OIDE LIMIT MITIGATION STUDIES

O. R. Blanco*, CERN, Geneva, Switzerland and LAL, U-PSUD, CNRS/IN2P3, Orsay, France

P. Bambade, LAL, U-PSUD, CNRS/IN2P3, Orsay, France

R. Tomas, CERN, Geneva., Switzerland

Abstract

Particle radiation when traversing a focusing quadrupole limits the minimum achievable beam size, known as the Oide limit. This effect may be compensated by a pair of multipoles which reduce the impact of the energy loss in the vertical beam size. Simulations in PLACET using the CLIC 3 TeV QD0 and L^* show a reduction of $(4.3 \pm 0.2)\%$ in the vertical beam size.

INTRODUCTION

In order to achieve higher luminosity, it is necessary to reduce the beam size to compensate the lower frequency collisions in linear accelerators compared with collider rings [1], radiation effects plays an important role in the presence of strong focusing required for the IP small beam size. This document addresses the radiation phenomenon called Oide effect [2].

The Oide effect is caused by the interaction of charged particles with the magnetic field from quadrupoles. Radiation in a focusing magnet, schematically represented as QD0 in Fig. 1, changes the energy of the particle and modifies the focusing effect. This results in a limit on the minimum beam size specially relevant in the vertical plane.
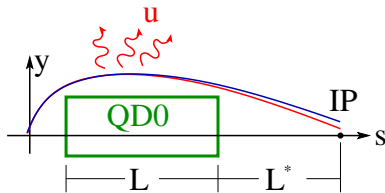

Figure 1: Designed particle trajectory in blue and the trajectory of a particle due to radiation in the quadrupole in red.

The beam size growth due to radiation is added quadratically to the linear beam size $\sigma_0^2 = \epsilon\beta$ where β represents the optical beta function and ϵ is the emittance. Therefore, $\sigma^2 = \sigma_0^2 + \sigma_{oide}^2$. The beam size contribution is [2],

$$\sigma_{oide}^2 = \frac{110}{3\sqrt{6\pi}} r_e \frac{\lambda_e}{2\pi} \gamma^5 F(\sqrt{k}L, \sqrt{k}L^*) \left(\frac{\epsilon}{\beta^*}\right)^{5/2} \quad (1)$$

where $F(\sqrt{k}L, \sqrt{k}L^*)$ is a double integral solved in [3], λ_e is the Compton wavelength of the electron, r_e is the electron radius, γ is the relativistic factor, β^* is the twiss parameter function at the observation point, in this case the IP; and, k, L, and L^* are the quadrupole gradient, the quadrupole length and the distance to the IP.

---
* blanco@lal.in2p3.fr

Although the total contribution to beam size depends on lattice and beam parameters, the minimum achievable beam size is given by [2],

$$\sigma_{y\,\min} = \left(\frac{7}{5}\right)^{\frac{1}{2}} \left[\frac{275}{3\sqrt{6\pi}} r_e \frac{\lambda_e}{2\pi} F(\sqrt{K}L, \sqrt{K}L^*)\right]^{\frac{1}{7}} (\epsilon_{Ny})^{\frac{5}{7}} \quad (2)$$

where $\epsilon_N = \gamma\epsilon$ is the normalized emittance, showing the independence from beam energy.

The only possibility to reduce the beam size is by changing the value of F, by modifying the magnet parameters, or to minimize the beam emittance. However, using the ILC 500 GeV [4], CLIC 500 GeV and CLIC 3 TeV [5] parameters, it is possible to conclude from Table 1 that the contribution to beam size is significant for CLIC 3 TeV.

Δy DUE TO RADIATION

Particle tracking from the imput of QD0 to the IP for CLIC 3 TeV with and without radiation, using PLACET [6], allows one to compute the effects of radiation on the six dimensional phase space. Figure 2 shows the current transverse distribution of particles at the IP. To compensate the adverse effects a compensation system would ideally remove the position change due to radiation $\Delta y = y_{\mathrm{rad}} - y_0$.
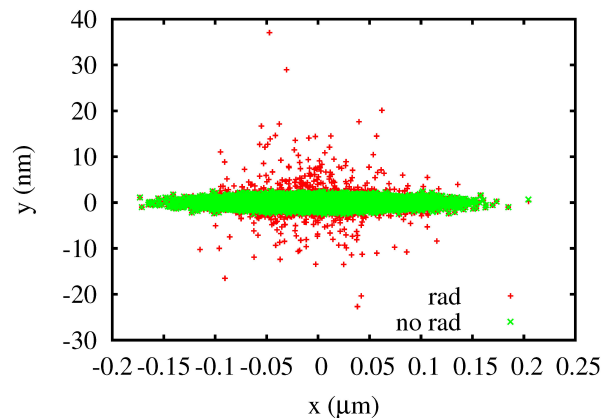

Figure 2: CLIC 3 TeV beam at the IP after tracking QD0 with and without radiation.

Although the average radiation effect is zero, $\langle \Delta y \rangle = 0$ because of the cubic term $(y_0')^3$ as stated by Oide [2], the correlation between $\Delta y, y'$ is not zero. The correlation expression is shown in Eq. (3).

$$\langle \Delta y, y_0' \rangle = \frac{2}{3} r_e \gamma^3 G(\sqrt{K}L, \sqrt{K}L^*)(y_0')^3 \quad (3)$$

Table 1: Vertical Beam Size and Radiation Beam Size Contribution for Three Lattices. ϵ_N is the normalized emittance, $\epsilon_N = \gamma\epsilon$.

Lattice	ϵ_N (nm)	γ (10^3)	σ_0 (nm)	k (m^{-2})	L (m)	L^* (m)	F	σ_{oide} (nm)	σ (nm)	σ_{min} (nm)
CLIC 3 TeV	20	2935.0	0.70	0.116	2.73	3.5	4.086	0.85	1.10	1.00
CLIC 500 GeV	25	489.2	2.3	0.077	3.35	4.3	4.115	0.08	2.3	1.17
ILC 500 GeV	40	489.2	5.7	0.170	2.20	4.3	9.567	0.04	5.7	1.85

where

$$G(\sqrt{K}L, \sqrt{K}L^*) = \int_0^{\sqrt{K}L} (\sin\phi + \sqrt{K}L^* \cos\phi)^2$$
$$\int_0^\phi (\sin\phi' + \sqrt{K}L^* \cos\phi')^2 d\phi' d\phi$$

$$(4)$$

Figure 3 shows the comparison between the correlation obtained from tracking and the theoretical evaluation of the previous expression.

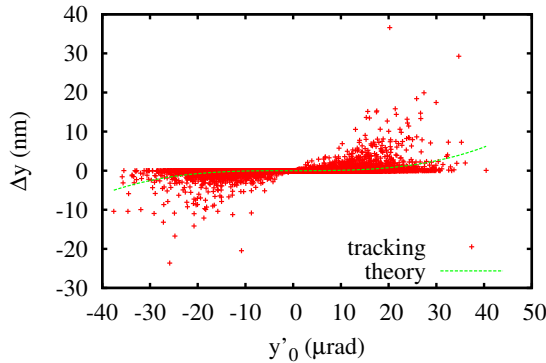Figure 3: Correlation between the phase space coordinates y, y' for CLIC 3 TeV from particle tracking and theoretical expression.

CORRECTORS

A pair of correctors, as in Fig. 4, is added to the strong focusing in order to mitigate the radiation effect. Particles that did not radiate along QD0 receive kicks in C1 and C0 cancelling one another. However, the C1 and C0 kicks do not cancel for particles that did radiate, this difference is used to correct only the particles trajectory change due to radiation.

The procedure consists in scan the best position and multipole gradient (s, k_i) for C0, and then set C1 at QD0 input to cancel the effect of C0. If two points with same β_y/β_x ratio are chosen, then the mutual cancellation of C1 and C0 correctors is limited only by the phase advance between them [7]. Figure 5 shows the horizontal and vertical β functions for CLIC 3 TeV in Final Doublet (FD) region, and their ratio.

The equal β_y/β_x ratio for C0 and C1 condition is difficult to fulfill because C0 should be too close to the IP. In addi-

tion, it will lead to correctors running at very high strengths perturbing the beam.

A second approach is to minimize the phase advance between correctors. Therefore they will be located on both faces of QD0. This has the advantage of correctors running at lower strengths thanks to the large β functions.
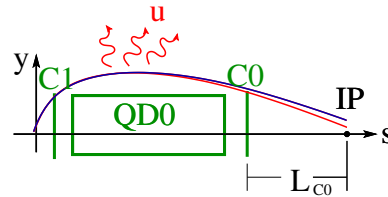

Figure 4: For the nominal trajectory in blue, the kick in C1 must cancel the kick in C0. For all particles that radiate in red, the difference in kicks should cancel Δy.
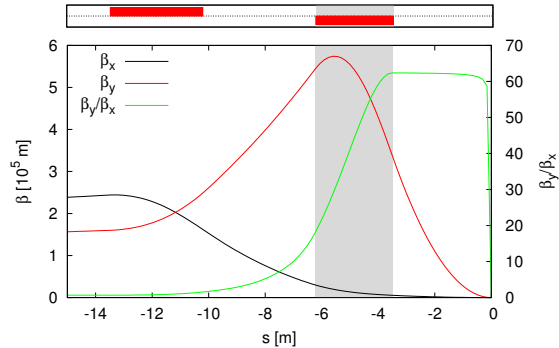

Figure 5: β functions and β_y/β_x ratio for CLIC 3 TeV FD, QF1 and QD0 in red on top. The dark area is occupied by QD0 and the IP is at $s = 0$.

Two octupoles (OD0,OD1) were tried as correctors (C0,C1) to substract the cubic fit. A CLIC 3 TeV nominal beam with no energy spread is generated at the IP and tracked back to the entrance of the C1 without radiation with both correctors off. This beam is used to study the Oide effect mitigation in QD0 using correctors by tracking to the IP with radiation.

The best result obtained with the octupole correctors is a vertical beam size reduction by $(-4.3 \pm 0.2)\%$ using OD0 only. Table 2 show the result of luminosity changes less than 10% for the case with no radiation in QD0, with radiation, with one corrector and with the two correctors obtained with Guinea Pig ++ [8].

The Oide effect contribution to vertical beam size affects very little the luminosity, and it is the same case when using

Table 2: Effect of Octupolar Correctors on the Beam Size, Total Luminosity and Peak Luminosity

	OD1		OD0		σ_x	σ_y	L_{tot}	L_{peak}
	L [m]	k_3 [m^{-4}]	L [m]	k_3 [m^{-4}]	[nm]	[nm]	[10^{34}cm$^{-2} \cdot s^{-1}$]	
NO RAD	0.01	0	0.01	0	47.45	0.69	7.7	2.9
RAD	0.01	0	0.01	0	47.45	1.18	7.5	2.7
RAD	0.01	0	0.01	-3900	47.45	1.13	7.4	2.7
RAD	0.01	1502	0.01	-3900	47.45	1.17	7.1	2.7

OD0 corrector. However, the case of two correctors OD1 and OD0 shows a drop. This has been attributed to the limited cancellation between correctors due to different β_y/β_x ratio and phase advance.

The possibility of slicing QD0 in two or three sections and mitigate the radiation effect with a pair of octupoles on each/any slice has been forseen to improve the matching betweeen C0 and C1 correctors.

CONCLUSIONS

Radiation in the final quad sets a limit on the vertical beamsize, this is called Oide effect. Only for CLIC 3 TeV this limit is significant therefore two possibilities have been explored to mitigate its contribution to beam size: double the length and reduce the QD0 gradient [3], or the integration of a pair of octupoles before and after QD0.

The best result with octupoles has been a vertical beam size reduction of $(4.3\pm0.2)\%$, with little impact on luminosity. The correction scheme is currently limited by the phase advance and β_y/β_x ratio. It may be possible to improve its performance by slicing QD0.

REFERENCES

[1] Delahaye, J. P., Guignard, G., Raubenheimer, T. & Wilson, I. Scaling laws for e$^+$/e$^-$ linear collider. *Nucl. Instr. and Meth. A* **421**, 369–405 (1998).

[2] Oide, Katsunobu. Synchrotron-Radiation Limit on the Focusing of Electron Beams. Phys. Rev. Lett. 61 Issue 15, Oct, 1988. Pages 1713 - 1715.

[3] Blanco, O.R., Tomás, R. and Bambade, P. Oide Effect and Radiation in Bending Magnets. CERN-OPEN-2014-051. CLIC-Note-1049. CERN, Geneva. Jun, 2014.

[4] The International Linear Collider Technical Design Report - Volume 3.I: Accelerator R&D in the Technical Design Phase, technical report. KEK-Report-2013-1. The full list of signatories is inside the Report. June, 2013.

[5] CLIC Conceptual Design Report, technical report. CERN-2012-007. CERN. 2012.

[6] PLACET.   https://savannah.cern.ch/projects/placet Link consulted 2013.04.17

[7] Xu, G. General conditions for self-cancellation of geometric aberrations in a lattice structure, *Phys. rev. st. acc. beams*, 8:104002, 10, October 2005.

[8] Schulte, D. *Beam-beam simulations with GUINEA-PIG*. March, 1999.