FIXED POINTS IN PRESENCE OF SPACE CHARGE IN CIRCULAR PARTICLE ACCELERATORS

S. S. Gilardoni, M. Giovannozzi, A. Huschauer, CERN, Geneva, Switzerland
S. Machida, C. R. Prior, S. L. Sheehy, STFC/RAL, Didcot, UK

Abstract

Recent measurements performed in the framework of the multi-turn extraction (MTE) studies at the CERN Proton Synchrotron showed a dependence of the position of beamlets obtained by crossing a stable transverse resonance on the total beam intensity. This novel observation has triggered a number of studies aiming at understanding the source of the observed effect. In this paper the results of numerical simulations performed in different conditions are discussed in detail.

INTRODUCTION

The Multi-Turn Extraction (MTE) at the CERN Proton Synchrotron (PS) is a novel technique [1, 2] proposed to replace the existing extraction from the PS to the Super Proton Synchrotron (SPS) so as to allow long runs compatible with delivering good quality high intensity beams.

The MTE extraction is based on beam trapping in stable islands of the transverse phase-space: the beam is split in five beamlets by crossing a fourth-order resonance and, once sufficiently separated in the horizontal plane, each beamlet is extracted on five consecutive PS turns towards the SPS. The adiabaticity of the trapping process requires a long extraction flat top. During the splitting process, realised at 14 GeV/c for the specific CERN application, the horizontal chromaticity and the beam $\Delta p/p$ are kept both as small as possible, to assure that the largest fraction of the particles crosses the resonance all at the same moment. The beam is kept bunched during the entire capture process to precisely control the radial position, i.e., the energy, for the different reasons described in [3]. The position of the beamlets at the end of the trapping process should depend only upon the distance in tune from the resonance and the amplitude-dependent tune shift, thus from the value of the quadrupolar, sextupolar and octupolar fields introduced to create and control the islands' size and separation.

During normal operation, the total beam intensity can vary between 1×10^{13} and 3×10^{13} protons per pulse, depending on the needs of the SPS users. The intensity is usually kept constant for a certain period of time, usually days, and changed upon users requests. During the MTE commissioning period, as reported in [3], it was noticed that the position of the beamlets, i.e., the position of the stable fixed point in the horizontal phase space, changes significantly and in a measurable way with the intensity. In particular, the final position of the beamlets was found to be a linear function of the total beam intensity, whereas the four beamlets and the beam core sizes did not show any measurable change, as shown for example in Fig. 1. The observed effect suggested
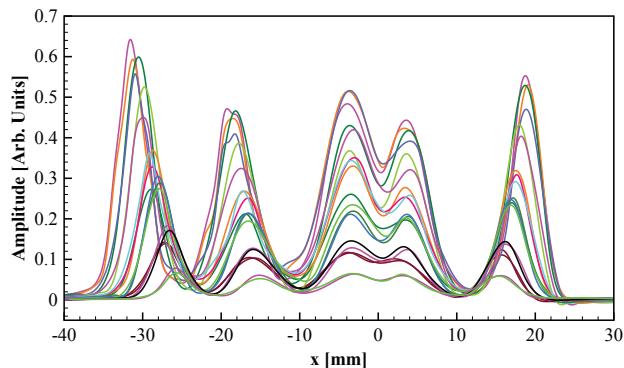

Figure 1: Example of islands profiles measured by a wire scanner and for different intensity [3]. The displacement of the islands centers with intensity is apparent.

as being generated by the superposition of two different components, i.e., the direct space charge between the beamlets and the beam core, and the beamlets interaction with the image charges on the vacuum chamber walls. Also the finite resistivity of the vacuum chamber was proposed to introduce a relevant effect.

This paper presents a first attempt to disentangle the different sources that might cause the observed change of beamlet position, and in particular space charge.

SIMPLE MODEL WITH DIRECT SPACE CHARGE

The incoherent tune shift due to the direct space charge is always negative so that the fixed points in the horizontal phase space move inward when the amplitude-dependent tune shift due to nonlinear elements has negative sign (Fig. 2). That is not the case of the experimental observation where the beamlets move outward with beam intensity [3].

SIMULATIONS WITH FROZEN SPACE CHARGE

In order to simulate the space charge effects, we consider the frozen space charge model of a coasting beam. The charge distribution and total charge of each beamlet are fixed at the beginning of the simulations and redistribution or beam loss is not considered. More specifically, the charge distribution is Gaussian in both horizontal and vertical directions and all the beamlets including that at the centre have the same charge. We calculate, however, the position of beamlets self-consistently including the space charge effects among them. When the distance among the beamlets in real space is more than the beam size of each of them, the
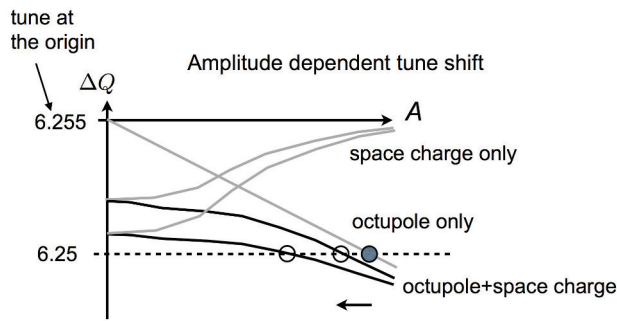
Figure 2: Diagram showing the tune shift due to octupole magnets and space charge as a function of particle amplitude. Since we chose the sign of the octupole such that the amplitude dependent tune shift becomes negative, the total tune shift of octupole and space charge is always negative. As a result, the addition of direct space charge moves the islands inward.

other beamlets can be seen as a static and point-like charge distribution, so that the assumed model is well justified.

As we normally do, we split space charge interaction into three parts: direct electric and magnetic, image charge (electric) and image current (magnetic). Image charges are created by the vacuum chamber and image currents are created by magnet poles.

Let us first consider the contributions from the direct space charge and current. Repulsive electric forces among particles with charges of the same sign weaken the restoring force so the betatron tune shift is negative. In simulations, direct electric and magnetic space charge are included as a sum of a space charge potential due to each beamlet. Macro particles receive kicks from all the real beamlets. Only the electric force is first calculated and then multiplied by a factor of $1-\beta^2$ to take into account the partial cancellation by the magnetic force.

Having the image charge of other beamlets on the other side of the vacuum chamber, the restoring force of the outer beamlets with respect to the oscillation centre becomes stronger. For simplicity parallel plates are assumed in order to model the vacuum chamber, which is indeed more similar to an ellipse. There is infinite layer of image charges with alternating signs. The image charge in the second layer acts to cancel the force of the first one, but not completely because the distance of the interaction is longer as shown in Fig. 3. The force from the image charge is modelled in simulations in the same way as the direct space charge. The frozen space charge potential is located at the position of the image charge with the correct sign of the charge. However, there is no factor of $-\beta^2$ from the magnetic force because the image current does not exist at the same position. Although the orbit of the beamlets oscillates around the ring, it does not change in time. Only the DC component of the magnetic fields is created, which can penetrate the metallic vacuum chamber wall. On the other hand, the DC component of magnetic fields makes an image current on the magnet pole
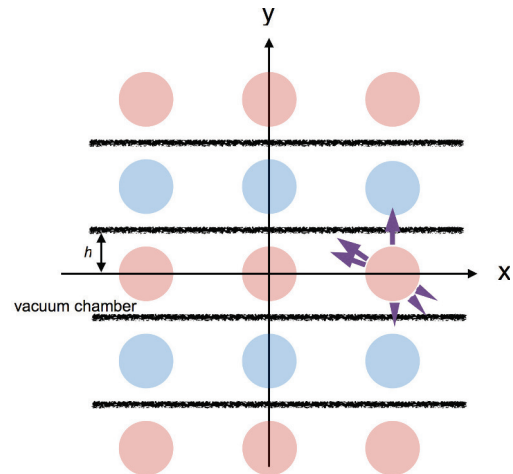

Figure 3: There are infinite series of the image charges with alternating sign. The force from all images with both signs have to be included. Nevertheless, it is clear that the image charge strengthens the restoring force of the betatron oscillation of the outer beamlets.

face. The forces from the image current are included in the similar way but only the term of the magnetic force, which is equivalent to the electric force multiplied by $-\beta^2$. Since the image current exists only at the magnets, the magnitude of the force is further reduced by another factor of F, which is the magnet packing factor and set to be 0.5 in this study.
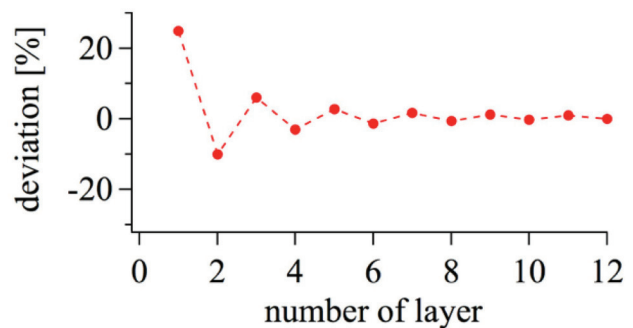

Figure 4: Deviation of the fixed point position with different numbers of the image charge/current layers on one side. In this test, the total number of particles is 2.0×10^{13}, the beam momentum is 14 GeV/c, the half vacuum chamber height is h=35 mm, and the half magnet pole face height is g=50 mm.

The sufficient number of layers of the image charge and current in simulation is determined by looking at the convergence of the fixed point position as a function of the number of layers. The deviation of the fixed point position is within 1% when more than 10 layers on one side (for image charge and image current separately) are included. When it was truncated at the odd layer, an attractive force between the real charge/current and the image charge/current with opposite sign dominates. The even layers with the same sign of charge/current tend to partially cancel the force by the inner layers, so as to make the zigzag shape (Figure 4).

5: Beam Dynamics and EM Fields

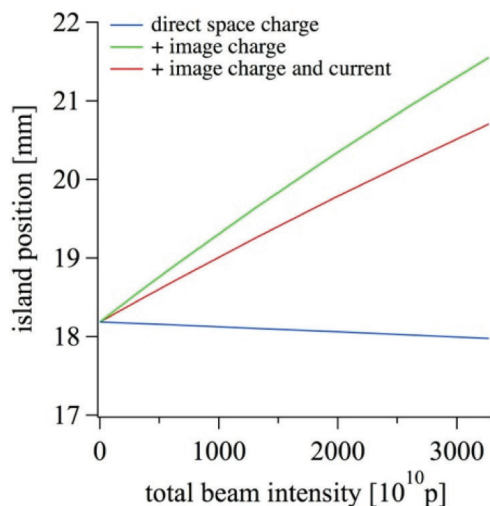D02 - Nonlinear Dynamics - Resonances, Tracking, Higher Order

Figure 5: Island position with total beam intensity. Direct space charge produces a negative tune shift and the island moves inward with intensity. The image charge only gives the positive tune shift and the island moves outward. The image current partially cancels the tune shift and the gradient is less than the case with the image charge only. Beam momentum is 14 GeV/c, half vacuum chamber height is is h=35 mm, and half magnet pole face height is g=50 mm.

Figure 5 shows the outermost island positions as a function of total beam intensity at the extraction momentum of 14 GeV/c. Three lines show the results with three different boundary conditions: direct electric and magnetic space charge without boundary condition, direct space charge and parallel plate vacuum chamber which produces the image charge, direct space charge and parallel place vacuum chamber and parallel magnet pole face which produces the image charge and current. Direct space charge only moves the island inward. The image charge, on the other hand, moves the island outward as we expected. The image current cancels the positive tune shift due to the image charge partially so that the gradient is less than the case with only the image charge. The phase space with all contributions is depicted in Fig. 6.
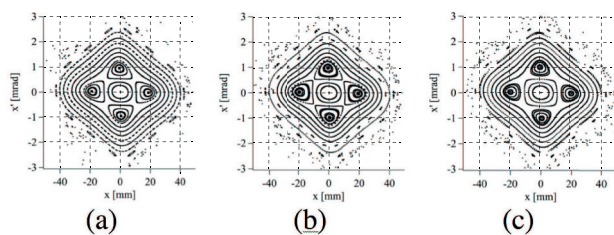

(a)   (b)   (c)

Figure 6: Horizontal phase space with different beam intensity. (a) at 0.67×10^{13}, (b) at 1.3×10^{13}, (c) at 2.0×10^{13}. All are at the extraction momentum of 14 GeV/c.

CONCLUSIONS AND FUTURE WORKS

Despite some simplifications of the frozen model approach, as well as the use of parallel plate geometry to represent the elliptical vacuum chamber, the image charges and image currents seem to be the main source of the observed variation of the beamlets' position with intensity. Further studies, both from the experimental as from the simulation point of view, will continue to improve the understanding of the process and to validate the result. It would be possible, as discussed in [4], to verify how the beamlets' positions versus intensity vary by realising the capture at different beam momenta, in particular to disentangle space-charge energy-dependent effects. A first attempt was already done at injection energy (1.4 GeV), where unfortunately the uncorrected linear coupling produced islands in the vertical plane [6], making the interpretation of the results more difficult. It would be possible to simplify the experiment for momenta between 2.5 and 5 GeV/c, and beyond transition energy, between about 7 and 14 GeV/c. Another possibility would be to implement both in simulations and in measurements the capture with different intensities between islands and core, and eventually cross a lower order resonance, like the 3rd order one as presented in [5]. In the last case, the core is naturally emptied since the origin of the horizontal phase space is an unstable fixed point, and the total beam intensity is equally shared between the islands, with only a very minor fraction of the beam left in the center. Always in this spirit, it would be possible to probe the phase space of the islands for different intensities by kicking a single bunch beam transversally within an island at a fixed amplitude. Then the island fixed point position can be reconstructed turn-by-turn by the trajectory system, as already done in [8]. On the theoretical and simulations point of view, work could be focused on a more quantitative prediction of the fix point position change with intensity. Also full 6D simulations with the PIC code PTC-PyOrbit [7] recently started, with the goal of first reproducing the capture process.

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