$\chi_{c0}(3915)$ as the lightest $c\bar{c}s\bar{s}$ state

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The state $\chi_{c0}(3915)$ has recently been demoted by the Particle Data Group from its previous status as the conventional $c\bar{c}$ 2^3P_0 state, largely due to the absence of expected $D\bar{D}$ decays. We propose that $\chi_{c0}(3915)$ is actually the lightest $c\bar{c}s\bar{s}$ state, and calculate the spectrum of such states using the diquark model, identifying many of the observed charmoniumlike states that lack open-charm decay modes as $c\bar{c}s\bar{s}$. Among other results, we argue that $Y(4140)$ is a $J^{PC} = 1^{++}$ $c\bar{c}s\bar{s}$ state that has been not been seen in two-photon fusion largely as a consequence of the Landau-Yang theorem.

I. INTRODUCTION

The past 13 years have been a time of remarkable growth in experimental reports of hadronic states, particularly in the charmonium and bottomonium sectors. Starting with Belle's observation of the $X(3872)$ in 2003 [1], almost 30 new states with masses lying in these regions have been reported. Until last year's observation of the baryonic P_c^+ states by LHCb [2], all of the observed states were mesonic. Since this counting does not include new conventional quarkonium states discovered in the interim, such as the $c\bar{c}$ $\chi_{c2}(2P)$ [3,4], all of these states are considered *exotic*. These additional exotic states have been suggested in numerous papers to be gluon hybrids, kinematical threshold effects, dimeson molecules, compact charmonium embedded in a light-quark cloud (*hadrocharmonium*), and diquark-antidiquark states (Ref. [5] gives an exhaustive recent review of work in these areas).

Evidence has steadily mounted that at least some of the mesonic exotics are *tetraquarks*, and the baryonic exotics are *pentaquarks*. For example, the $Z^+(4430)$ state first observed in 2008 [6] is charmoniumlike but also charged, so that its minimum valence quark content is $c\bar{c}u\bar{d}$. But the confirmation experiment by LHCb [7] also measured a rapid phase variation of the $Z^+(4430)$ production amplitude near the peak mass, which is characteristic of true

resonant scattering behavior. Similar observations were carried out for the P_c^+ states [2].[1]

The definitive separation of exotic from conventional states is not always trivial, however. The $X(3872)$ has the same $J^{PC} = 1^{++}$ as the yet-unseen $c\bar{c}$ state $\chi_{c1}(2P)$, but its mass is several tens of MeV lower than expected. Moreover, $\Gamma_{X(3872)} < 1.2$ MeV, while the $\chi_{c1}(1P)$, its ostensible radial ground state, has a width $\Gamma = 0.84$ MeV, almost as large. The $\chi_{c1}(1P)$ has a mass 360 MeV lower, so one expects the $\chi_{c1}(2P)$ to have all of the $1P$ state's decay modes (as well as many additional ones), but with much more phase space, and hence a substantially larger width.

The $J^{PC} = 0^{++}$ state $\chi_{c0}(3915)$ is an even trickier example. Its mass lies very close to the quark-potential model predictions for that of the yet-unseen $c\bar{c}$ state $\chi_{c0}(2P)$. As described in detail below, it is produced in $\gamma\gamma$ fusion, as one would expect for the $\chi_{c0}(2P)$, and $\chi_{c0}(3915)$ was briefly hailed by the Particle Data Group (PDG) [12] as the missing $c\bar{c}$ state $\chi_{c0}(2P)$. However, the current absence of the expected dominant $D^{(*)}\bar{D}^{(*)}$ decay modes speaks against a $c\bar{c}$ interpretation, and indeed, also against a $c\bar{c}q\bar{q}$ interpretation ($q = u$, d).

In this work, we therefore propose that $\chi_{c0}(3915)$ is the lightest hidden-charm, hidden-strangeness ($c\bar{c}s\bar{s}$) tetraquark state. Our analysis is performed assuming the diquark-antidiquark model first proposed in Ref. [13] and applied to $c\bar{c}s\bar{s}$ states in Ref. [14] (where the lightest $c\bar{c}s\bar{s}$ state was indeed found to have $J^{PC} = 0^{++}$). Since the advent of those two papers, many new exotic states have been observed, and the model was improved recently to reflect the new data in Ref. [15]. Our analysis, therefore, develops this improved

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[1]The rapid phase variation alone is not universally accepted as decisive evidence of a resonance, and more discriminating tests have been proposed [8–11].

version of the diquark model for $c\bar{c}s\bar{s}$ states, under the assumption that $\chi_{c0}(3915)$ is their ground state.

Along the way we predict the full spectrum of $c\bar{c}s\bar{s}$ states, noting several whose properties match those of observed exotics remarkably well. For example, the $Y(4140)$ observed in B decays appears as an enhancement in the $J/\psi\phi$ spectrum, exactly as expected for a $c\bar{c}s\bar{s}$ state, but it has not yet appeared in $\gamma\gamma$ fusion experiments. Our model neatly accommodates a $J^{PC} = 1^{++}$ state at 4140 MeV, which is forbidden by the Landau-Yang theorem [16,17] from coupling to a two-photon state.

This paper is organized as follows. In Sec. II we review the measured properties of the $\chi_{c0}(3915)$ to motivate the proposal that it and several other exotics may be $c\bar{c}s\bar{s}$ states. Section III introduces the diquark-antidiquark model used and develops its spectrum of $c\bar{c}s\bar{s}$ states. We analyze our results in Sec. IV by comparing to the known exotics spectrum, pointing out both successes and shortcomings of the results. In Sec. V we present a brief discussion and conclude.

II. $\chi_{c0}(3915)$ and Other Potential $c\bar{c}s\bar{s}$ States

An understanding of the exotic charmoniumlike spectrum remains elusive, to say the least, from both experimental and theoretical viewpoints (see Ref. [5] for a thorough review and Ref. [18] for perspectives on future prospects.). With respect to the current work, the most interesting state is $\chi_{c0}(3915)$, which was discovered by Belle in 2005 [19] as a $J/\psi\omega$ enhancement in the process $B \to J/\psi\omega K$ [and was originally labeled $Y(3940)$], and confirmed by *BABAR* [20,21]. However, Belle found no evidence for $D^{*0}\bar{D}^0$ decays of the state [22]. In 2010, Belle discovered [23] the state $X(3915)$ in $\gamma\gamma \to J/\psi\omega$, and *BABAR* subsequently confirmed the result [24], establishing furthermore that the state has $J^{PC} = 0^{++}$, so that its name under the conventional scheme should be χ_{c0}. However, again, no evidence for a peak near 3915 MeV in $D^{(*)}\bar{D}^{(*)}$ was found in $B \to D^{(*)}\bar{D}^{(*)}K$ decays at Belle [25] or *BABAR* [26]. The shared $J/\psi\omega$ decay mode and proximity in mass and width for these two states has led them to be identified as the same state, currently called $\chi_{c0}(3915)$. Its mass and width are currently given as [12]

$$M = 3918.4 \pm 1.2 \text{ MeV}, \quad \Gamma = 20 \pm 5 \text{ MeV}. \quad (1)$$

In fact, the establishment of $J^{PC} = 0^{++}$ for $\chi_{c0}(3915)$ immediately suggested that the state is actually the first radial excitation $\chi_{c0}(2P)$ of the known conventional charmonium state $\chi_{c0}(1P)$, the $2P$ state mass being predicted in quark potential models to lie in the range 3842–3916 MeV [27–29]. The $2P$ identification was also briefly espoused by the Particle Data Group (PDG) [12] (in its online form). However, this identification was questioned by Refs. [29–31]; their objections amount to: (i) the mass splitting between the established $\chi_{c2}(2P)$ (3927 MeV) and

$\chi_{c0}(3915)$ is rather smaller than expected from quark potential models; (ii) the true $c\bar{c}$ $\chi_{c0}(2P)$ should decay copiously to $D^{(*)}\bar{D}^{(*)}$ (the $D^0\bar{D}^{*0}$ threshold lies at 3872 MeV, and the $D^0\bar{D}^0$ threshold lies at 3730 MeV); (iii) as a charmonium-to-charmonium process, the decay $\chi_{c0}(2P) \to J/\psi\omega$ is Okubo-Zweig-Iizuka (OZI) suppressed and would be expected to occur less frequently than is observed. In fact, Ref. [31] showed that the tension between (ii) and (iii) if $\chi_{c0}(3915)$ is assumed to be $\chi_{c0}(2P)$ leads to incompatible bounds on the branching fraction $\mathcal{B}(\chi_{c0}(2P) \to J/\psi\omega)$. As a result of these objections, the PDG currently refers to the state as $\chi_{c0}(3915)$.

Some comments regarding the J^{PC} assignment in $\gamma\gamma$ fusion are in order. If the photons are both transversely polarized, then the Landau-Yang theorem [16,17] forbids the resonance from having spin one. Of course, the photons at Belle and *BABAR* are produced from e^+e^- collisions, and longitudinally polarized off-shell photons can evade this constraint. However, the photon virtuality in this case scales with m_e, which is much smaller than the other mass scales in the process. The difference between the longitudinal and timelike photon polarizations (the latter of which gives an exactly vanishing contribution to physical amplitudes due to the Ward identity) then vanishes with m_e, meaning that longitudinal photon contributions also vanish in this limit. Noting both P and C conservation in QED and using Bose symmetry, the allowed quantum numbers for resonances formed in $e^+e^- \to \gamma\gamma \to X$ are therefore indeed either 0^{++} or 2^{++}.

The $\chi_{c0}(3915)$ therefore appears to be a supernumerary 0^{++} charmoniumlike state, and very likely a 4-quark state (the lowest 0^{++} hybrid computed by lattice QCD being expected to lie many hundreds of MeV higher [32]). It is most natural to suppose that $\chi_{c0}(3915)$ has the flavor structure of an isosinglet: $c\bar{c}(u\bar{u} - d\bar{d})/\sqrt{2}$. Indeed, searches for signals of charged partner states $c\bar{c}u\bar{d}$ or $c\bar{c}d\bar{u}$ [33,34] in the same energy range [actually designed to look for $X(3872)$ isospin partners] have produced no clear signal. Furthermore, such a 4-quark state would seem to have no obvious barrier for decaying into $D\bar{D}$,[2] and only have a relatively small p-wave barrier for decay into $D\bar{D}^*$. The absence of observed open-charm decays of $\chi_{c0}(3915)$ poses a real problem for the 4-quark interpretation.

We propose, therefore, a rather radical solution: The $\chi_{c0}(3915)$ is a $c\bar{c}s\bar{s}$ state, hence naturally an isosinglet that eschews open-charm decays. It lies just below the $D_s^+\bar{D}_s^-$ threshold (3937 MeV) as well as the $J/\psi\phi$ threshold (4116 MeV), and therefore the only OZI-allowed decay (in that no new flavors in a quark-antiquark pair are

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[2]The only known exception to this statement is if the state is a molecule of two mesons held together primarily through 0^- exchanges, such as by π and η. In that case, Lorentz symmetry plus P conservation of strong interactions forbids decay into two 0^- mesons.

created or destroyed) open to it is $\eta_c\eta$ (threshold 3531 MeV).[3] We present a calculation of this width in Sec. IV and argue that it naturally accommodates the value in Eq. (1). The observed decay mode $J/\psi\omega$ actually appears to be quite suppressed, being either due to ω-ϕ mixing that is less than ideal (so that ω contains a small amount of valence $s\bar{s}$, and ϕ contains a small amount of valence $q\bar{q}$), or double OZI-suppression ($s\bar{s}\to g\to q\bar{q}$). Furthermore, we assert that $\chi_{c0}(3915)$ is the lightest $c\bar{c}s\bar{s}$ state; the only lighter charmoniumlike exotic is $X(3872)$, and it decays freely into open-charm states.

A number of higher exotic states have properties amenable to a $c\bar{c}s\bar{s}$ description, by virtue of having neither obvious isospin partners nor observed open-charm decays. Including the $\chi_{c0}(3915)$, nine states share these properties: $Y(4008)$, $Y(4140)$, $Y(4230)$, $Y(4260)$, $Y(4274)$, $X(4350)$, $Y(4360)$, and $Y(4660)$. This list includes four of the five states, $Y(4008)$, $Y(4260)$, $Y(4360)$, and $Y(4660)$, observed using initial-state radiation (ISR) production in e^+e^- annihilation, and therefore necessarily carrying $J^{PC}=1^{--}$; the fifth, $Y(4630)$, decays to $\Lambda_c^+\bar{\Lambda}_c^-$. $Y(4008)$ and $Y(4260)$ have been seen only in decays containing a J/ψ, while $Y(4360)$ and $Y(4660)$ have been seen only in decays containing a $\psi(2S)$. ISR states curiously also do not appear as obvious peaks in the $R(e^+e^-\to\text{hadrons})$ ratio, unlike the conventional 1^{--} charmonium states J/ψ, $\psi(2S)$, $\psi(3770)$, $\psi(4040)$, $\psi(4160)$, and $\psi(4415)$ [12] (Indeed, a local minimum of R appears around 4260 MeV.) If this effect reflects the relative difficulty of making extra particles in e^+e^- annihilation at energies where α_s is small [i.e., with $\alpha_s(m_c)\simeq 0.3$, producing not just $c\bar{c}$, but $c\bar{c}g$ or $c\bar{c}q\bar{q}$], then the production of $c\bar{c}s\bar{s}$ would presumably be even further suppressed due to a mass effect.

The $Y(4140)$, $Y(4274)$, and $X(4350)$ are even better $c\bar{c}s\bar{s}$ candidates, since they are observed as $J/\psi\phi$ enhancements. The $Y(4140)$ was first reported by CDF in the process $B\to J/\psi\phi K$ in 2009 [36], and presented with higher statistics by them in 2011 [37], with other observations in this channel provided by D0 [38] and CMS [39], while LHCb has not yet seen the state, but the disagreement is only at the level of 2σ [40]. Along the way, Refs. [37,39] observed in the same channel the enhancement called $Y(4274)$. Belle, however, using the production mode $\gamma\gamma\to J/\psi\phi$, saw neither $Y(4140)$ nor $Y(4274)$, but instead discovered a new state, $X(4350)$ [41]. A possible explanation for the absence of $Y(4140)$ and $Y(4274)$ in $\gamma\gamma$ production is of course the Landau-Yang theorem, granted that neither state is $J^{PC}=0^{++}$ nor 2^{++}. A study of $Y(4140)$, $Y(4274)$, and $X(4350)$ as $c\bar{c}s\bar{s}$ states using QCD sum rules (but leading to rather different J^{PC} assignments) appears in Ref. [42], while Ref. [43] is a quark-model study predicting $Y(4140)$ to be 1^{++} and notes the importance of the $\eta_c\eta$ mode.

Lastly, the $Y(4230)$ is an enhancement seen in the process $e^+e^-\to\chi_{c0}\omega$ [44]. Should it turn out to be a $c\bar{c}s\bar{s}$ state, its $\chi_{c0}\omega$ decay must proceed through the same ω-ϕ mixing or double-OZI suppression mechanism as suggested for $\chi_{c0}(3915)$.

III. DIQUARK MODELS

Interest in diquark-antidiquark models for light scalar mesons has a long and interesting history (see, e.g., Ref. [45] for a review). The decay patterns for such states obtained from the OZI rule are discussed in Ref. [46], and those from instanton-induced decays are discussed in Ref. [47]. Here, however, we focus on an approach obtained from simple Hamiltonian considerations.[4]

The "type-I" diquark model of Ref. [13] is defined in terms of a Hamiltonian with local spin-spin couplings combined with spin-orbit and purely orbital terms. The orbital angular momentum operator $\mathbf{L}$ refers to the excitation between the diquark-antidiquark pair, while orbital excitations within each diquark are ignored. Specializing (for notational simplicity) to 4-quark systems with hidden charm $[cq_1][\bar{c}\bar{q}_2]$, the Hamiltonian reads

$$H = m_{[cq_1]} + m_{[\bar{c}\bar{q}_2]} + H_{SS}^{qq} + H_{SS}^{q\bar{q}} + H_{SL} + H_L, \quad (2)$$

where $m_{[cq_1]}$ and $m_{[\bar{c}\bar{q}_2]}$ are the diquark masses, H_{SS}^{qq} refers to spin-spin couplings between two quarks (or antiquarks) and therefore refers to spin-spin couplings within either the diquark or antidiquark:

$$H_{SS}^{qq} = 2\kappa_{[cq_1]}\mathbf{s}_c\cdot\mathbf{s}_{q_1} + 2\kappa_{[\bar{c}\bar{q}_2]}\mathbf{s}_{\bar{c}}\cdot\mathbf{s}_{\bar{q}_2}, \quad (3)$$

$H_{SS}^{q\bar{q}}$ couples quarks to antiquarks, and therefore induces interactions between the diquark and the antidiquark:

$$\begin{aligned} H_{SS}^{q\bar{q}} &= 2\kappa_{c\bar{q}_2}\mathbf{s}_c\cdot\mathbf{s}_{\bar{q}_2} + 2\kappa_{c\bar{c}}\mathbf{s}_c\cdot\mathbf{s}_{\bar{c}} \\ &\quad + 2\kappa_{q_1\bar{c}}\mathbf{s}_{q_1}\cdot\mathbf{s}_{\bar{c}} + 2\kappa_{q_1\bar{q}_2}\mathbf{s}_{q_1}\cdot\mathbf{s}_{\bar{q}_2}, \end{aligned} \quad (4)$$

and H_{SL} and H_L are the spin-orbit and purely orbital terms, respectively:

$$\begin{aligned} H_{SL} &= -2a(\mathbf{s}_{[cq_1]}\cdot\mathbf{L} + \mathbf{s}_{[\bar{c}\bar{q}_2]}\cdot\mathbf{L}) = -2a\mathbf{S}\cdot\mathbf{L}, \\ H_L &= \frac{B_c}{2}\mathbf{L}^2, \end{aligned} \quad (5)$$

where $\mathbf{S}$ is the total quark spin operator. The "type-II" diquark model [15] is defined by neglecting all spin-spin couplings between a quark of the diquark and an antiquark of the antidiquark, i.e., effectively by setting $H_{SS}^{q\bar{q}} = 0$. The dynamics binding tetraquark states can be very different from that binding conventional hadrons, so one should not expect a "universal" Hamiltonian to hold for all hadrons.

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[3]Note that no exotic to η_c decays have yet been observed [35].

[4]For example, studies of tetraquarks by allowing for flavor breaking through chromomagnetic interactions have a long history in the literature [48].

The most natural basis in which to describe the diquark-antidiquark states is one in which the good quantum numbers are the four quark spins s_c, $s_{\bar{s}}$, s_{q_1}, $s_{\bar{q}_2}$, diquark spins $s_{[cq_1]}$, $s_{[\bar{c}\bar{q}_2]}$, total quark spin S, orbital angular momentum L, and total angular momentum J. One can also recouple the quark spins into $s_{c\bar{c}}$, $s_{q_1\bar{q}_2}$ using the Wigner $9j$ symbol [49]. With $q_1 = q_2 = s$,

$$\langle ((s_s s_c) s_{[cs]}, (s_{\bar{s}} s_{\bar{c}}) s_{[\bar{c}\bar{s}]}, JM | (s_s s_{\bar{s}}) s_{s\bar{s}}, (s_c s_{\bar{c}}) s_{c\bar{c}}, JM \rangle$$

$$= \sqrt{(2s_{[cs]} + 1)(2s_{[\bar{c}\bar{s}]} + 1)(2s_{s\bar{s}} + 1)(2s_{c\bar{c}} + 1)}$$

$$\times \begin{Bmatrix} s_s & s_c & s_{[cs]} \\ s_{\bar{s}} & s_{\bar{c}} & s_{[\bar{c}\bar{s}]} \\ s_{s\bar{s}} & s_{c\bar{c}} & J \end{Bmatrix}. \tag{6}$$

This basis is particularly convenient for identifying the charge conjugation (C) quantum number of the states:

$$C = (-1)^{s_{c\bar{c}} + s_{s\bar{s}} + L}. \tag{7}$$

The $c\bar{c}s\bar{s}$ tetraquark states have received a dedicated study only in the type-I model [14], some years ago when many known exotic charmoniumlike states had not yet been observed. Should $c\bar{c}s\bar{s}$ tetraquark states be produced, their natural OZI-allowed decays are the open-charm, open-strangeness modes $D_s^{(*)}\bar{D}_s^{(*)}$ (if kinematically possible), or hidden-charm, hidden-strangeness decays such as $J/\psi\phi$, $\eta_c\eta$, etc., depending upon the J^{PC} of the state. In particular, open-charm decays $D^{(*)}\bar{D}^{(*)}$ are expected to be suppressed because they are doubly OZI suppressed: The $s\bar{s}$ pair must annihilate and a $q\bar{q}$ pair must be created. As discussed above, no less than nine of the exotic charmoniumlike candidates have not (yet) been seen to have open-charm decays: $\chi_{c0}(3915)$, $Y(4008)$, $Y(4140)$, $Y(4230)$, $Y(4260)$, $Y(4274)$, $X(4350)$, $Y(4360)$, and $Y(4660)$. Furthermore, no exotic candidate has yet been seen to decay to $D_s^{(*)}\bar{D}_s^{(*)}$. The presence of possible $c\bar{c}s\bar{s}$ states also ameliorates one of the more awkward problems of tetraquark models: If hidden-charm tetraquarks contain light quarks, then one expects either near-degenerate quartets $\{c\bar{c}u\bar{u}, c\bar{c}d\bar{d}, c\bar{c}u\bar{d}, c\bar{c}d\bar{u}\}$ or an isosinglet-isotriplet combination of these states, all carrying the same J^{PC}. The original $X(3872)$ exotic discovered at Belle [1] is a $J^{PC} = 1^{++}$ state widely believed to be $c\bar{c}q\bar{q}$, but dedicated searches for such partner states [33,34] have produced no signal.[5] Of

course, any states believed to be $c\bar{c}s\bar{s}$ do not present this problem.

Implicit in these diquark models is the assumption of the validity of a Hamiltonian approach, which in turn implies a single relevant time coordinate (as the conjugate variable to the Hamiltonian), and hence a common rest frame for the component quarks. In reality, the quarks can move relativistically, especially since the exotic states are generally created in b-quark decays or colliders, in processes accompanied by the release of large amounts of energy. In particular, the spin of a particle is measured in its rest frame, and therefore the meaning of a spin-spin operator becomes obscured in highly relativistic systems. If needed, the mathematical way forward is to employ a helicity formalism, as was most famously expounded in Ref. [51].

From a dynamical point of view, one can imagine the heavy-quark diquark and antidiquark to be fairly compact objects (tenths of a fm)[6] that achieve a substantial separation (1 fm or more) due to the large energy release, before being forced to hadronize due to confinement. In this "dynamical diquark picture" [53], the implicit rest-frame approximations of Refs. [13,15] are not wholly satisfactory, but they should nevertheless provide a lowest-order set of expectations for the spectrum of fully dynamical tetraquark states produced via the diquark-antidiquark mechanism. Moreover, the dynamical diquark picture explains why exotics have only become clearly visible in the heavy-quark sector: In the light-quark sector, the diquark-antidiquark pair never achieves sufficient separation for clear identification. In the intermediate $s\bar{s}$ case, one may discern some hints of diquark structure [54,55].

Diquark structure, via the attractive channel of two color-**3** quarks into a color-**3̄** diquark, has also successfully been used to explain the P_c^+ pentaquark states, both in the original formulation [56] and the dynamical picture [57].

With the formalism established, it is a simple matter to enumerate the $[cs][\bar{c}\bar{s}]$ diquark-antidiquark states and compute their masses using Eqs. (2)–(5). One finds the six s-wave and 14 p-wave states listed in Table I. The results in Table I of Ref. [14] are analogous, but once again, use a different model (as well as different numerical inputs). The mass formula obtained in the type-II model is concise. Since $q_1 = q_2 = s$, the diquark masses are equal, and only one distinct spin-spin coupling $\kappa_{[cs]}$ appears:

$$M = m_{[cq_1]} + m_{[\bar{c}\bar{q}_2]} + \frac{B_c}{2}L(L + 1)$$
$$+ a[L(L + 1) + S(S + 1) - J(J + 1)]$$
$$+ \kappa_{[cs]}[s_{[cs]}(s_{[cs]} + 1) + s_{[\bar{c}\bar{s}]}(s_{[\bar{c}\bar{s}]} + 1) - 3]. \tag{8}$$

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[5]In the case of $X(3872)$, the absence of obvious charged partners can be related to differing distances to the isospin-partner neutral and slightly higher charged $D\bar{D}^*$ thresholds. For example, the formation of the $X^{\pm}$ might be suppressed by a Feshbach-type mechanism, as described in [50]. Alternately, the natural level of the X isotriplet states might be sufficiently high compared to the largely isosinglet $X(3872)$ that they may have escaped detection to date due to having large widths.

[6]In contrast, light-quark diquarks can be rather larger $[O(1)$ fm]; for a lattice calculation, see [52].

Abbreviating

$$M_0 \equiv m_{[cq_1]} + m_{[\bar{c}\bar{q}_2]} - 3\kappa_{[cs]},$$
$$\tilde{B} \equiv B_c + 2a,$$
$$\alpha \equiv 2a,$$
$$k \equiv 2\kappa_{[cs]}, \tag{9}$$

one immediately obtains

$$M = M_0 + \frac{\tilde{B}}{2}L(L+1) + \frac{\alpha}{2}[S(S+1) - J(J+1)]$$
$$+ \frac{k}{2}[s_{[cs]}(s_{[cs]} + 1) + s_{[\bar{c}\bar{s}]}(s_{[\bar{c}\bar{s}]} + 1)], \tag{10}$$

from which the mass expressions given in the last column of Table I follow. The additional type-I terms of Eq. (3) could also be computed, for example, by first diagonalizing the states in a more convenient basis, using recoupling formulas like Eq. (6); however, as seen in Ref. [14], the typical contributions from these terms appear to be no more than about 20 MeV, which we can treat as a systematic uncertainty in our mass predictions. This uncertainty is indicated henceforth by the use of the symbol "$\simeq$."

TABLE I. All s- and p-wave $c\bar{c}s\bar{s}$ diquark-antidiquark states. In the cases $s_{[cs]} = 1$, $s_{[\bar{c}\bar{s}]} = 0$, linear combinations with $s_{[cs]} = 0$, $s_{[\bar{c}\bar{s}]} = 1$ states are understood to combine as needed [using Eqs. (6) and (7)] to form eigenstates of C. State names used in Ref. [15] are also listed, and masses are obtained from Eq. (10).

State	J^{PC}	$s_{[cs]}$	$s_{[\bar{c}\bar{s}]}$	S	L	Mass
X_0	$\lvert 0^{++}\rangle_1$	0	0	0	0	M_0
X_0'	$\lvert 0^{++}\rangle_2$	1	1	0	0	$M_0 + 2k$
X_1	$\lvert 1^{++}\rangle$	1	0	1	0	$M_0 + k$
Z	$\lvert 1^{+-}\rangle_1$	1	0	1	0	$M_0 + k$
Z'	$\lvert 1^{+-}\rangle_2$	1	1	1	0	$M_0 + 2k$
X_2	$\lvert 2^{++}\rangle$	1	1	2	0	$M_0 + 2k$
	$\lvert 0^{--}\rangle$	1	0	1	1	$M_0 + \tilde{B} + \alpha + k$
	$\lvert 0^{-+}\rangle_1$	1	0	1	1	$M_0 + \tilde{B} + \alpha + k$
	$\lvert 0^{-+}\rangle_2$	1	1	1	1	$M_0 + \tilde{B} + \alpha + 2k$
Y_1	$\lvert 1^{--}\rangle_2$	0	0	0	1	$M_0 + \tilde{B} - \alpha$
Y_2	$\lvert 1^{--}\rangle_1$	1	0	1	1	$M_0 + \tilde{B} + k$
	$\lvert 1^{-+}\rangle_1$	1	0	1	1	$M_0 + \tilde{B} + k$
	$\lvert 1^{-+}\rangle_2$	1	1	1	1	$M_0 + \tilde{B} + 2k$
Y_3	$\lvert 1^{--}\rangle_3$	1	1	0	1	$M_0 + \tilde{B} - \alpha + 2k$
Y_4	$\lvert 1^{--}\rangle_4$	1	1	2	1	$M_0 + \tilde{B} + 2\alpha + 2k$
	$\lvert 2^{-+}\rangle_1$	1	0	1	1	$M_0 + \tilde{B} - 2\alpha + k$
	$\lvert 2^{-+}\rangle_2$	1	1	1	1	$M_0 + \tilde{B} - 2\alpha + 2k$
	$\lvert 2^{--}\rangle_1$	1	0	1	1	$M_0 + \tilde{B} - 2\alpha + k$
	$\lvert 2^{--}\rangle_2$	1	1	2	1	$M_0 + \tilde{B} + 2k$
	$\lvert 3^{--}\rangle$	1	1	2	1	$M_0 + \tilde{B} - 3\alpha + 2k$

Using the results of Table I, one can quickly establish the mass hierarchy of states. Assuming only that $k > 0$ [expected from Eq. (3) to hold, inasmuch as vector diquarks are heavier than scalar diquarks], the lightest s-wave state is $X_0 = \lvert 0^{++}\rangle_1$, which we naturally identify with the 0^{++} state $\chi_{c0}(3915)$, and hence $M_0 = 3918.4$ MeV [12]$\approx$3920 MeV. One also expects $\tilde{B} \geq 0$, or else orbitally excited states would actually be lower in mass than s-wave states. Lastly, the spin-orbit coefficient α was argued in Ref. [15] to be positive, so that masses increase with L and S [as seen in Eq. (8)]; an interesting feature of this choice, as noted in Ref. [58], is that with this inverted spin-orbit coupling, states of higher J but other quantum numbers the same are lighter [compare, e.g., $Y_4 = \lvert 1^{--}\rangle_4$, $\lvert 2^{--}\rangle_2$, and $\lvert 3^{--}\rangle$].

IV. ANALYSIS

The strategy for the fit is now quite straightforward. The $c\bar{c}s\bar{s}$ spectrum depends upon four parameters: the multiplet base mass M_0, the orbital excitation coefficient $\tilde{B}$, the spin-orbit coefficient α, and the diquark spin-spin coefficient k. We have noted that nine candidate exotics may be used to fix these parameters, and that the s- and p-wave bands consist of 20 states. Therefore, the goal is to achieve a fit that predicts as many of the nine exotics as possible, while not predicting any of the $20 - 9 = 11$ states with unseen J^{PC} values to occur in mass regions where they likely would already have been observed.

A. Which 1^{--} states are $c\bar{c}s\bar{s}$?

Of particular note is that only four 1^{--} states occur in the s and p waves; Ref. [15] notes that one more 1^{--} state, labeled Y_5, occurs in the f wave ($s_{[cs]} = s_{[\bar{c}\bar{s}]} = 1$, $S = 2$, $L = 3$), but it is most likely much heavier than the others considered here due to its high orbital excitation. That being said, at least four 1^{--} candidate states have already been observed in ISR processes: $Y(4008)$, $Y(4260)$, $Y(4360)$, and $Y(4660)$ [although $Y(4008)$ has only been seen by Belle [59,60]]. In addition, the $Y(4230)$ seen by BESIII in $e^+e^- \to \chi_{c0}\omega$ [44] is necessarily a 1^{--} state if formed in the s wave. On the other hand, lattice calculations, while still not in full agreement, concur that no more than one 1^{--} charmonium hybrid should occur below 4.5 GeV (see, e.g., Ref. [32], which predicts it to lie at a mass of 4216 ± 7 MeV).

Also of note is that the neutral states so far lacking open-charm decays appear to fall into at least two distinct classes based upon their widths: Only $Y(4008)$ and $Y(4260)$ have widths > 100 MeV. One may suppose that one or both of these states are $c\bar{c}q\bar{q}$ (hence possessing many more open channels and thus a larger width) or $c\bar{c}g$ hybrids (so that OZI suppression of $s\bar{s}$ annihilation is absent). The $Y(4260)$ has been observed in the 6-quark modes $J/\psi\pi\pi$ and $J/\psi K^+ K^-$, which speaks against a hybrid structure, and

the $J/\psi\pi\pi$ channel speaks against a $c\bar{c}s\bar{s}$ structure [but see further discussion on $Y(4008)$ later in this section]. In addition, a recent study [61] has calculated that the rate for the radiative transition $Y(4260) \to \gamma X(3872)$ not only suggests that $Y(4260)$ is a $c\bar{c}q\bar{q}$ state like $X(3872)$, but also that both states are compatible with having the same diquark-antidiquark wave function, except that $Y(4260)$ carries an additional unit of orbital angular momentum.

B. s-wave states

Of little ambiguity is the necessity of assigning $\chi_{c0}(3915)$ the role of the $c\bar{c}s\bar{s}$ band ground state $X_0 = |0^{++}\rangle_1$, which according to Table I immediately fixes the parameter $M_0 \simeq 3920$ MeV. The full set of mass predictions is presented in Table II.

Beyond this start, however, hints from the exotic state decay modes become essential. Perhaps the other states most essential to describe as $c\bar{c}s\bar{s}$ are those observed to decay into $J/\psi\phi$, namely, $Y(4140)$, $Y(4274)$, and $X(4350)$. Assuming that $Y(4140)$ is the $X_1 = |1^{++}\rangle$, then using Table I one chooses $k \simeq 220$ MeV, which not only resolves the absence of this state from $\gamma\gamma$ production via the Landau-Yang theorem, but also allows immediate prediction of all the other masses in the s-wave band. In particular, one finds a degenerate state $Z = |1^{+-}\rangle_1$ at 4140 MeV and another $Z' = |1^{+-}\rangle_2$ at 4360 MeV; note that the known neutral isotriplet $J^{PC}=1^{+-}$ states $Z_c^0(4025)$,

TABLE II. Predictions for s-wave (the first six entries) and p-wave (the remaining 14 entries) $c\bar{c}s\bar{s}$ diquark-antidiquark state masses (in MeV), following the notation from Table I. Inputs are labeled by "*". A "?" indicates when more than one good assignment is possible. In terms of Eq. (10), the parameter fit values in MeV are $M_0 = 3920$, $k = 220$, $\tilde{B} = \alpha = 90$.

State		Predicted mass	Observed
X_0	$\|0^{++}\rangle_1$	3920	$\chi_{c0}(3915)$*
X_0'	$\|0^{++}\rangle_2$	4360	$X(4350)$?
X_1	$\|1^{++}\rangle$	4140	$Y(4140)$*
Z	$\|1^{+-}\rangle_1$	4140	
Z'	$\|1^{+-}\rangle_2$	4360	
X_2	$\|2^{++}\rangle$	4360	$X(4350)$?
	$\|0^{--}\rangle$	4320	$Y(4274)$?
	$\|0^{-+}\rangle_1$	4320	$Y(4274)$?
	$\|0^{-+}\rangle_2$	4540	
Y_1	$\|1^{--}\rangle_2$	3920	$Y(4008)$
Y_2	$\|1^{--}\rangle_1$	4230	$Y(4230)$*
	$\|1^{-+}\rangle_1$	4230	
	$\|1^{-+}\rangle_2$	4450	
Y_3	$\|1^{--}\rangle_3$	4360	$Y(4360)$*
Y_4	$\|1^{--}\rangle_4$	4630	$Y(4660)$
	$\|2^{-+}\rangle_1$	4050	
	$\|2^{-+}\rangle_2$	4270	$Y(4274)$?
	$\|2^{--}\rangle_1$	4050	
	$\|2^{--}\rangle_2$	4450	
	$\|3^{--}\rangle$	4180	

$Z_c^0(3900)$ lie rather lower in mass. Additionally, one finds two more degenerate states at 4360 MeV, $X_0' = |0^{++}\rangle_2$ and $X_2 = |2^{++}\rangle$. Either of these is an excellent candidate for the $X(4350)$ found in $\gamma\gamma$ production.

Returning to the $Y(4140)$, one may use Eq. (6) to find that the state X_1 has solely $s_{c\bar{c}} = s_{s\bar{s}} = 1$ content. At the quark level, one expects $\gamma\gamma$ fusion to produce one of the quark-antiquark pairs first (and necessarily with $J^{PC} = 0^{++}$ or 2^{++}), and the other pair to be produced as the result of bremsstrahlung from one of the initial quarks. Thus, even at the quark level, one sees the production of such a state to be problematic.

According to Table I or II, the s-wave states are highly degenerate and obey a simple equal-spacing rule (in k). Note that no s-wave state therefore carries a mass close to that of $Y(4274)$, reported by CDF [37] as $4274.4^{+8.4}_{-6.7} \pm 1.9$ MeV, and by CMS [39] as $4313.8 \pm 5.3 \pm 7.3$ MeV. Fitting to the p-wave states requires input from the ISR state masses, as discussed below. Then, the sole potential candidate for the first mass is $|2^{-+}\rangle_2$ at 4270 MeV, while the second mass can be accommodated by either $|0^{--}\rangle$ or $|0^{-+}\rangle_1$ at 4320 MeV. In the first case, a lighter $|2^{-+}\rangle_1$ state occurs at 4050 MeV, which lies below the 4116 MeV $J/\psi\phi$ threshold and therefore could easily have escaped detection to now. In all cases, however, the fact that none of these states have $J^{PC} = 0^{++}$ or 2^{++} means that they cannot be created in $\gamma\gamma$ fusion, in agreement with observation.

Before leaving the s-wave band, let us note interesting properties of the $\chi_{c0}(3915)$ under this assignment. We have seen in the previous section that its mass lies just below the $D_s^+ \bar{D}_s^-$ threshold 3937 MeV. However, it is extremely problematic to identify $\chi_{c0}(3915)$ as a $D_s\bar{D}_s$ molecule (which was proposed in Ref. [62]) held together by meson exchanges, again using a fact noted in the previous section: D_s^+ and $\bar{D}_s^-$ are $J^P = 0^-$ states, and coupling to a 0^- meson (presumably η) is forbidden by Lorentz symmetry plus P invariance. Should $\chi_{c0}(3915)$ prove to be a $c\bar{c}s\bar{s}$ state, it is almost certainly not a hadronic molecule. The closeness of the $\chi_{c0}(3915)$ mass to the $D_s^+ \bar{D}_s^-$ threshold need not be considered an unnatural coincidence, as the so-called "cusps" due to such thresholds have been shown to be effective in attracting nearby states, in particular for heavy-quark states [63,64].

Second, we have noted that the only OZI-allowed and phase-space allowed decay mode for a $c\bar{c}s\bar{s}$ state of this mass is $\eta_c\eta$. We propose that this is the dominant $\chi_{c0}(3915)$ decay mode. The recombination of quark spins for the X_0 state according to Eq. (6) gives

$$X_0 = \frac{1}{2}|s_{c\bar{c}} = 0, s_{s\bar{s}} = 0\rangle + \frac{\sqrt{3}}{2}|s_{c\bar{c}} = 1, s_{s\bar{s}} = 1\rangle, \quad (11)$$

meaning that the J/ψ modes, if kinematically allowed, are more probable by a factor 3. Likewise, the η wave function is only fractionally $s\bar{s}$:

$$\eta = \frac{1}{\sqrt{6}}(|u\bar{u}\rangle + |d\bar{d}\rangle - 2|s\bar{s}\rangle). \tag{12}$$

The decay $\chi_{c0}(3915) \to \eta_c\eta$ is otherwise a simple two-body decay of a scalar to two (pseudo)scalars, and therefore its width is of the form

$$\Gamma = |\mathcal{M}|^2 \frac{p}{8\pi M^2}, \tag{13}$$

where M is the $\chi_{c0}(3915)$ mass and $p = 665.0$ MeV is the magnitude of the spatial momentum for the two-body decay. The invariant amplitude $\mathcal{M}$ is seen to have dimensions of mass; with $\Gamma = 20$ MeV, one finds $|\mathcal{M}| = 3.4$ GeV. When the suppression factors suggested by Eqs. (11) and (12) are removed, the "natural" amplitude for the process is about 8.3 GeV, a substantial number that suggests the sole decay already observed, $\chi_{c0}(3915) \to J/\psi\omega$, can occur at a reasonable rate if the ω contains a phenomenologically acceptable $s\bar{s}$ component. For example, if the nonideal mixing ϵ of ω is parametrized as

$$\omega = \cos\epsilon \frac{1}{\sqrt{2}}(|u\bar{u}\rangle + |d\bar{d}\rangle) + \sin\epsilon|s\bar{s}\rangle, \tag{14}$$

then using Eq. (11) and the same value of $|\mathcal{M}|$, one finds $\Gamma(\chi_{c0}(3915) \to J/\psi\omega) = 29.9\sin^2\epsilon$ MeV, which for, e.g., $\epsilon = 10^{-3}$ gives $\Gamma = 29.9$ eV.

As mentioned above, the size of the $J/\psi\omega$ branching fraction for $\chi_{c0}(3915)$, given in Ref. [23] in the form

$$\Gamma(\chi_{c0}(3915) \to \gamma\gamma) \times \mathcal{B}(\chi_{c0}(3915) \to J/\psi\omega)$$
$$= (61 \pm 17 \pm 8) \text{ eV}, \tag{15}$$

is considered too large to be compatible with the expected size of OZI-suppressed decays of conventional charmonium. If $\chi_{c0}(3915)$ is a $c\bar{c}s\bar{s}$ state, then OZI violation is evaded if the decay mode is accomplished through the presence of a small valence $s\bar{s}$ component in the ω, which means nonideal ω-ϕ mixing. This effect has been considered in heavy-quark decays such as $D_s^+ \to \omega e^+\nu_e$ [65]. It might, however, be more complicated in the 4-quark environment in the sense that ω-ϕ mixing influenced by final-state interactions can have a significantly different strength than in exclusive processes in which ω is the only hadron present.

C. p-wave states

Let us now turn to the p waves. We have already fixed two of the four model parameters, M_0 and k, from the s waves. When including the p waves, we find that the fits best representing the known spectrum and introducing fewer light unknown states leave out $Y(4260)$ and keep $Y(4008)$. We have remarked above that these are the two widest neutral charmoniumlike states, and are therefore the best candidates for $c\bar{c}q\bar{q}$, and also that the mode $Y(4260)$ in

particular is very unlikely to be purely $c\bar{c}s\bar{s}$. Therefore, in the fit we present in Table II, the $Y(4260)$ is excluded.

It should however be noted that the $Y(4008)$, which has only been seen by Belle [59,60] is even wider ($M = 3890.8 \pm 40.5 \pm 11.5$ MeV, $\Gamma = 254.5 \pm 39.5 \pm 13.6$ MeV, according to Ref. [60]), and like $Y(4260)$, decays to $J/\psi\pi\pi$ (indeed, they are seen together in the same experiment). However, note that the central value for the $Y(4008)$ mass actually lies lower than that of the $\chi_{c0}(3915)$ and well above the thresholds for the p-wave $c\bar{c}s\bar{s}$ modes $\eta_c\eta$ (again, 3531 MeV) and $J/\psi\eta$ (3645 MeV), as well as the ω-ϕ mixing modes, $\eta_c\omega$ (3766 MeV) and $J/\psi\omega$ (3880 MeV). However, $Y(4008)$ lies well below the $J/\psi\phi$ threshold (4116 MeV) but $Y(4260)$ lies well above it; if $Y(4260)$ contained a substantial $c\bar{c}s\bar{s}$ component, presumably its $J/\psi\phi$ mode would have been prominently observed.

The closeness of the $Y(4008)$ and $\chi_{c0}(3915)$ masses has an additional peculiar effect. If one identifies $Y(4008)$ as the lightest $J^{PC} = 1^{--}$ $c\bar{c}s\bar{s}$ state $Y_1 = |1^{--}\rangle_2$, then the fit in Table II gives $\tilde{B} = \alpha$, or using Eq. (9), $B_c = 0$ in the original notation of Eq. (5), which means that the only orbital coupling appears through the spin-orbit term.

In fact, the actual fit in Table II does not choose $Y(4008)$ as an input, but rather chooses $Y(4230) = Y_2 = |1^{--}\rangle_1$ and $Y(4360) = Y_3 = |1^{--}\rangle_3$ to fix $\tilde{B} = \alpha = 90$ MeV. Then, the prediction of $Y(4008)$ as Y_1 and $Y(4660)$ as $Y_4 = |1^{--}\rangle_4$ is noteworthy. An additional feature commending this choice is that Eq. (6) can again be used to show that Y_2 contains only terms in which $s_{c\bar{c}} = s_{s\bar{s}} = 1$, very much in agreement with the $Y(4230)$ so far being seen only in the $\chi_{c0}\omega$ channel: The preferred decay mode would be $\chi_{c0}\phi$, but its threshold is 4434 MeV, so again we suggest that $Y(4230)$ is a $c\bar{c}s\bar{s}$ state that can decay via ω-ϕ mixing.

Since the $D_s^{(*)+}\bar{D}_s^{(*)-}$ thresholds occur at 3937, 4081, and 4224 MeV, one would expect these "fall-apart" modes to be the dominant ones for many of these states, particularly higher ones such as $Y(4660)$. However, it is worth noting that the best current data for $e^+e^- \to D_s^{(*)+}\bar{D}_s^{(*)-}$ [66] is only sensitive to the conventional charmonium ψ states; none of the exotics have yet been seen to decay to charm-strange states. Moreover, should the dynamical diquark picture [53] hold, such that more highly energetic states entail greater separation of the diquarks and therefore suppressed hadronization matrix elements, one then has a natural mechanism for suppressing their decay widths beyond naive expectations.

Lastly, this work presents only one of many possible fits to the known exotic states lacking open-charm decays. Several other possibilities can occur, such as, e.g., identifying the high-mass 1^{--} $Y(4660)$ state as the first in the f-wave ($L = 3$) band ($s_{[cs]} = s_{[\bar{c}\bar{s}]} = 1$, $S = 2$, called Y_5 in Ref. [15]).

V. CONCLUSIONS

Based on interesting patterns in the phenomenology of the charmoniumlike states observed to date, we propose

that the $J^{PC} = 0^{++}$ state $\chi_{c0}(3915)$ is the lightest $c\bar{c}s\bar{s}$ state. Its lack of observed $D^{(*)}\bar{D}^{(*)}$ decays argue against it being either the conventional $c\bar{c}$ state $\chi_{c0}(2P)$ or a light-quark containing $c\bar{c}q\bar{q}$ exotic state, and its single known decay mode $J/\psi\omega$ can be understood as the ω having a small (nonideal mixing) $s\bar{s}$ component.

Furthermore, as a $c\bar{c}s\bar{s}$ state lying slightly below the $D_s\bar{D}_s$ threshold, the $\chi_{c0}(3915)$ is very unlikely to be a loosely bound molecule, and we therefore analyze it as a diquark-antidiquark state. Indeed, a state with $J^{PC} = 0^{++}$ in the mass region ~ 3900 MeV is precisely where the lightest $c\bar{c}s\bar{s}$ state was expected in previous studies. Importantly, even if $\chi_{c0}(3915)$ turns out not to be $c\bar{c}s\bar{s}$, states with this quark content should certainly appear in the same mass range as some of those already observed. To emphasize: One expects $c\bar{c}s\bar{s}$ states to occur in the same range as other charmoniumlike states; and even if the particular assignments in this paper are later disfavored, the analysis leading to Table I still holds.

Under the current hypothesis, however, some remarkable identifications arise. The $Y(4140)$, a $J/\psi\phi$ enhancement seen in B decays, is naturally a 1^{++} $c\bar{c}s\bar{s}$ state which, by the Landau-Yang theorem, is naturally absent from $\gamma\gamma$ production experiments (as is the case). The $X(4350)$, $Y(4274)$, and several of the $J^{PC} = 1^{--}$ Y states arise naturally at masses predicted for $c\bar{c}s\bar{s}$ states, and no unwanted extra states that would already likely have been observed appear to occur.

The most flexible part of the identification—both experimentally and theoretically—occurs in the 1^{--} sector: If so many of these states are $c\bar{c}s\bar{s}$, what has happened to the expected $c\bar{c}q\bar{q}$ states? We have argued that $Y(4260)$ is almost certainly $c\bar{c}q\bar{q}$ and is quite broad; one can imagine that the higher ones are broader still, and thus difficult to discern. Indeed, the very broad $Y(4008)$ might also be $c\bar{c}q\bar{q}$, and either the true lowest 1^{--} $c\bar{c}s\bar{s}$ state is obscured by it, or does not occur until it appears as $Y(4230)$. In any case, subsequent experiments will certainly clarify the true nature of the full spectrum, and $c\bar{c}s\bar{s}$ states will certainly play a role.

During the finalization of this paper, D0 announced [67] the observation of a new state in the channel $B_s^0\pi^{\pm}$, while a preliminary analysis by LHCb found no evidence for such a state [68]. Such a novel exotic flavor structure, a tetraquark with only one heavy quark ($b\bar{s}u\bar{d}$ for π^+), is expected to produce two states close in mass (with $J^P = 0^+$, 1^+) due to heavy-quark fine structure. In particular, if confirmed, it would be the first tetraquark not simply of the $b\bar{b}q\bar{q}$ or $c\bar{c}q\bar{q}$ type, which makes studies of new flavor structures like $c\bar{c}s\bar{s}$ all the more timely. Indeed, Ref. [67] suggests the same type of tetraquark paradigm as discussed here as being the most likely structure.

ACKNOWLEDGMENTS

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