tau} e^{-} \bar{\nu}_e)} \simeq N_c = 3, \quad (1.2b)$$

which should be compared with the formal experimental averages [1] :

$$Br(\tau^- \rightarrow \nu_{\tau} e^{-} \bar{\nu}_e) = (17.7 \pm 0.4)\%, \quad (1.3a)$$

$$Br(\tau^- \rightarrow \nu_{\tau} \mu^{-} \bar{\nu}_{\mu}) = (17.8 \pm 0.4)\%, \quad (1.3b)$$

$$R_H = (3.50 \pm 0.07)\%. \quad (1.3c)$$

The agreement is fairly good. Notice that the measured tau hadronic width provides strong evidence for the colour degree of freedom. We will discuss later whether the QCD dynamics is able to explain the difference between the measured value of R_H and the lowest order prediction $R_H = N_c$.

For the leptonic modes, the τ -decay partial width is easily computed, with the result (neutrinos are assumed to be massless)

$$\Gamma(\tau^- \rightarrow \nu_{\tau} l^{-} \bar{\nu}_l) = \frac{G_F^2 m_{\tau}^5}{192\pi^3} f(m_l^2/m_{\tau}^2) r, \quad (l = e, \mu), \quad (1.4)$$

where $f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \log x$. The factor r takes into account radiative corrections not included in the Fermi coupling constant G_F , and the non-local structure of the W -propagator. These effects have been shown [2] to be small, $r \simeq 0.996$. Eq.(1.4) gives a relation between the tau-lifetime and the electronic branching ratio,

$$Br(\tau^- \rightarrow \nu_{\tau} e^{-} \bar{\nu}_e) = \tau_r / (1.600 \times 10^{-12} s). \quad (1.5)$$

Using the world average measured τ -lifetime [1], $\tau_r = (3.03 \pm 0.08) \times 10^{-13} s$, one gets the prediction $Br(\tau^- \rightarrow \nu_{\tau} e^{-} \bar{\nu}_e)_{th} = (18.9 \pm 0.5)\%$, which is about two standard deviations higher than the measured branching fraction (1.3a). Given the present limit [3] of $m_{\nu} < 35 MeV$ (95% C.L.), this small discrepancy cannot be due to a non-zero value of the tau-neutrino mass. The agreement is slightly better in the muonic channel; taking into account the phase-space mass correction, $f(m_{\mu}^2/m_{\tau}^2) = 0.9728$, one predicts $Br(\tau^- \rightarrow \nu_{\tau} \mu^{-} \bar{\nu}_{\mu})_{th} = (18.4 \pm 0.5)\%$, which compares reasonably well with the measured value (1.3b). Note, however, that in both cases the experimental branching fractions are below the values extracted theoretically from the measured lifetime.

The semileptonic decay modes of the tau, $\tau^- \rightarrow \nu_{\tau} H^-$, probe the matrix element of the left-handed charged current between the vacuum and the final hadronic state H^- ,

$$\langle H^- | d_{\theta} \gamma^{\mu} (1 - \gamma_5) u | 0 \rangle. \quad (1.6)$$

For the decay modes with lowest multiplicity, $\tau^- \rightarrow \nu_{\tau} \pi^-$ and $\tau^- \rightarrow \nu_{\tau} K^-$, the relevant matrix elements are already known from the measured decays $\pi^- \rightarrow \mu^{-} \bar{\nu}_{\mu}$ and $K^- \rightarrow \mu^{-} \bar{\nu}_{\mu}$. In the Cabibbo allowed modes with $J^P = 1^-$, the matrix element of the vector charged current can also be obtained, through an isospin rotation, from the isovector part of the e^+e^- annihilation cross-section into hadrons, which measures the hadronic matrix element of the $I = 1$ component of the electromagnetic current. For all these modes, the corresponding τ -decay widths can then be easily computed and the predictions agree quite well with the data.

The exclusive τ -decays into final hadronic states with $J^P = 1^+$ or Cabibbo suppressed modes with $J^P = 1^-$ cannot be predicted with the same degree of confidence. We can only make model-dependent estimates, with an accuracy which depends on our ability to handle the strong interactions at low energies. That just indicates that the decay of the τ -lepton is providing us with new experimental hadronic information. Due to their semileptonic character, the hadronic tau-decay data can then be a unique and extremely useful tool to learn about the couplings of the low-lying mesons to the weak currents.

An overview of different aspects of strong interaction phenomena which can be tested using tau-decay data has been already given in reference [4]. To avoid unnecessary repetition, I'm going to concentrate here on the total τ -hadronic width, which can be accurately predicted [5] [6] using standard QCD-methods. I will show how the quantity R_H , defined in eq.(1.2b), can be used to infer a value of the QCD-scale $\Lambda_{\overline{MS}}$. In the following, I will present an update of the theoretical analysis of this quantity, taking into account the new results [7] on the $O(\alpha_s^2)$ correction to the vector current spectral function.

2. CURRENT CORRELATORS

It is convenient to consider the two-point correlation functions

$$\begin{aligned} i \int d^4x e^{iqx} < 0 | T(V^\mu(x)_{ij} V^\nu(0)^\dagger_{ij}) | 0 > \\ \equiv (-g^{\mu\nu} q^2 + q^\mu q^\nu) \Pi_V^{(1)}(q^2)_{ij} + q^\mu q^\nu \Pi_V^{(0)}(q^2)_{ij}, \end{aligned} \quad (2.1a)$$

$$\begin{aligned} i \int d^4x e^{iqx} < 0 | T(A^\mu(x)_{ij} A^\nu(0)^\dagger_{ij}) | 0 > \\ \equiv (-g^{\mu\nu} q^2 + q^\mu q^\nu) \Pi_A^{(1)}(q^2)_{ij} + q^\mu q^\nu \Pi_A^{(0)}(q^2)_{ij}, \end{aligned} \quad (2.1b)$$

associated with the vector $V^\mu(x)_{ij} = \bar{q}_i \gamma^\mu q_j$ and axial $A^\mu(x)_{ij} = \bar{q}_i \gamma^\mu \gamma_5 q_j$ currents. Here, $i, j = 1, 2, 3$ denote the light quark flavours (u,d,s), and the different Lorentz structures on the r.h.s. correspond to $J = 1$ and $J = 0$ angular momentum.

We can write down a spectral decomposition for these correlators,

$$\Pi_{V,A}^{(1,0)}(q^2)_{ij} = \int_0^\infty ds \frac{\frac{1}{\pi} \text{Im} \Pi_{V,A}^{(1,0)}(s)_{ij}}{s - q^2 - i\epsilon} + \text{subtractions}. \quad (2.2)$$

The corresponding spectral functions govern the hadronic τ -decay width, which can be written as an integral over the invariant mass of the final hadrons,

$$R_H = 12\pi \int_0^{m_\tau^2} ds \frac{ds}{m_\tau^2} \left(1 - \frac{s}{m_\tau^2}\right)^2 \{ (1 + \frac{2s}{m_\tau^2}) \text{Im} \Pi^{(1)}(s) + \text{Im} \Pi^{(0)}(s) \}, \quad (2.3)$$

where $(J = 0, 1)$

$$\begin{aligned} \text{Im} \Pi^{(J)}(s) \equiv \cos^2 \theta_c \{ \text{Im} \Pi_V^{(J)}(s)_{12} + \text{Im} \Pi_A^{(J)}(s)_{12} \} \\ + \sin^2 \theta_c \{ \text{Im} \Pi_V^{(J)}(s)_{13} + \text{Im} \Pi_A^{(J)}(s)_{13} \}. \end{aligned} \quad (2.4)$$

In the chiral limit ($m_u = m_d = m_s = 0$), the longitudinal vector spectral functions vanish, while the corresponding axial ones contain only the pole contribution from the Goldstone bosons, ($f \approx f_\pi \approx 93.3 \text{ MeV}$)

$$\frac{1}{\pi} \text{Im} \Pi_V^{(0)}(s)_{12} = \frac{1}{\pi} \text{Im} \Pi_V^{(0)}(s)_{13} = 0, \quad (2.5a)$$

$$\frac{1}{\pi} \text{Im} \Pi_A^{(0)}(s)_{12} = \frac{1}{\pi} \text{Im} \Pi_A^{(0)}(s)_{13} = 2f^2 \delta(s). \quad (2.5b)$$

In principle the vector and axial-vector spectral functions are calculable in QCD, but in practice we are still very far away from being able to do that in the low energy region. The semileptonic τ -decays offer the possibility to obtain experimentally these spectral functions, for s -values below m_τ^2 , from the invariant mass distribution of the final mesons [8] [4].

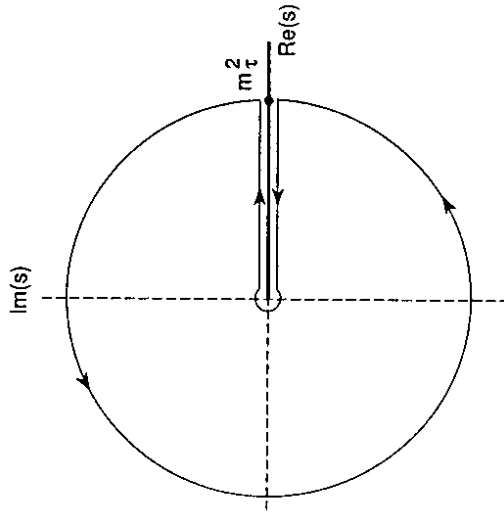


Fig.2. Integration contour in the complex s -plane, used to obtain eq.(2.6).

The imaginary parts of the two-point correlation functions (2.1) are proportional to their discontinuities along the positive real s -axis; therefore, using the contour shown in figure 2, eq. (2.3) can be expressed as an integral around the circle $|s| = m_\tau^2$,

$$R_H = 6\pi i \int_{|s|=m_\tau^2} \frac{ds}{m_\tau^2} \left(1 - \frac{s}{m_\tau^2}\right)^2 \{ (1 + \frac{2s}{m_\tau^2}) \Pi^{(1)}(s) + \Pi^{(0)}(s) \}. \quad (2.6)$$

Even if we are not able to compute the spectral functions at very low values of s , analyticity allows us to calculate integrals from zero to some value s_0 of these spectral functions, provided s_0 is big enough for a perturbative calculation to be reliable at this scale. In our case $s_0 = m_\tau^2$, which seems to be sufficiently big ($\alpha_s(m_\tau^2)/\pi \approx 0.1$). We will discuss in section 4 how sizeable the potential non-perturbative contributions are at this scale.

3. PERTURBATIVE CALCULATION

The light quark masses are much smaller than m_τ ; therefore, we can work in the chiral limit ($m_u = m_d = m_s = 0$) to simplify the calculation. The tiny quark mass corrections will be estimated in the next section. The kind of diagrams contributing to the correlation functions (2.1) are shown in figure 3.

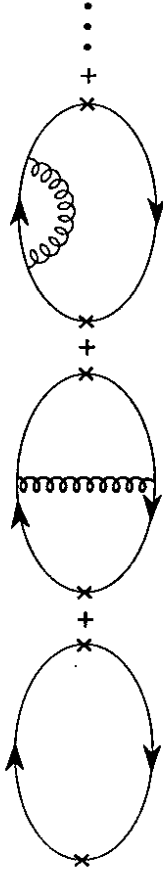


Fig.3. Feynman diagrams contributing to the correlation functions (2.1).

As we are working with non-singlet currents, the two vertices must be joined by a quark trace at all orders in α_s . Therefore, the chiral invariance of the strong interactions implies $\Pi_V^{(1)}(s)_{ij} = \Pi_A^{(1)}(s)_{ij}$ ($i \neq j$) at any finite order in α_s [9]. Moreover, for massless quarks the result is flavour independent.

In the case of the electromagnetic vector current, $J_{em}^\mu \equiv \sum_f Q_f \bar{q}_f \gamma^\mu q_f$ ($f = u, d, s$), the associated two-point correlation function has already been computed to four-loop accuracy. The corresponding spectral function governs the e^+e^- annihilation into hadrons:

$$\begin{aligned} R(s) &\equiv \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = 12\pi \text{Im} \Pi_{J_{em}}(s) \\ &= 3 \left(\sum_f Q_f^2 \right) \left\{ 1 + \frac{\alpha_s(s)}{\pi} + F_3 \left(\frac{\alpha_s(s)}{\pi} \right)^2 + F_4 \left(\frac{\alpha_s(s)}{\pi} \right)^3 + O\left(\left(\frac{\alpha_s(s)}{\pi} \right)^4 \right) \right\} \\ &\quad + \left(\sum_f Q_f \right)^2 \left\{ \Delta_4 \left(\frac{\alpha_s(s)}{\pi} \right)^3 + O\left(\left(\frac{\alpha_s(s)}{\pi} \right)^4 \right) \right\}. \end{aligned} \quad (3.1)$$

The $O(\alpha_s^2)$ coefficient is well known [10] [11] [12],

$$F_3 = 1.986 - 0.115n_f, \quad (3.2)$$

where n_f denotes the number of flavours. A calculation of the $O(\alpha_s^3)$ corrections was reported in ref. [13], where a very large value of F_4 was claimed ($F_4 = 67.340$ for $n_f = 3$). This could cast doubts on the meaning of the perturbative expansion in powers of α_s . However, some errors have been discovered [14] [7] in the computer program used in this calculation, which make the result of ref. [13] incorrect. The four-loop corrections to $R(s)$ have been re-evaluated recently, with the result [7]

$$F_4 = -6.637 - 1.200n_f - 0.005n_f^2, \quad (3.3a)$$

$$\Delta_4 = -1.240. \quad (3.3b)$$

The new value of F_4 is smaller by an order of magnitude and has the opposite sign from the one given in ref. [13].

The Δ_4 -term, which is proportional to $(\sum_f Q_f)^2$, results from the QCD analogues of the QED light-by-light diagrams (see figure 4). Terms of such a type appear only at the four-loop level and in higher orders. This kind of diagrams, with a separate quark trace attached to each current, are only present for singlet currents. They do not contribute to the hadronic decay of the τ -lepton.

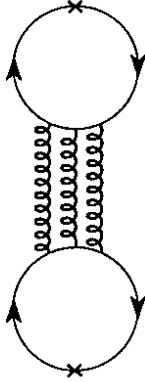


Fig.4. Feynman diagram contributing to the Δ_4 -term in eq.(3.1).

Taking away the factor $(\sum_f Q_f^2)$ in the first term of the r.h.s. of eq.(3.1) and doing trivial phase-space integrations, it is straightforward to get the perturbative prediction for the hadronic τ -decay width. One gets [5] [6],

$$\begin{aligned} R_H^{\text{pert}} &= 3 \left\{ 1 + \frac{\alpha_s(m_\tau^2)}{\pi} + \left(\frac{\alpha_s(m_\tau^2)}{\pi} \right)^2 \left[F_3 - \frac{19}{24} \beta_1 \right] \right. \\ &\quad \left. + \left(\frac{\alpha_s(m_\tau^2)}{\pi} \right)^3 \left[F_4 - \frac{19}{12} F_3 \beta_1 - \frac{19}{24} \beta_2 + 288 \beta_1^2 \right] + O\left(\left(\frac{\alpha_s(m_\tau^2)}{\pi} \right)^4 \right) \right\}, \end{aligned} \quad (3.4)$$

where $\beta_1 = (2n_f - 33)/6$ and $\beta_2 = (19n_f - 153)/12$ are the first two coefficients of the QCD β -function.

Using the results (3.2) and (3.3) with $n_f = 3$, we have finally:

$$R_H^{pert} = 3 \left\{ 1 + \frac{\alpha_s(m_\tau^2)}{\pi} + 5.2035 \left(\frac{\alpha_s(m_\tau^2)}{\pi} \right)^2 + 26.376 \left(\frac{\alpha_s(m_\tau^2)}{\pi} \right)^3 + O\left(\left(\frac{\alpha_s(m_\tau^2)}{\pi} \right)^4 \right) \right\}. \quad (3.5)$$

4. QUARK-MASS CORRECTIONS AND NON-PERTURBATIVE CONTRIBUTIONS

Although the perturbative expansion (3.5) is very well behaved, one should worry about the size of possible non-perturbative contributions at the scale m_τ . Non-perturbative effects, which are usually parametrized by a set of vacuum expectation values of quark and gluon fields, can be included as power corrections to the perturbative QCD result [15], ($Q^2 \equiv -q^2$)

$$\Pi(Q^2) = \Pi(Q^2)_{pert} + \frac{1}{4\pi^2} \sum_{n=2} \frac{C_{2n} < O_{2n} >}{(Q^2)^n}. \quad (4.1)$$

For the $J^P = 1^- (i, j) = (1, 2)$ correlator, the leading non-perturbative correction is given, in the $m_u = m_d \equiv m$ limit, by

$$C_4 < O_4 >_V = \frac{\pi}{3} < \alpha_s G^2 > + 4\pi^2 m < \bar{u}u + \bar{d}d >. \quad (4.2)$$

Due to the conservation of the vector current the $J^P = 0^+$ correlator is zero in the isospin limit. For the axial two-point function, the Goldstone nature of the pion makes it difficult to separate the $J^P = 1^+$ and $J^P = 0^-$ correlators, so it is better to work with their sum; the leading non-perturbative term is given in this case by

$$C_4 < O_4 >_A = \frac{\pi}{3} < \alpha_s G^2 > - 4\pi^2 m < \bar{u}u + \bar{d}d >. \quad (4.3)$$

The hadronic τ -decay data can be used to obtain information on these vacuum condensates (see ref. [4] for details). The analysis of the ARGUS data [16] made in ref. [17] has given the results:

$$C_4 < O_4 >_V = (0.025 - 0.11) GeV^4, \quad C_4 < O_4 >_A = (0.045 - 0.10) GeV^4, \quad (4.4a)$$

$$C_6 < O_6 >_V = -(0.16 - 0.24) GeV^6, \quad C_6 < O_6 >_A = (0.16 - 0.28) GeV^6, \quad (4.4b)$$

$$C_8 < O_8 >_V = (0.28 - 0.55) GeV^8, \quad C_8 < O_8 >_A = -(0.36 - 0.54) GeV^8. \quad (4.4c)$$

Similar results have been obtained in ref. [18]. Although the errors are quite big, the typical scale is clearly below 1 GeV. We should then expect the non-perturbative corrections

to be smaller than $(1 GeV/\sqrt{Q^2})^4$, which is about 10% at the τ -mass scale. We will see in the following that for R_H they are in fact a factor of 10 smaller than that.

Using eq.(2.6) and Cauchy's theorem, it is easy to calculate the effect of inverse power corrections on R_H . One finds [6]

$$R_H = 3 \left\{ 1 + \frac{\alpha_s(m_\tau^2)}{\pi} + 5.2035 \left(\frac{\alpha_s(m_\tau^2)}{\pi} \right)^2 + 26.376 \left(\frac{\alpha_s(m_\tau^2)}{\pi} \right)^3 + O\left(\left(\frac{\alpha_s(m_\tau^2)}{\pi} \right)^4 \right) \right\} + \frac{C_2 < O_2 >_{V+A}}{m_\tau^2} - 3 \frac{C_6 < O_6 >_{V+A}}{m_\tau^6} - 2 \frac{C_8 < O_8 >_{V+A}}{m_\tau^8}. \quad (4.5)$$

Here, $C_2 < O_2 >_{V+A}$ stands for the small perturbative quark mass effects,

$$C_2 < O_2 >_{V+A} = -8 [\cos^2 \theta_c (m_u^2 + m_d^2) + \sin^2 \theta_c (m_u^2 + m_s^2)], \quad (4.6)$$

where $m_i \equiv m_i(m_\tau^2) = \hat{m}_i / (\log(m_\tau / \Lambda_{\overline{MS}}))^{-2/\beta_i}$ ($i = u, d, s$) are the running quark masses evaluated at the m_τ -scale. Using [19] $\hat{m}_u = (8.2 \pm 1.5) MeV$, $\hat{m}_d = (14.4 \pm 1.5) MeV$, $\hat{m}_s = (290 \pm 50) MeV$ and [1] $\sin \theta_c = 0.22$, the numerical effect of this mass correction is

$$\delta_2 \equiv \frac{C_2 < O_2 >_{V+A}}{m_\tau^2} = - \begin{pmatrix} 0.43 \pm 0.15 \\ 0.54 \pm 0.19 \\ 0.65 \pm 0.22 \end{pmatrix} \%, \quad \text{for } \Lambda_{\overline{MS}} = \begin{pmatrix} 0.1 \\ 0.2 \\ 0.3 \end{pmatrix} GeV. \quad (4.7)$$

Due to the phase space factors in eq.(2.6), the leading non-perturbative condensates of dimension four do not contribute to R_H ¹. Non-perturbative effects are then suppressed by additional powers of the tau mass. The only contributions which survive the integration (2.6) are the ones from condensates of dimension 6 and 8. Moreover, as shown in eqs.(4.4), for these condensates there is a strong cancellation between the vector and the axial-vector contributions. This cancellation can be understood in the dimension six-case as being due to the γ_5 -flip which implies $C_6 < O_6 >_A = -\frac{11}{7} C_6 < O_6 >_V$.

Taking also into account information on the value of $C_6 < O_6 >_V$ coming from other sources (e^+e^- cross-sections and charmonium) [20], the size of the leading non-perturbative effect was estimated in ref. [6] to be

$$\delta_6 \equiv -3 \frac{C_6 < O_6 >_{V+A}}{m_\tau^6} = \frac{12 C_6 < O_6 >_V}{7 m_\tau^6} \approx -(0.6 - 1.3) \%. \quad (4.8)$$

The sign difference between $C_8 < O_8 >_V$ and $C_8 < O_8 >_A$ is not understood at present. These condensates are usually assumed to be dominated by four-gluon operators [21], which should give the same contribution to the vector and axial-vector channels. Theoretical estimates [22] also suggest that the size of these condensates should be much smaller than the values shown in eq.(4.4c). It would be interesting to do again the analysis of refs. [17] and [18], when better τ -data become available, to see if the results (4.4c) are confirmed. In any case, the total effect of the dimension-8 condensates on R_H is certainly smaller than the error in the dimension-6 contribution, and therefore can be neglected.

¹ This is no longer true when two-loop corrections to the Wilson coefficient C_4 are taken into account. The numerical effect of these contributions is however negligible.

5. DETERMINATION OF $\Lambda_{\overline{MS}}$

The small size of the non-perturbative contributions allows us to make an accurate prediction of R_H in terms of $\alpha_s(m_\tau^2)$. This can be used to infer a value of the QCD-scale $\Lambda_{\overline{MS}}$ from the measured tau hadronic-width. In order to do that, we still need to take into account the small effect of higher order electroweak corrections, which amount (including the $|V_{ud}|^2 + |V_{us}|^2 \approx 0.9979$ mixing factor) to a multiplicative factor [2] [23] $r_H = 1.0225 \pm 0.0050$ on the r.h.s. of eq. (4.5).

Table 1 shows the results obtained for different values of $\Lambda_{\overline{MS}}$. The complete three-loop calculation of $\alpha_s(m_\tau^2)$ has been used to evaluate the different perturbative terms in eq.(4.5). The quoted errors are due to the small uncertainties associated with the electroweak corrections and the non-perturbative contributions.

Table 1

$\Lambda_{\overline{MS}}$ (MeV)	$\alpha_s(m_\tau^2)/\pi$	R_H
100	0.06111	3.29 ± 0.02
200	0.07818	3.40 ± 0.02
300	0.09418	3.52 ± 0.02

The ratio R_H is related to the total τ -decay width through the equation

$$\Gamma(\tau^-) = \Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e) \{1.9728 + R_H\}. \quad (5.1)$$

Therefore, R_H can be obtained experimentally either from the leptonic branching ratios or from the lifetime measurement, if the theoretical prediction (1.4) for the leptonic width is assumed. One finds

$$R_H^{exp} = \begin{cases} 3.68 \pm 0.13, & \text{from } Br(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e); \\ 3.49 \pm 0.12, & \text{from } Br(\tau^- \rightarrow \nu_\tau \mu^- \bar{\nu}_\mu); \\ 3.31 \pm 0.14, & \text{from } \tau\text{-lifetime}; \end{cases} \quad (5.2)$$

which shows once again the discrepancy between the lifetime and the leptonic branching ratio measurements mentioned in section 1. One can also estimate R_H by directly summing the measured exclusive widths of the different hadronic channels; this gives a smaller value [6] $R_H^{exp} = 3.22 \pm 0.10$, reflecting the well-known missing one-prong problem. If one does a formal average of the three R_H -values in eq.(5.2), one gets $R_H^{exp} = 3.50 \pm 0.07$, which seems to favour a big value of $\Lambda_{\overline{MS}}$ around $300 MeV$. Given the present

experimental discrepancies, however, we can only conclude that the lifetime and exclusive decay measurements require $\Lambda_{\overline{MS}} \sim 100 MeV$, while higher values for the QCD-scale are preferred by the experimental leptonic branching ratios. Future high precision experiments will certainly clarify the present disagreements among different measurements, allowing for a more accurate determination of $\Lambda_{\overline{MS}}$.

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