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Mass dependence of branching ratios into HNL for FairShip

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Abstract

In this note we show how we have calculated the mass dependent branching ratios of 2 and 3 body decays into HNL. We also discuss how these ratios were implemented in FairShip.

Contents

1 Introduction

In the nuMSM([1], [2]) HNL particles can be produced in the following (semi) leptonic decays:

- 2 body decays of B/D pseudoscalar mesons
- 3 body decays of B/D pseudoscalar mesons (with pseudoscalars in the final state)
- 3 body decays of B/D pseudoscalar mesons (with vector mesons in the final state)
- 2 body decays of τ leptons
- 3 body decays of τ leptons
- 3 body decays of charm/beauty hyperons (Λ, Ξ and Ω)

The purpose of this note is to give a comprehensive list of the most important channels in these categories, and to calculate the dependence of their branching ratios on the HNL mass. This calculation is done with the toy Monte Carlo written by one of us (EG). EvH added extensions for vector mesons, 3 body tau decays and hyperons.

The mass dependent branching ratios are used to dynamically configure the Pythia 8 step in the FairShip simulation program as a function of the HNL mass.

The details of this implementation are discussed.

2 Charm/beauty pseudoscalar meson 2 body decays

The branching ratios of pseudoscalar meson leptonic decays into sterile neutrinos are calculated using formula (B1) from [1]:

$$\frac{d\mathrm{Br}(\mathrm{H}^+ \to \mathrm{l}^+_\alpha \mathrm{N})}{dE_N} = \tau_H \cdot \frac{G_F^2 f_H^2 M_H M_N^2}{8\pi} |V_H|^2 |U_\alpha|^2 \cdot \left(1 - \frac{M_N^2}{M_H^2} + 2\frac{M_l^2}{M_H^2} + \frac{M_l^2}{M_N^2}\left(1 - \frac{M_l^2}{M_H^2}\right)\right)$$
$$\times \sqrt{\left(1 + \frac{M_N^2}{M_H^2} - \frac{M_l^2}{M_H^2}\right)^2 - 4\frac{M_N^2}{M_H^2}} \cdot \delta\left(E_N - \frac{M_H^2 - M_l^2 + M_N^2}{2M_H}\right) \quad (1)$$

where τ_H is the meson life-time [3]. In the calculation of the M_N dependent branching ratios, we have taken the values for f_H from [1].

The channels in this category and their braching ratios are shown in Tables 1 and 2. The plots show the dependence of the branching ratios on the HNL mass. The branching ratio is shown in units of the total U^2 coupling. The BR value for $\mathrm{M_N} = 1$ GeV is shown for reference.

Table 1: D meson 2 body decays into HNL.

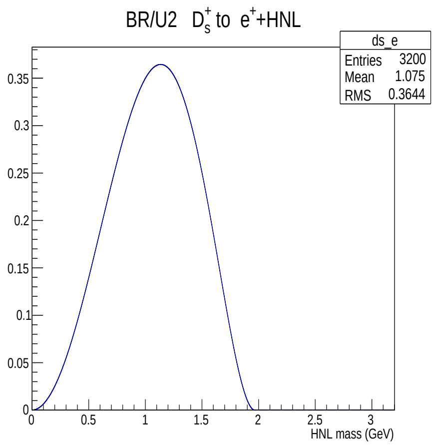
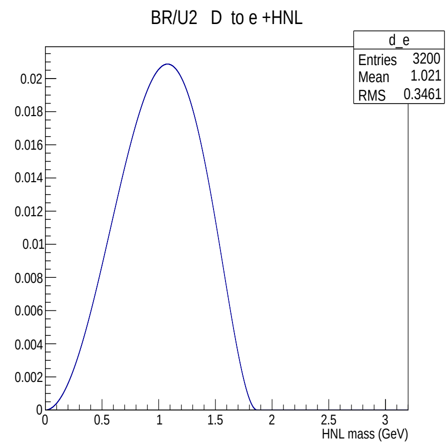
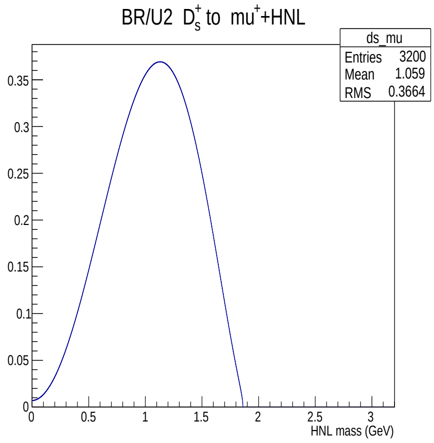
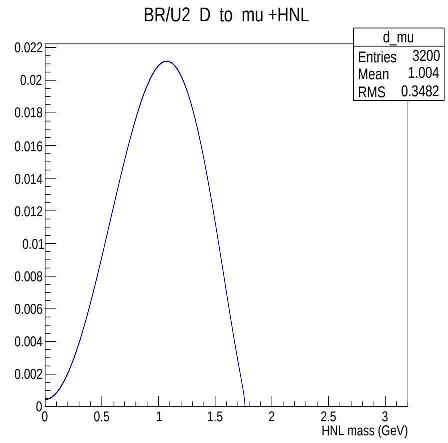
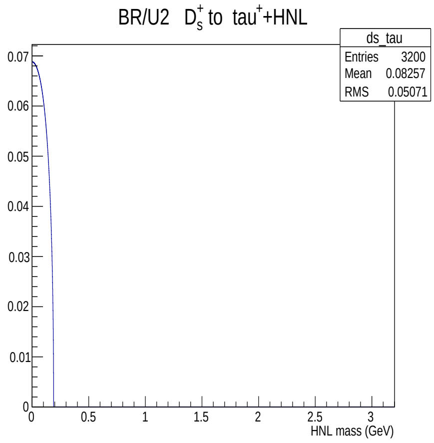
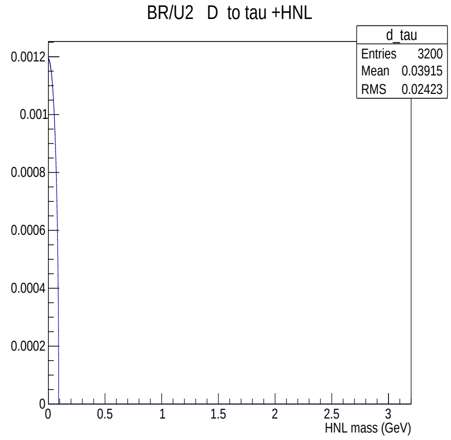
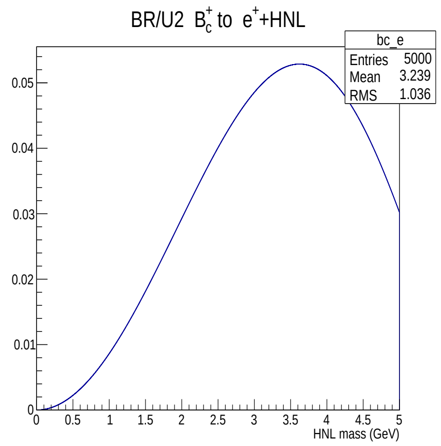
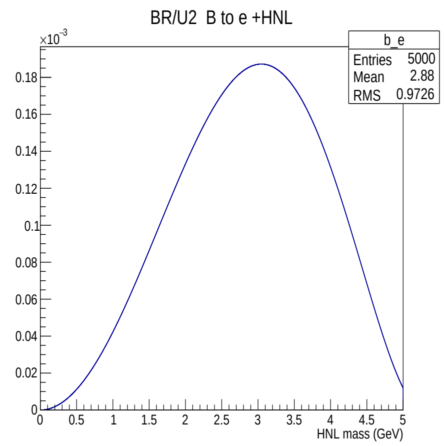
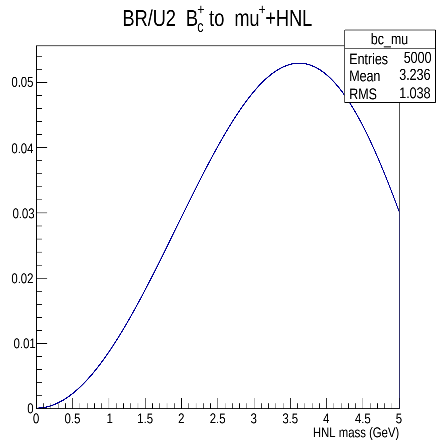
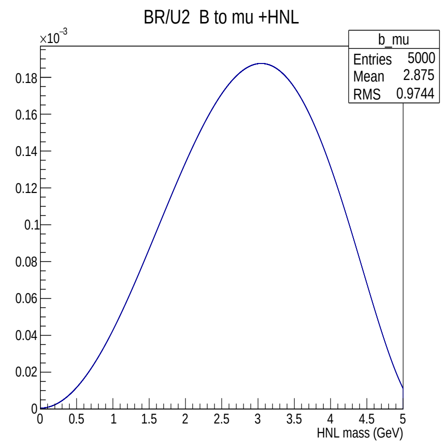
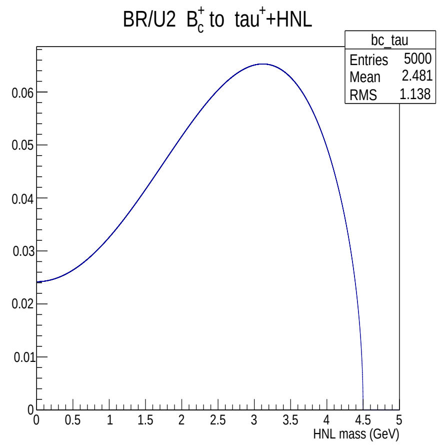
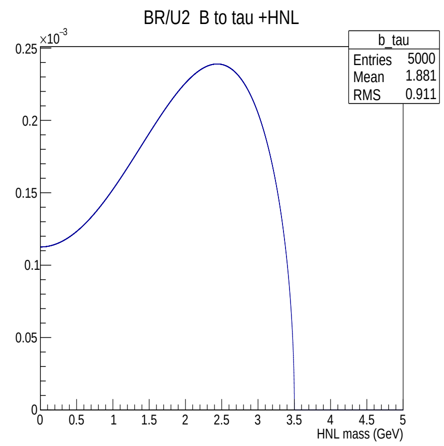
<table>
<tr><th>Channel</th><th>M_N dependence</th><th>Channel</th><th>M_N dependence</th></tr>
<tr><td>$D_s^+ \rightarrow e^+ + N$ Br/U^2 for M_N = 1 GeV : 0.34957</td><td>
</td><td>$D^+ \rightarrow e^+ + N$ Br/U^2 for M_N = 1 GeV : 0.02054</td><td>
</td></tr>
<tr><td>$D_s^+ \rightarrow \mu^+ + N$ Br/U^2 for M_N = 1 GeV : 0.35517</td><td>
</td><td>$D^+ \rightarrow \mu^+ + N$ Br/U^2 for M_N = 1 GeV : 0.02087</td><td>
</td></tr>
<tr><td>$D_s^+ \rightarrow \tau^+ + N$ Br/U^2 for $M_N = 0.05$ GeV : 0.06719</td><td>
</td><td>$D^+ \rightarrow \tau^+ + N$ Br/U^2 for $M_N = 0.05$ GeV : 0.00103</td><td>
</td></tr>
</table>

Table 2: B meson 2 body decays into HNL.

Channel	M_N dependence	Channel	M_N dependence
$B_c^+ \rightarrow e^+ + N$ Br/U^2 for M_N = 1 GeV : 0.00860		$B^+ \rightarrow e^+ + N$ Br/U^2 for M_N = 1 GeV : 4.2144e−05	
$B_c^+ \rightarrow \mu^+ + N$ Br/U^2 for M_N = 1 GeV : 0.00870		$B^+ \rightarrow \mu^+ + N$ Br/U^2 for M_N = 1 GeV : 4.2648e−05	
$B_c^+ \rightarrow \tau^+ + N$ Br/U^2 for M_N = 1 GeV : 0.03255		$B^+ \rightarrow \tau^+ + N$ Br/U^2 for M_N = 1 GeV : 0.0001522	

3 Charm/beauty pseudoscalar 3 body decays

The branching ratios of pseudoscalar meson three body semileptonic decays are calculated using formula (B2) from [1]:

$$\frac{d\mathrm{Br}(\mathrm{H} \to \mathrm{H'l_\alpha^+N})}{dE_N} = \tau_H \cdot |U_\alpha|^2 \cdot \frac{|V_{HH'}|^2 G_F^2}{64\pi^3 M_H^2} \times \int dq^2 \Bigg(f_-^2(q^2) \cdot \left(q^2(M_N^2 + M_l^2) - (M_N^2 - M_l^2)^2 \right)$$
$$+ 2f_+(q^2)f_-(q^2) \left(M_N^2(2M_H^2 - 2M_{H'}^2 - 4E_N M_H - M_l^2 + M_N^2 + q^2) + M_l^2(4E_N M_H + M_l^2 - M_N^2 - q^2) \right)$$
$$f_+^2(q^2) \Bigg((4E_N M_H + M_l^2 - M_N^2 - q^2)(2M_H^2 - 2M_{H'}^2 - 4E_N M_H - M_l^2 + M_N^2 + q^2)$$
$$- (2M_H^2 + 2M_{H'}^2 - q^2)(q^2 - M_N^2 - M_l^2) \Bigg) \Bigg) \tag{2}$$

where $q^2 = (p_l + p_N)^2$ is momentum of lepton pair, $V_{HH'}$ is the corresponding entry of CKM matrix and $f_+(q^2)$, $f_-(q^2)$ are dimensionless hadronic form factors; we have listed the references below. To obtain this branchng ratio, we must integrate over q^2 and E_N. Their limits are given by the phasespace for 3 body decays:

- q^2 limits: $[(M_l + M_N)^2, (M_H - M_{H'})^2]$
- E_N limits: $[M_N, (M_H^2 + M_N^2 - (M_l + M_{H'})^2)/2M_H]$

For $D \to H' + l + \nu$ decays, the form factors f_+ and f_- can be approximated by a pole model as follows [4] ($f_+(0) = 0.747$):

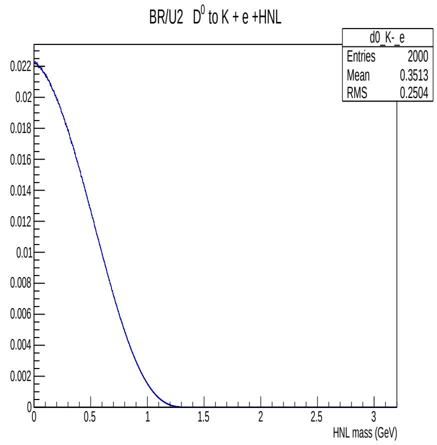
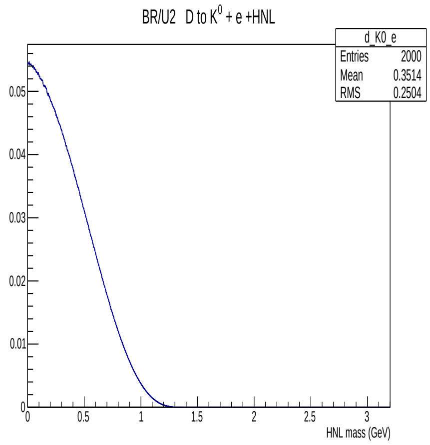
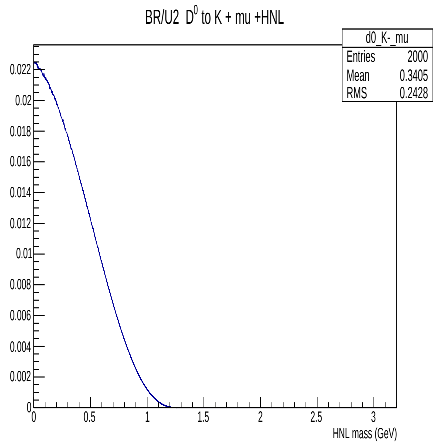
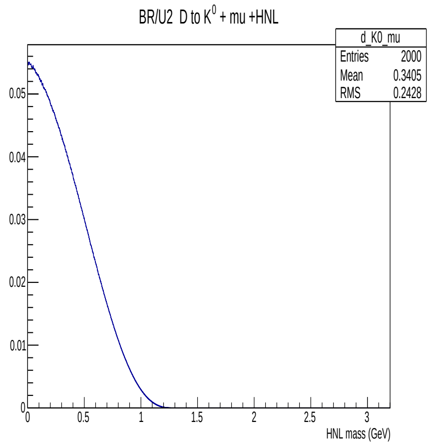
$$f_+(q^2) = \frac{f_+(0)}{(1 - q^2/M_V^2)} \tag{3}$$

$$f_0(q^2) = \frac{f_0(0)}{(1 - q^2/M_S^2)} \tag{4}$$

$$f_0(q^2) = f_+(q^2) + \frac{q^2}{M_D^2 - M_K^2} f_-(q^2) \tag{5}$$

The pole masses are for the $D^*(2010)$ (vector) and $D_0^*(2400)$ (scalar) resonances. The branching ratios for D meson three body semileptonic decays are shown in Table 3. The plots show the dependence of the branching ratio on the HNL Mass. The branching ratio is shown in units of the total U^2 coupling. The BR value for $\mathrm{M_N} = 1$ GeV is shown for reference.

Table 3: Charm 3 body decays into HNL.

Channel	$\mathrm{M_N}$ dependence	Channel	$\mathrm{M_N}$ dependence
$D^0 \to K^- + e^+ + N$ Br/U^2 for $\mathrm{M_N} =$ 1 GeV : 0.0014663		$D \to \overline{K^0} + e^+ + N$ Br/U^2 for $\mathrm{M_N} =$ 1 GeV : 0.0035893	
$D^0 \to K^- + \mu^+ + N$ Br/U^2 for $\mathrm{M_N} =$ 1 GeV : 0.0011479		$D \to \overline{K^0} + \mu^+ + N$ Br/U^2 for $\mathrm{M_N} =$ 1 GeV : 0.0028062	

For B decays, the same formulas apply. The pole masses are for the $B_c^*(6400)$ (vector) and $D_s^*(2110)$ (scalar) resonances.

For $B \to D + l + \nu$ decays, see [8] (equation 43: $f_+(0) = f_0(0) = 0.66$).

For $B_s \to D_s + l + \nu$ decays, see [6] (Table 1, $f_+(0) = f_0(0) = 0.57$).

For $B_c \to B^0/B_s^0 + l + \nu$ decays, see [7] (Table II, $f_+(0) = f_0(0) = -0.58(B_c \to B), f_+(0) = f_0(0) = -0.61(B_c \to B_s)$).

The branching ratios for the channels in this category are shown in Tables 4 and 5. The plots show the dependence of the branching ratio on the HNL Mass. The branching ratio is shown in units of the total U^2 coupling. The BR value for $\mathrm{M_N} = 0.5, 1$ GeV is shown for reference.

Table 4: Beauty meson pseudoscalar 3 body decays into HNL (part 1).

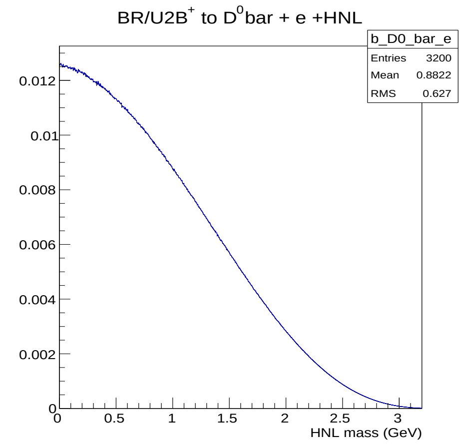
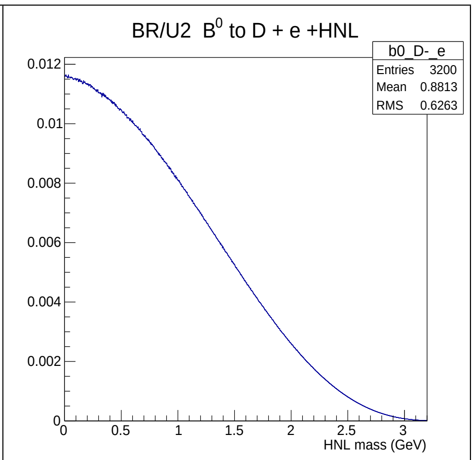
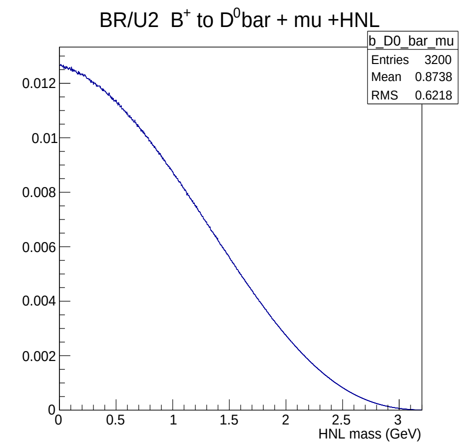
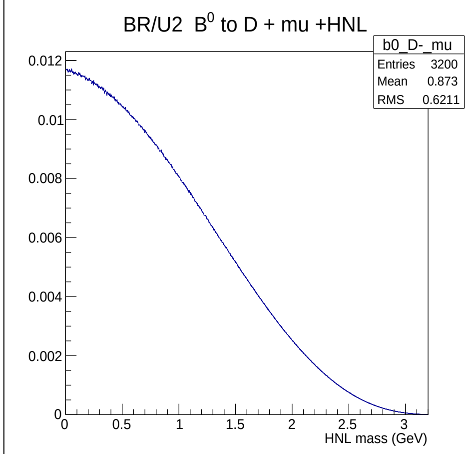
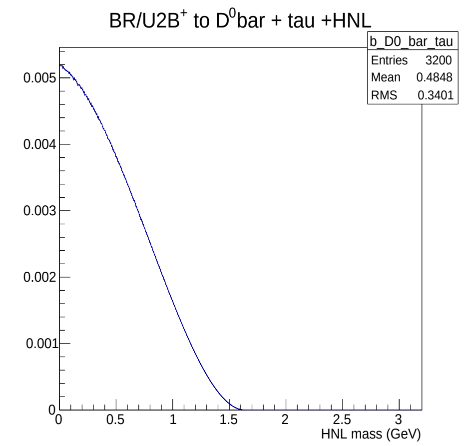
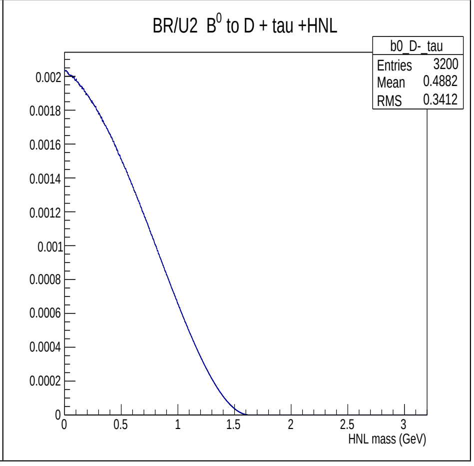
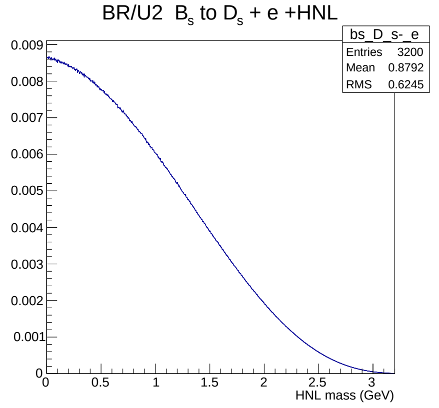
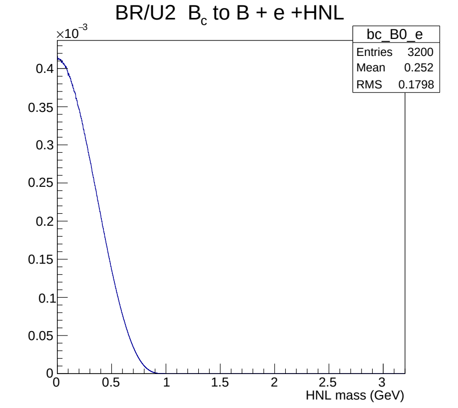
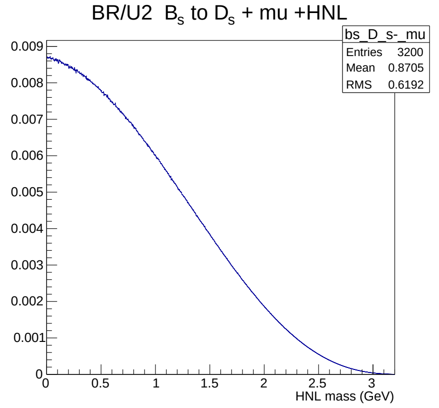
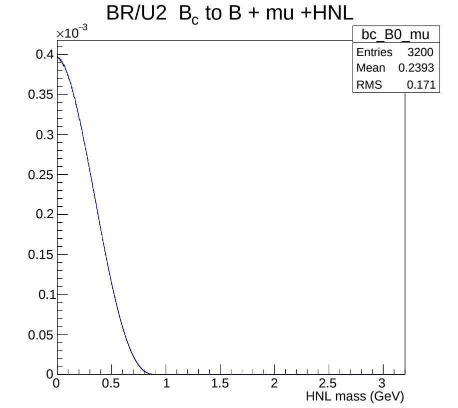
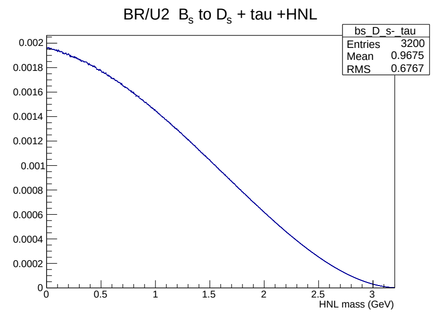
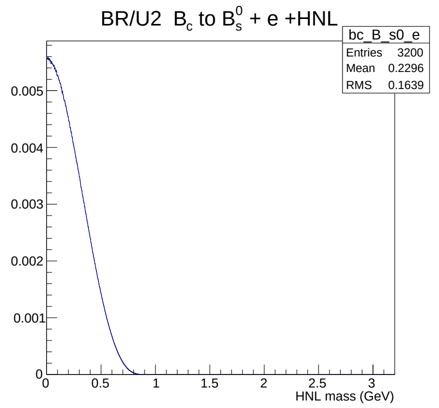
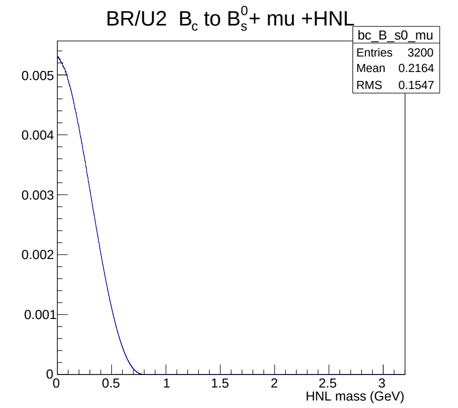
$B^+ \to \overline{D^0} + e^+ + N$ Br/U^2 for M$_\text{N}$ = 1 GeV : 0.00888713		$B^0 \to D^- + e^+ + N$ Br/U^2 for M$_\text{N}$ = 1 GeV : 0.00818813	
$B^+ \to \overline{D^0} + \mu^+ + N$Br/U^2 for M$_\text{N}$ = 1 GeV : 0.00877759		$B^0 \to D^- + \mu^+ + N$Br/U^2 for M$_\text{N}$ = 1 GeV : 0.008044587	
$B^+ \to \overline{D^0} + \tau^+ + N$Br/U^2 for M$_\text{N}$ = 1 GeV : 0.00164433		$B^0 \to D^- + \tau^+ + N$ Br/U^2 for M$_\text{N}$ = 1 GeV : 0.001502365	

Table 5: Beauty meson pseudoscalar 3 body decays into HNL (part 2).

$B_s^0 \to D_s^- + e^+ + N$ Br/U^2 for M$_\mathrm{N}$ = 1. GeV : 0.00604794		$B_c^+ \to B^0 + e^+ + N$Br/U^2 for M$_\mathrm{N}$ = 0.5 GeV : 0.00013768	
$B_s^0 \to D_s^- + \mu^+ + N$ Br/U^2 for M$_\mathrm{N}$ = 1 GeV : 0.00598956		$B_c^+ \to B^0 + \mu^+ + N$ Br/U^2 for M$_\mathrm{N}$ = 0.5 GeV : 0.00011536	
$B_s^0 \to D_s^- + \tau^+ + N$ Br/U^2 for M$_\mathrm{N}$ = 1 GeV : 0.0030619			
$B_c^+ \to B_s^0 + e^+ + N$ Br/U^2 for M$_\mathrm{N}$ = 0.5 GeV : 0.00144535		$B_c^+ \to B_s^0 + \mu^+ + N$ Br/U^2 for M$_\mathrm{N}$ = 0.5 GeV : 0.0011272	

4 Charm/beauty 3 body decays (vector mesons)

The branching ratios of pseudoscalar meson three body semileptonic decays into vector mesons V are calculated using formula (B3) from [1]:

$$\frac{d\mathrm{Br}(\mathrm{H} \to \mathrm{V} \mathrm{l}_\alpha \mathrm{N}}{dE_N} = \tau_H \cdot |U_\alpha|^2 \cdot \frac{|V_{HV}|^2 G_F^2}{32\pi^3 M_H^2} \times \int dq^2 \left(\frac{f_2^2}{2} \left(q^2 - M_N^2 - M_l^2 + \omega^2 \frac{\Omega^2 - \omega^2}{M_V^2} \right) \right.$$
$$+ \frac{f_5^2}{2} \left(M_N^2 + M_l^2 \right) \left(q^2 - M_N^2 + M_l^2 \right) \left(\frac{\Omega^4}{4M_V^2} - q^2 \right) + 2 f_3^2 M_V^2 \left(\frac{\Omega^4}{4M_V^2} - q^2 \right) \left(M_N^2 + M_l^2 - q^2 + \omega^2 \frac{\Omega^2 - \omega^2}{M_V^2} \right)$$
$$+ 2 f_3 f_5 \left(M_N^2 \omega^2 + \left(\Omega^2 - \omega^2 \right) M_l^2 \right) \left(\frac{\Omega^4}{4M_V^2} - q^2 \right) + 2 f_1 f_2 \left(q^2 \left(2\omega^2 - \Omega^2 \right) + \Omega^2 \left(M_N^2 - M_l^2 \right) \right)$$
$$+ \frac{f_2 f_5}{2} \left(\omega^2 \frac{\Omega^2}{M_V^2} \left(M_N^2 - M_l^2 \right) + \frac{\Omega^4}{M_V^2} M_l^2 + 2 \left(M_N^2 - M_l^2 \right)^2 - 2q^2 \left(M_N^2 + M_l^2 \right) \right)$$
$$+ f_2 f_3 \left(\Omega^2 \omega^2 \frac{\Omega^2 - \omega^2}{M_V^2} + 2\omega^2 \left(M_l^2 - M_N^2 \right) + \Omega^2 \left(M_N^2 - M_l^2 - q^2 \right) \right)$$
$$\left. + f_1^2 \left(\Omega^4 \left(q^2 - M_N^2 + M_l^2 \right) - 2 M_V^2 \left(q^4 - \left(M_N^2 - M_l^2 \right)^2 \right) + 2\omega^2 \Omega^2 \left(M_N^2 - q^2 - M_l^2 \right) + 2\omega^4 q^2 \right) \right) , \tag{6}$$

where $\omega^2 = M_H^2 - M_V^2 + M_N^2 - M_l^2 - 2M_H E_N$ and $\Omega^2 = M_H^2 - M_V^2 - q^2$; form factors $f_i(q^2)$ can be expressed via standard axial form factors $A_0(q^2), A_1(q^2), A_2(q^2)$ and vector form factor $V(q^2)$ as

$$f_1 = \frac{V}{M_H + M_V} , \quad f_2 = (M_H + M_V) \cdot A_1 , \quad f_3 = -\frac{A_2}{M_H + M_V} ,$$
$$f_4 = \left(M_V \left(2A_0 - A_1 - A_2 \right) + M_H \left(A_2 - A_1 \right) \right) \cdot \frac{1}{q^2} , \quad f_5 = f_3 + f_4 ,$$

that can be found in literature; we have listed the references below.

To obtain the branching ratios, we must integrate over q^2 and E_N. Their limits are given by the phase space for 3 body decays (see Section 3).

For B/D decays into vector mesons we need standard axial form factors A_0, A_1, A_2 and the vector form factor V. The q^2 dependence of form factors A_0 and V is given by the formula below(see [5] eqns (9) and (10)):

$$f(q^2) = \frac{f(0)}{(1 - q^2/M^2)(1 - \sigma_1 q^2/M^2 + \sigma_2 q^4/M^4)} \tag{7}$$

$M = M_P$ for form factor A_0 and $M = M_V$ for form factor V. For form factors A_1 and A_2:

$$f(q^2) = \frac{f(0)}{(1 - \sigma_1 q^2/M_V^2 + \sigma_2 q^4/M_V^4)} \tag{8}$$

$f(0)$ and $\sigma_{1,2}$ are fit parameters obtained from lattice gauge theory. We can take $\sigma_2 = 0$ (loss of accuracy 1%).

For $D \to V + l + \nu$ decays, we use $V = 1.03, A0 = 0.76, A1 = 0.66, A2 = 0.49, \sigma_1(V) = 0.27, \sigma_1(A0) = 0.17, \sigma_1(A1) = 0.3, \sigma_1(A2) = 0.67$ ([5] Table IV). The pole masses are for the $D^*(2007)$ (vector) and $D_s(1969)$ (scalar) resonances.

For $B \to D^*$ decays, we use $V = 0.76, A0 = 0.69, A1 = 0.66, A2 = 0.62, \sigma_1(V) = 0.57, \sigma_1(A0) = 0.58, \sigma_1(A1) = 0.78, \sigma_1(A2) = 1.4$ ([5] Table IV). The pole masses are for the $B_c^*(6842)$ (vector) and $B_c(6277)$ (scalar) resonances.

For $B_s \to D_s^{*-}$ decays, we use $V = 0.95, A0 = 0.67, A1 = 0.70, A2 = 0.75, \sigma_1(V) = 0.372, \sigma_1(A0) = 0.35, \sigma_1(A1) = 0.463, \sigma_1(A2) = 1.04$ ([9] Table I). The pole masses are $B_c^*(6842)$.

For $B_c \to B_s^*/B^*$ decays, the form factors can be approximated (see [9] eqn. 6.3) by the form:

$$f(q^2) = \frac{f(0)}{1 - q^2/m_{fit}^2 - \delta \cdot (q^2/m_{fit}^2)^2} \tag{9}$$

We have taken (from [9] Table VIII): $m_{fit}^2, \delta(V) = 1.76^2, -0.052; (A0) = 1.86^2, 0.13; (A1) = 3.44^2, -1.07; (A2) = 1.73^2, 0.09$.

The branching ratios for the channels in this category are shown in Tables 6 and 7. the plots show the dependence of the branching ratio on the HNL Mass. The branching ratio is shown in units of the total U^2 coupling. The BR value for $\mathrm{M_N} = 0.5$ or 1 GeV is shown for reference.

Table 6: Charm/beauty meson vector 3 body decays into HNL (part 1).

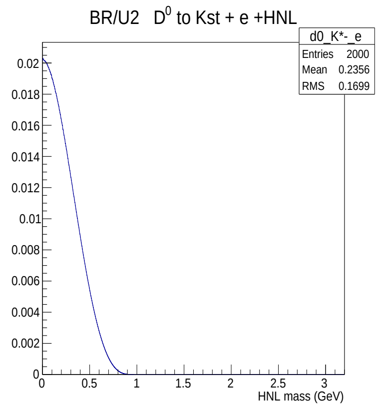
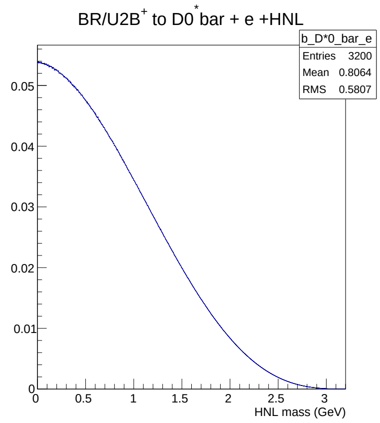
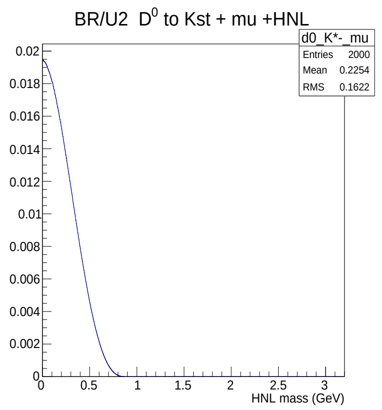
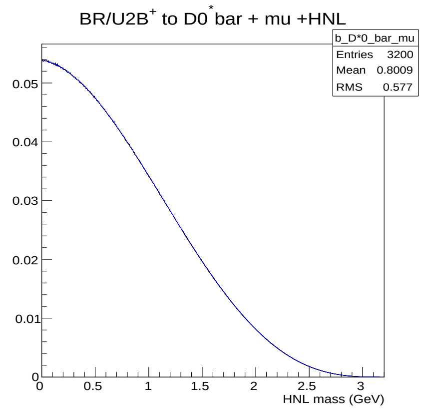
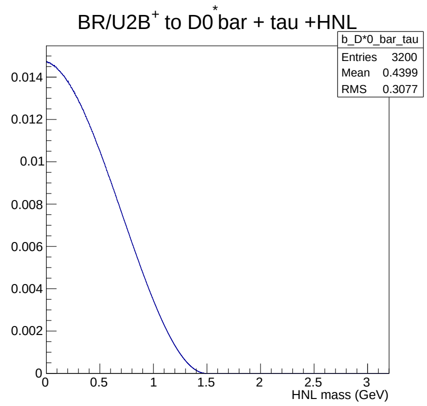
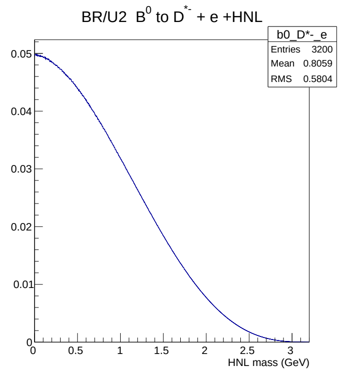
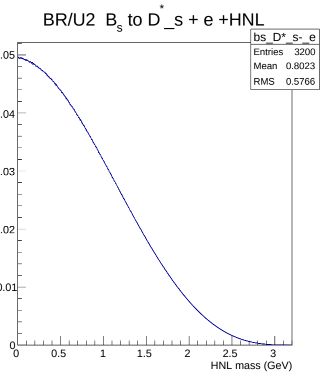
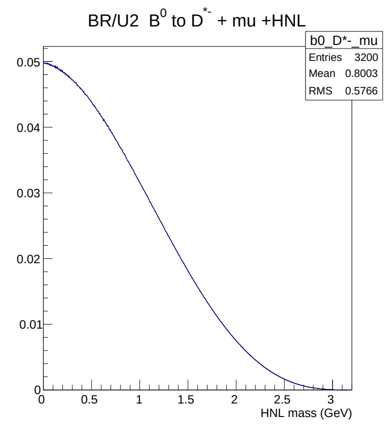
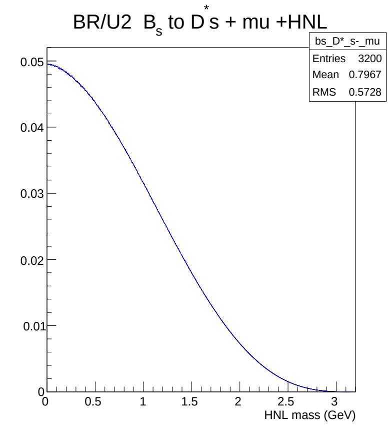
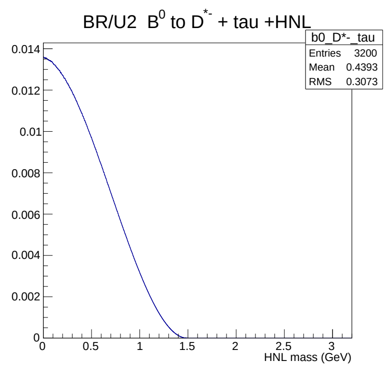
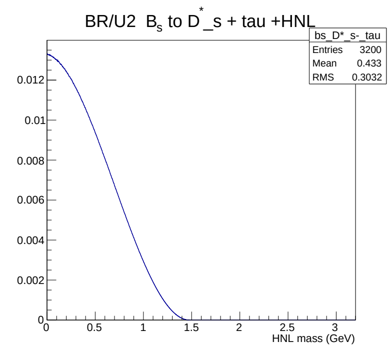
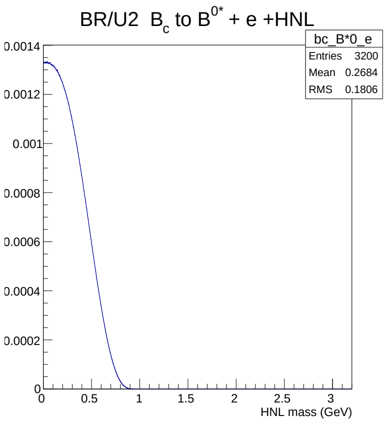
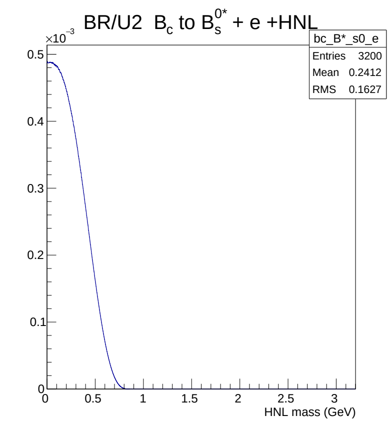
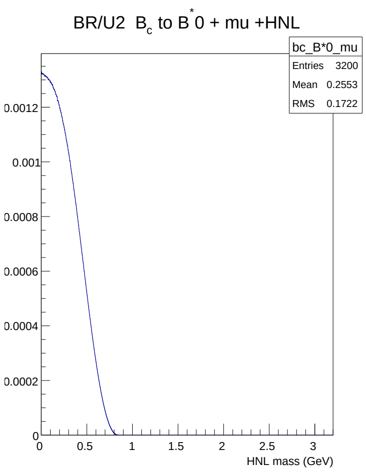
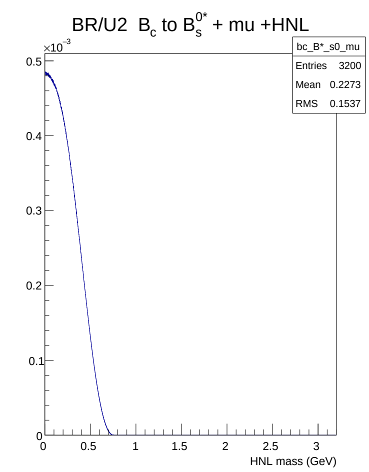
<table>
<tr><td>Channel</td><td>$\mathrm{M_N}$ dependence</td><td>Channel</td><td>$\mathrm{M_N}$ dependence</td></tr>
<tr><td>$D^0 \to K^{*-} + e^+ + N$ Br/U^2 for $\mathrm{M_N}$ = 0.5 GeV : 0.0054702</td><td>
</td><td>$B^+ \to \overline{D^{*0}} + e^+ + N$ Br/U^2 for $\mathrm{M_N}$ = 1 GeV : 0.034673</td><td>
</td></tr>
<tr><td>$D^0 \to K^{*-} + \mu^+ + N$ Br/U^2 for $\mathrm{M_N}$ = 0.5 GeV : 0.0046451</td><td>
</td><td>$B^+ \to \overline{D^{*0}} + \mu^+ + N$ Br/U^2 for $\mathrm{M_N}$ = 1 GeV : 0.034257</td><td>
</td></tr>
<tr><td></td><td></td><td>$B^+ \to \overline{D^{*0}} + \tau^+ + N$ Br/U^2 for $\mathrm{M_N}$ = 1 GeV : 0.006958</td><td>
</td></tr>
<tr><td>$B^0 \to D^{*-} + e^+ + N$ Br/U^2 for $\mathrm{M_N}$ = 1 GeV : 0.032048</td><td>
</td><td>$B_s^0 \to D_s^{*-} + e^+ + N$ Br/U^2 for $\mathrm{M_N}$ = 1 GeV : 0.0318154</td><td>
</td></tr>
</table>

Table 7: Charm/beauty meson vector 3 body decays into HNL (part 2).

Channel	M_N dependence	Channel	M_N dependence
$B^0 \rightarrow D^{*-} + \mu^+ + N$ Br/U^2 for M_N = 1 GeV : 0.031679		$B^0_s \rightarrow D^{*-}_s + \mu^+ + N$ Br/U^2 for M_N = 1 GeV : 0.0315129	
$B^0 \rightarrow D^{*-} + \tau^+ + N$ Br/U^2 for M_N = 1 GeV : 0.00319026		$B^0_s \rightarrow D^{*-}_s + \tau^+ + N$ Br/U^2 for M_N = 1 GeV : 0.0029561	
$B^+_c \rightarrow B^{*0} + e^+ + N$ Br/U^2 for M_N = 1 GeV : 0.00059944		$B^+_c \rightarrow B^{*0}_s + e^+ + N$ Br/U^2 for M_N = 1 GeV : 0.0001639	
$B^+_c \rightarrow B^{*0} + \mu^+ + N$ Br/U^2 for M_N = 1 GeV : 0.00052505		$B^+_c \rightarrow B^{*0}_s + \mu^+ + N$ Br/U^2 for M_N = 1 GeV : 0.0001330	

5 3 body decays of hyperons (Λ, Ξ and Ω)

The differential branching ratios of baryon semileptonic decays with massive sterile neutrinos in the final state are given by [2] (here the Λ decays are shown; formulas for Ξ and Ω are obtained by appropriate substitutions):

$$\begin{aligned}
\frac{d\Gamma}{dE_\Lambda} = & \frac{G^2|V_{cs}|^2}{64\pi^3}\frac{\sin^2\theta_{lx}}{q^4}\frac{\sqrt{E_\Lambda^2-M_\Lambda^2}}{M_{\Lambda_c}}\sqrt{\frac{q^4-2q^2(m_{\nu_x}^2+m_l^2)+(m_l^2-m_{\nu_x}^2)^2}{q^4}} \\
& \times\Big[\frac{1}{3}\big(2q^4-q^2(m_{\nu_x}^2+m_l^2)-(m_l^2-m_{\nu_x}^2)^2\big)\big((f_1^2+g_1^2)(4M_\Lambda^2q^2-12Pq\,q^2+8(Pq)^2) \\
& \quad -12(f_1^2-g_1^2)M_\Lambda M_{\Lambda_c}q^2+24\frac{M_\Lambda}{M_{\Lambda_c}}(f_1f_2+g_1g_2)(Pq-q^2)q^2-24(f_1f_2-g_1g_2)Pq\,q^2 \\
& \quad -\frac{f_2^2+g_2^2}{M_{\Lambda_c}^2}(12Pq\,q^4-8(Pq)^2q^2+4M_\Lambda^2q^4)-6\frac{M_\Lambda}{M_{\Lambda_c}}(f_2^2-g_2^2)q^4\big) \\
& +\big((m_l^2-m_{\nu_x}^2)^2-q^2(m_{\nu_x}^2+m_l^2)\big)\big((f_1^2+g_1^2)(4M_\Lambda^2q^2+4Pq\,q^2-8(Pq)^2) \\
& \quad -4(f_1^2-g_1^2)q^2M_\Lambda M_{\Lambda_c}-8\frac{M_\Lambda}{M_{\Lambda_c}}(f_1f_3+g_1g_3)Pq\,q^2-8(f_1f_3-g_1g_3)Pq\,q^2 \\
& \quad +4\frac{f_3^2}{M_{\Lambda_c}^2}q^2(Pq-M_\Lambda(M_{\Lambda_c}+M_\Lambda))+4\frac{g_3^2}{M_{\Lambda_c}^2}q^2(Pq+M_\Lambda(M_{\Lambda_c}-M_\Lambda))\big)\Big],
\end{aligned} \tag{10}$$

where Pq and q^2 are functions of E_Λ:

$$Pq = M_\Lambda^2 - E_\Lambda M_{\Lambda_c}$$

and

$$q^2 = M_\Lambda^2 + M_{\Lambda_c}^2 - 2M_{\Lambda_c}E_\Lambda.$$

As for the pseudoscalar decays described in the previous chapters, we use a dipole approximation for the form factors [10]:

$$f_i(q^2) = \frac{f_i(0)}{\left(1-\frac{q^2}{m_V^2}\right)^2}, \tag{11}$$

$$g_i(q^2) = \frac{g_i(0)}{\left(1-\frac{q^2}{m_A^2}\right)^2}, \tag{12}$$

where for charmed baryons the pole masses are $m_V = 2.11$ GeV, $m_A = 2.54$ GeV, and for beauty baryons $m_V = 6.34$ GeV, $m_A = 6.73$ GeV.

The values of $f_i(0)$ and $g_i(0)$ for the transitions $\Lambda_b \to \Lambda_c$ and $\Lambda_c \to \Lambda$, $\Xi_b \to \Xi_c$, $\Xi_c \to \Xi$ and $\Omega_b \to \Omega_c$ are taken from Ref. [10] (Table I) see Table 8. The values of $f_2(0)$,

Table 8: Form factors for the transitions between charm and beauty hyperons from Ref. [10].

Form factors	$\Lambda_b \to \Lambda_c$	$\Lambda_c \to \Lambda$	$\Xi_b \to \Xi_c$	$\Xi_c \to \Xi$	$\Omega_b \to \Omega_c$
$f_1(0)$	0.53	0.29	0.54	0.31	0.72
$f_2(0)$	0.12	0.14	0.14	0.19	0.68
$f_3(0)$	0.02	0.03	0.02	0.04	0.36
$g_1(0)$	0.58	0.38	0.58	0.39	0.2
$g_2(0)$	0.02	0.03	0.03	0.06	0.0.01
$g_3(0)$	0.13	0.19	0.16	0.24	0.06

$f_3(0)$, $g_1(0)$, $g_2(0)$, $g_3(0)$ have the opposite sign as compared to Ref. [6] because of the different parametrization of the matrix element.

The branching ratios for the channels in this category are shown in Tables 9, 10 and 11. Notice the good agreement between Figure 1 of [2] and the decays $\Lambda_c^+ \to \Lambda^0 + e^+ + N$ and $\Lambda_b^0 \to \Lambda_c^+ + e^+ + N$ in Tables 9. The plots show the dependence of the branching ratio on the HNL Mass. The branching ratio is shown in units of the total U^2 coupling. The BR value for $\mathrm{M_N} = 1$ GeV is shown for reference.

Table 9: Charm/beauty hyperon decays into HNL (part 1).

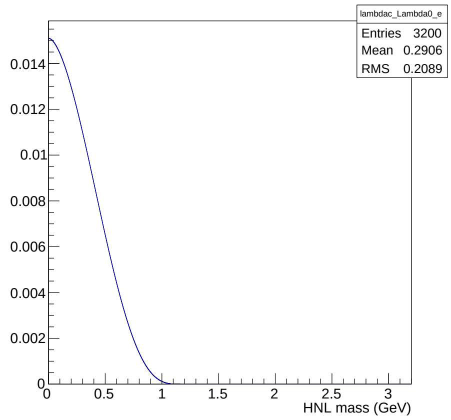
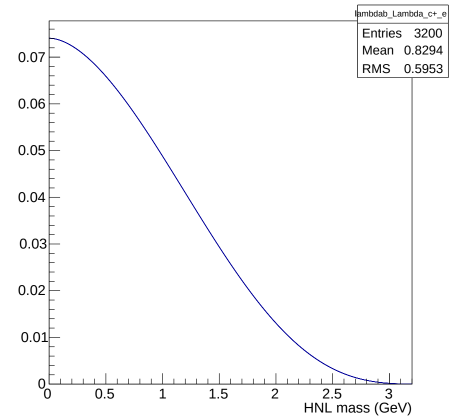
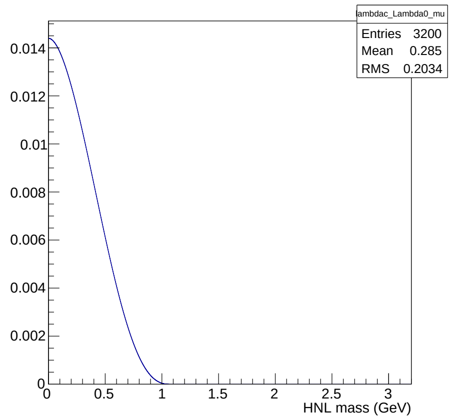
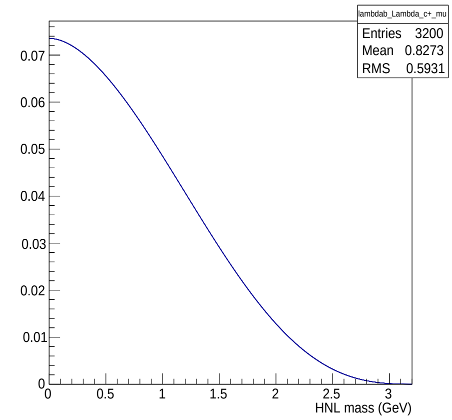
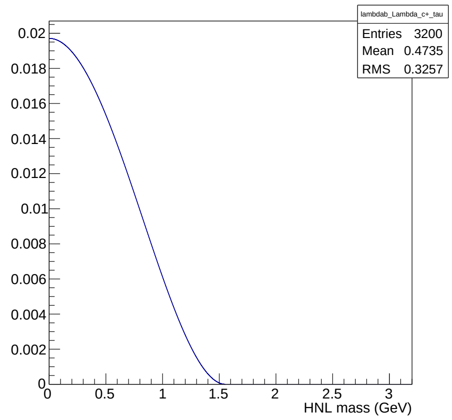
<table>
<tr><th>Channel
Br/U^2 for $\mathrm{M_N}$ = 1 GeV</th><th>$\mathrm{M_N}$ dependence</th><th>Channel
Br/U^2 for $\mathrm{M_N}$ = 1 GeV</th><th>$\mathrm{M_N}$ dependence</th></tr>
<tr><td>$\Lambda_c^+ \to \Lambda^0 + e^+ + N$
0.000122862</td><td>
</td><td>$\Lambda_b^0 \to \Lambda_c^+ + e^+ + N$
0.04885236</td><td>
</td></tr>
<tr><td>$\Lambda_c^+ \to \Lambda^0 + \mu^+ + N$
4.42535222694e-05</td><td>
</td><td>$\Lambda_b^0 \to \Lambda_c^+ + \mu^+ + N$
0.0485582</td><td>
</td></tr>
<tr><td></td><td></td><td>$\Lambda_b^0 \to \Lambda_c^+ + \tau^+ + N$
0.00614271</td><td>
</td></tr>
</table>

Table 10: Charm/beauty hyperons decays into HNL (part 2).

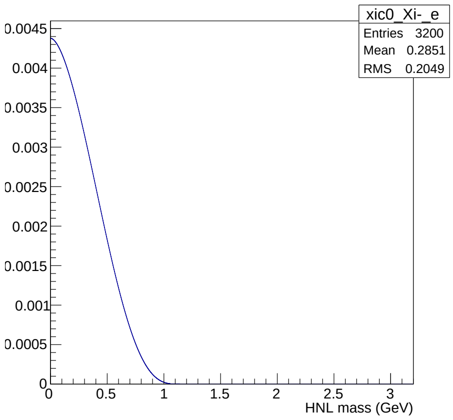
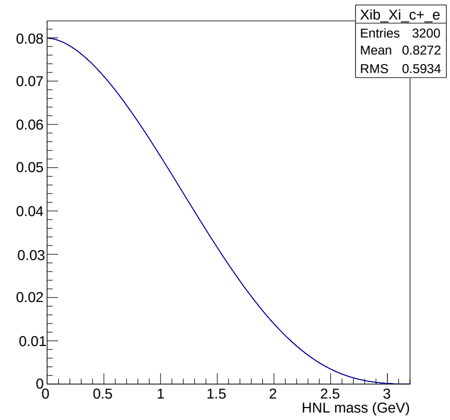
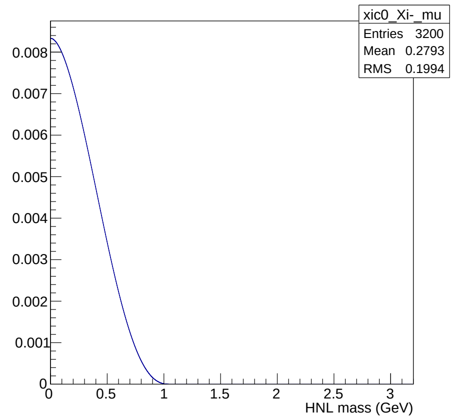
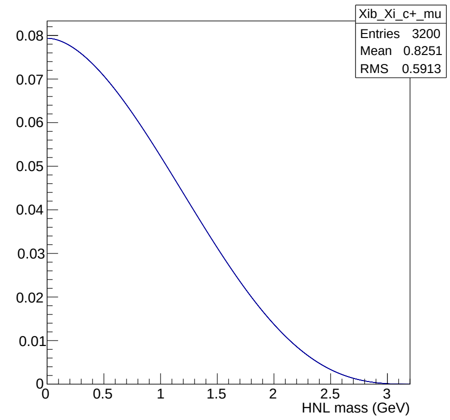
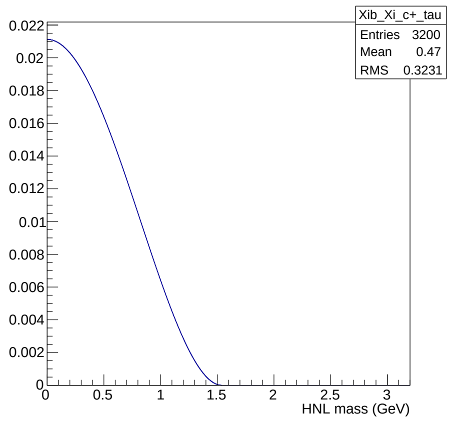
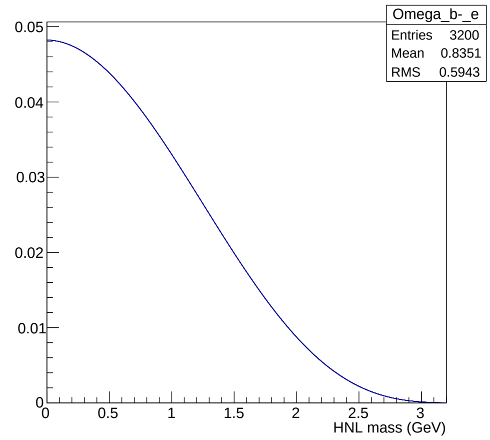
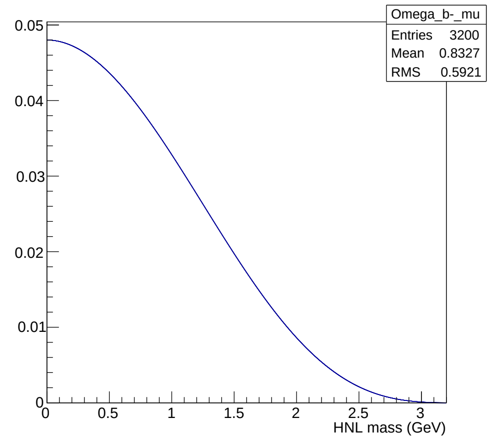
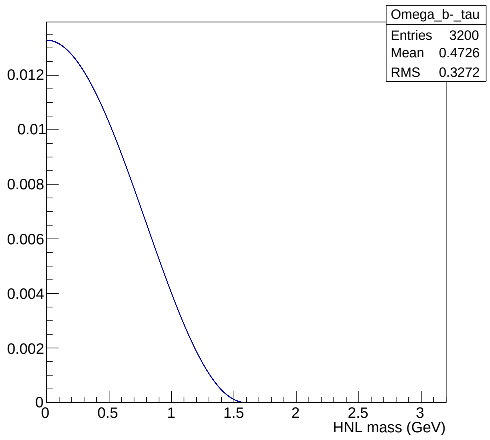
Channel $\mathrm{Br/U^2}$ for $\mathrm{M_N}$ = 1 GeV	$\mathrm{M_N}$ dependence	Channel $\mathrm{Br/U^2}$ for $\mathrm{M_N}$ = 1 GeV	$\mathrm{M_N}$ dependence
$\Xi_c^0 \to \Xi^- + e^+ + N$ 2.43084139627e-05		$\Xi_b^0 \to \Xi_c^+ + e^+ + N$ 0.052657266	
$\Xi_c^0 \to \Xi^- + \mu^+ + N$ 5.68015577956e-06		$\Xi_b^0 \to \Xi_c^+ + \mu^+ + N$ 0.05233910	
		$\Xi_b^0 \to \Xi_c^+ + \tau^+ + N$ 0.006444799	

Table 11: Charm/beauty hyperons decays into HNL (part 3).

Channel	BR/U2 for $M_N = 1$GeV	M_N dependence
$\Omega_b^- \to \Omega_c^0 + e^+ + N$	0.0330596	
$\Omega_b^- \to \Omega_c^0 + \mu^+ + N$	0.0328731	
$\Omega_b^- \to \Omega_c^0 + \tau^+ + N$	0.0040285	

6 τ 2 body decays into HNL

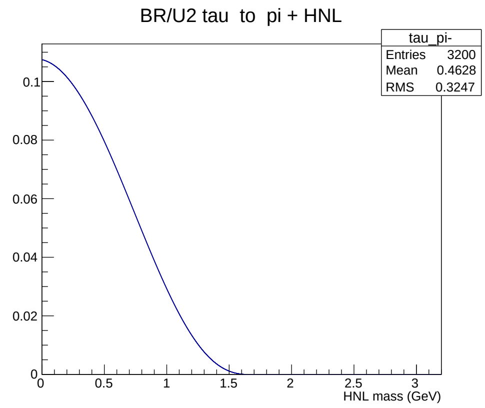
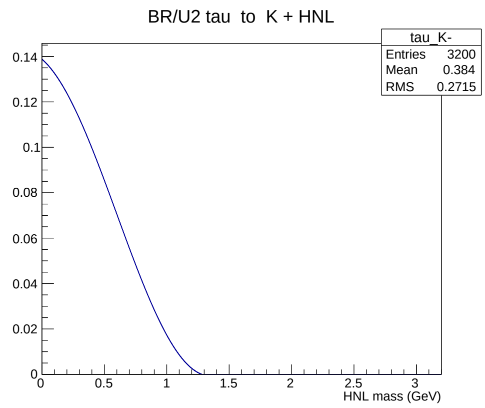
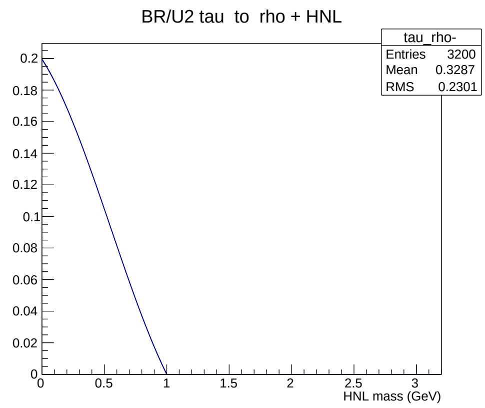
The branching ratios of two body decays of τ-leptons into heavy neutrino and meson are calculated using formulas (B4) from [1]:

$$\frac{d\text{Br}\,(\tau \to \text{HN})}{dE_N} = \tau_\tau \cdot \frac{|U_\tau|^2}{16\pi} G_F^2 |V_H|^2 f_H^2 M_\tau^3 \cdot \left(\left(1 - \frac{M_N^2}{M_\tau^2}\right)^2 - \frac{M_H^2}{M_\tau^2}\left(1 + \frac{M_N^2}{M_\tau^2}\right)\right)$$
$$\times \sqrt{\left(1 - \frac{(M_H - M_N)^2}{M_\tau^2}\right)\left(1 - \frac{(M_H + M_N)^2}{M_\tau^2}\right)} \cdot \delta\left(E_N - \frac{M_\tau^2 - M_H^2 + M_N^2}{2M_\tau}\right),$$
$$\frac{d\text{Br}\,(\tau \to \rho\text{N})}{dE_N} = \tau_\tau \cdot \frac{|U_\tau|^2}{8\pi} \frac{g_\rho^2}{M_\rho^2} G_F^2 |V_{ud}|^2 M_\tau^3 \cdot \left(\left(1 - \frac{M_N^2}{M_\tau^2}\right)^2 + \frac{M_\rho^2}{M_\tau^2}\left(1 + \frac{M_N^2 - 2M_\rho^2}{M_\tau^2}\right)\right)$$
$$\times \sqrt{\left(1 - \frac{(M_\rho - M_N)^2}{M_\tau^2}\right)\left(1 - \frac{(M_\rho + M_N)^2}{M_\tau^2}\right)} \cdot \delta\left(E_N - \frac{M_\tau^2 - M_\rho^2 + M_N^2}{2M_\tau}\right),$$

where τ_τ is the τ-lepton life-time.

The branching ratios for the channels in this category are shown in Table 12. The plots show the dependence of the branching ratio on the HNL Mass. The branching ratio is shown in units of the total U^2 coupling. The BR value for $\text{M}_\text{N} = 1$ GeV is shown for reference.

Table 12: τ lepton 2 body decays into HNL.

Channel	BR/U2 for $M_N = 1$ GeV	Plot
$\tau \to \pi + N$	0.0293826	
$\tau \to K + N$	0.0174661	
$\tau \to \rho + N$	0.00028824	

7 τ 3 body decays into HNL

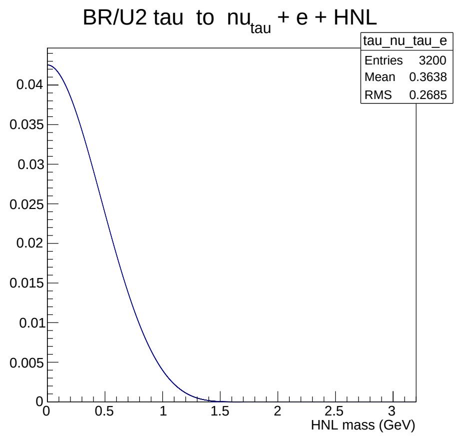
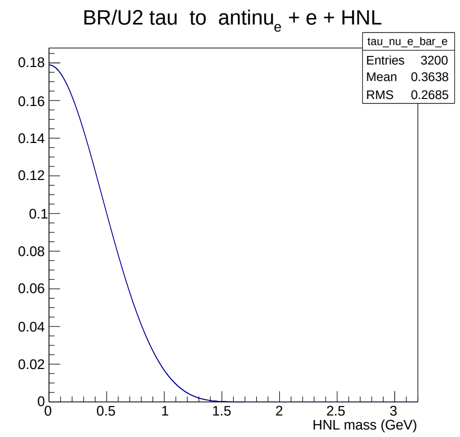
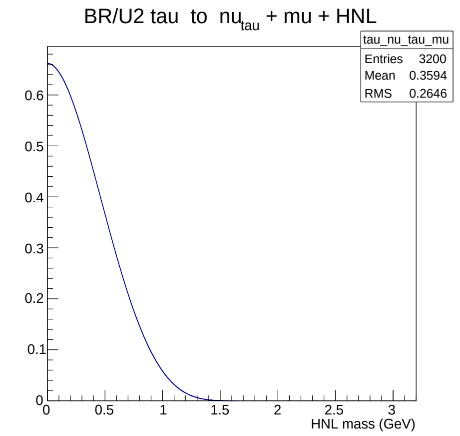
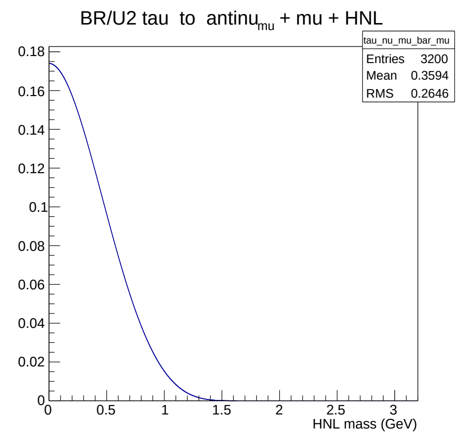
The branching ratios of three body decays of τ-leptons into heavy neutrino and leptons are calculated using formulas (B5) from [1].

$$\frac{d\mathrm{Br}\left(\tau\to\nu_\tau \mathrm{l}_\alpha \mathrm{N}\right)}{dE_N} = \tau_\tau\cdot\frac{|U_\alpha|^2}{2\pi^3}G_F^2 M_\tau^2\cdot E_N\left(1+\frac{M_N^2-M_l^2}{M_\tau^2}-2\frac{E_N}{M_\tau}\right)\left(1-\frac{M_l^2}{M_\tau^2+M_N^2-2E_N M_\tau}\right)\sqrt{E_N^2-M_N^2}\,,$$

$$\frac{d\mathrm{Br}\left(\tau\to\bar\nu_\alpha \mathrm{l}_\alpha \mathrm{N}\right)}{dE_N} = \tau_\tau\cdot\frac{|U_\tau|^2}{4\pi^3}G_F^2 M_\tau^2\left(1-\frac{M_l^2}{M_\tau^2+M_N^2-2E_N M_\tau}\right)^2\sqrt{E_N^2-M_N^2}$$
$$\times\left(\left(M_\tau-E_N\right)\left(1-\frac{M_N^2+M_l^2}{M_\tau^2}\right)-\left(1-\frac{M_l^2}{M_\tau^2+M_N^2-2E_N M_\tau}\right)\left(\frac{\left(M_\tau-E_N\right)^2}{M_\tau}+\frac{E_N^2-M_N^2}{3M_\tau}\right)\right)$$

The branching ratios are shown in Tables 13. The plots show the dependence of the branching ratio on the HNL Mass. The branching ratio is shown in units of the total U^2 coupling. The BR value for $\mathrm{M_N} = 1$ GeV is shown for reference.

Table 13: τ lepton 3 body decays into HNL.

Channel Br/U^2 for M$_\mathrm{N}$ = 1 GeV	M$_\mathrm{N}$ dependence	Channel Br/U^2 for M$_\mathrm{N}$ = 1 GeV	M$_\mathrm{N}$ dependence
$\tau \to \nu_\tau + e + N$ 0.0040114829		$\tau \to \overline{\nu_e} + e + N$ 0.01687158	
$\tau \to \nu_\tau + \mu + N$ 0.058464213		$\tau \to \overline{\nu_\mu} + \mu + N$ 0.0153724085	

8 Charm and Beauty branching ratios into HNL per meson

The production fractions for the various Charm mesons are given in Table 14 (see [12], Table I). The first table contains the measured fractions. As the measured fractions add up to 0.93, the second column contains the scaled fractions so that they add up to 1. The third column contains the values after cascade (see [13]). Since the D_s meson has the highest HNL branching fraction, its production value after the cascade has been reduced to 0.077, which is the measured value. This results in a slight increase for the other mesons. The values that are used by Pythia are shown in column 4. The D_s reduction factor is a parameter for the simulation script; we are waiting for confirmation to change it to 0.083. For Beauty mesons, the fractions are given in Table 15. In the case of Beauty, the

Table 14: Charm production fractions

Meson	Production Fraction from Hera II [12]	From cascade	In FairShip
fD^+	0.234	0.204	0.207
fD^0	0.588	0.622	0.632
fD_s	0.088	0.104	0.088
$f\Lambda_c$	0.079	0.066	0.067
$f\Omega + f\Sigma + f\Xi$ (14% $of\ f\Lambda_c$)	0.011	0.0022	0.0022

measurements are on less firm ground. Nevertheless, in the first column we quote the values from the literature [14]. In the second column we quote the values after the cascade. These values are used as input for Pythia.

Table 15: Beauty production fractions

Meson	Production Fraction	From cascade
fB^+	0.35	0.417
fB^0	0.35	0.418
fB_s	0.11	0.113
fB_c	0.002	-
$f\Lambda_b$	0.108	0.047
$f\Xi_b^-$	0.04	0.0023
$f\Xi_b^0$		0.0022
$f\Omega_b$	0.00003	

By summing the branching ratios divided by U^2, as a function of the $\mathrm{M_N}$, for each meson and multiplying them by the branching fractions (used as input by Pythia) one can get an idea of the relative importance for each meson. The τ lepton also decays into HNL (see Sec. 6, 7). Since it is produced in decays of the D_s, the τ lepton branching ratios, multiplied by BR($D_s \rightarrow \tau + \nu_\tau$) (5.43%) are added to the D_s branching ratios. These ratios are shown in Fig. 1 (Charm) and Fig. 2 (Beauty). The legend is ordered from high to low branching ratio. The B_c production fraction is small, but unknown. In the calculation of Fig. 2 we show two curves, one for $fB_c = 0.002$ (blue) and one for $fB_c = 0.00002$ (red). At large masses B_c could dominate, but there is a large uncertainty due to the strong dependence on fB_c.

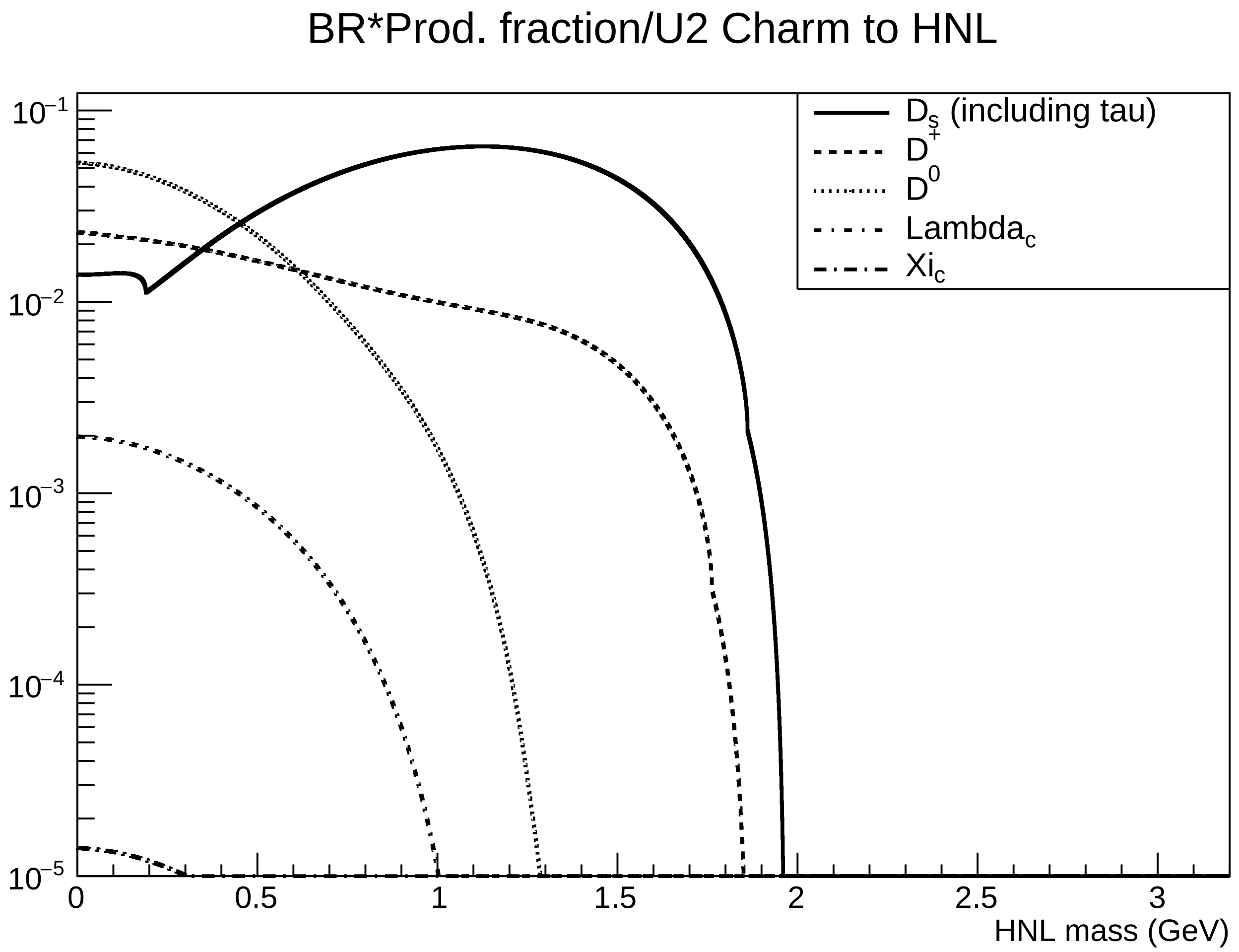


Figure 1: BR*Production fraction over U^2 for Charm to HNL

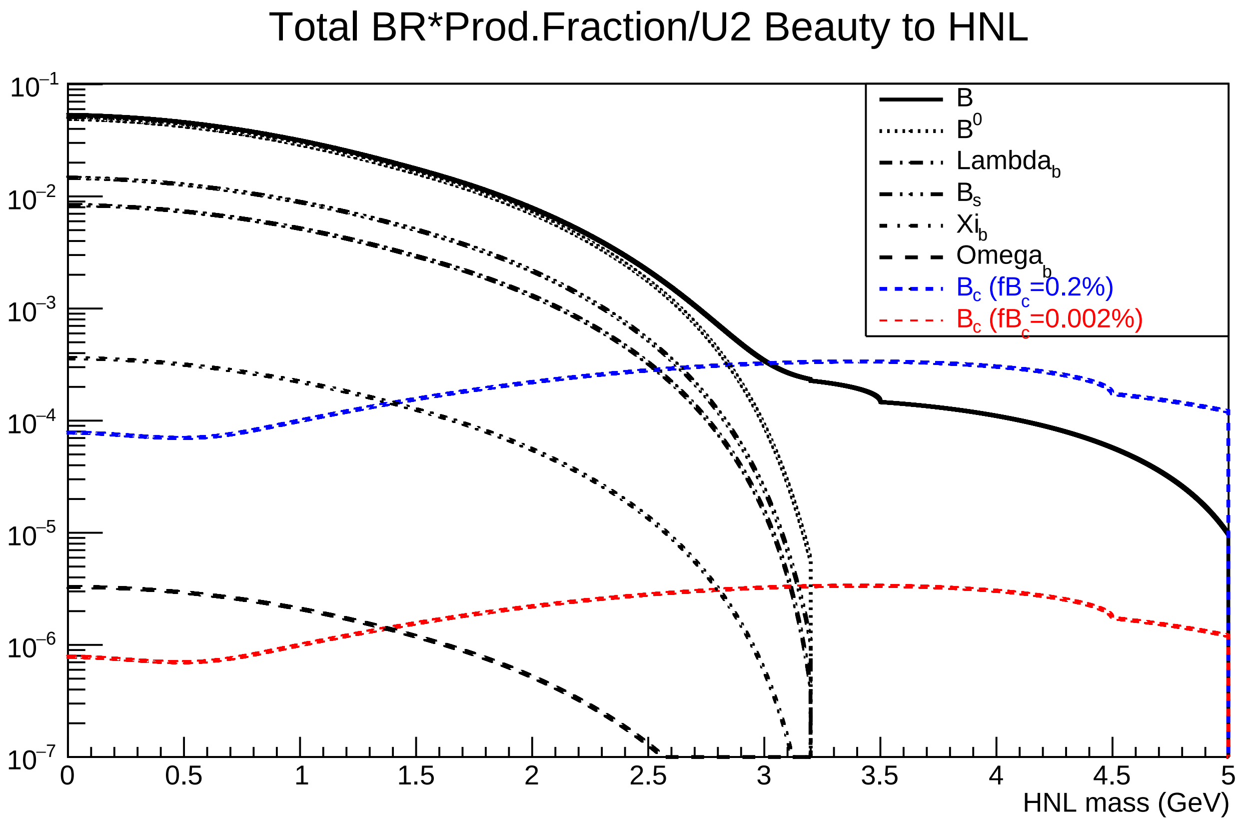


Figure 2: BR*Production fraction over U^2 for Beauty to HNL

9 Integration into FairShip and Conclusions

The FairShip simulation is run by the script `$FAIRSHIP/macro/run_simScript.py`, which takes the HNL mass M_N and its couplings to the Standard Model flavours U_e, U_μ, U_τ as input parameters. The Pythia 8 configuration is done by the file `$FAIRSHIP/python/pythia8_conf.py`.

In the simulations for the TP (the 'OLD' procedure), the `pythia8_conf.py` file was constructed by substituting all neutrinos in semi leptonic B/D decays by a HNL. Channels without HNLs were forbidden and the original semileptonic branching ratios were kept. The origin of reconstructed charm signals using the 'OLD' procedure for $M_N = 1$ GeV is shown in Table 16. To make the mass dependent branching ratios available to `run_Simscript.py`

Table 16: Source of reconstructed signal (in percent)

Meson	Source with 'OLD' procedure	Source with 'NEW' procedure
D^0	59.9	1.3
D^+	19	10.7
D_s	10.3	80.1
Λ_c	10.5	
Ξ_c	0.3	
τ		7.8

so that `Pythia 8` can be configured with the correct branching ratios (while keeping their relative proportions, i.e. the 'NEW' procedure) we proceed as follows.

The histograms containing the M_N dependent branching fractions are collected into a single root file, `branchingratios.root`. It can be found at `http://ship.web.cern.ch/ship/branchingratios.root`.

To avoid storing the binary root file in git, it is converted to an ascii file, `$FAIRSHIP/shipgen/branchingratios.dat`.

The `pythia8_conf.py` script reads the ascii histogram file, gets the branching ratios at the specified M_N for each channel. It sums the branching ratios per meson and looks for the maximum sum of branching ratios. The branching ratios are then divided by this maximum sum so that the meson with the maximum is normalized to 1, i.e. it decays into an HNL with 100% probability (to avoid the generation of uninteresting events). For the other mesons, we add an unphysical channel with branching ratio $1 - \Sigma\text{BR}/\text{maximum}(\Sigma\text{BR})$. This ensures that the relative proportions between meson decays are preserved.

For example, for $M_N = 1$GeV, couplings=[0.447e-9,7.15e-9,1.88e-9] and charm, the Pythia 8 configuration is as follows:

```
P8gen.UseRandom3()
P8gen.SetMom(400)
P8gen.SetParameters("130:mayDecay  = off")
P8gen.SetParameters("310:mayDecay  = off")
P8gen.SetParameters("3122:mayDecay = off")
P8gen.SetParameters("HardQCD::hardccbar  = on")
```

```
P8gen.SetParameters("9900015:new = N2 N2 2 0 0 1.0 0.0 0.0 0.0 58428064.2056  0   1   0   1   0")
P8gen.SetParameters("9900015:isResonance = false")
P8gen.SetParameters("Next:numberCount    =  0")
P8gen.SetParameters("9900015:mayDecay = on")
P8gen.SetHNLId(9900015)
P8gen.SetParameters("4132:new Xi_c0  Xi_cbar0     2 0 0 2.47088 0.00000 0.00000 0.00000 3.36000e-02 0 1 0 1 0")
P8gen.SetParameters("4132:addChannel      1  3.58822454991e-06  22     -11       9900015 3312")
P8gen.SetParameters("4132:addChannel      1  2.60669405168e-05  22     -13       9900015 3312")
P8gen.SetParameters("4132:addChannel      1  0.999970344835    0       22      22")
P8gen.SetParameters("421:new D0      Dbar0        1 0 0 1.86486 0.00000 0.00000 0.00000 1.22900e-01  1 0 1 0")
P8gen.SetParameters("421:addChannel       1  0.000231019503771  22     -11       9900015  -321")
P8gen.SetParameters("421:addChannel       1  0.00289294768576   22     -13       9900015  -321")
P8gen.SetParameters("421:addChannel       1  0.99687603281     0       22        22")
P8gen.SetParameters("15:new  tau-    tau+         2 -3 0 1.77682 0.00000 0.00000 0.00000 8.71100e-02 0 1 0 1 0")
P8gen.SetParameters("15:addChannel        1  0.0185887706956    1521   9900015   -211")
P8gen.SetParameters("15:addChannel        1  0.0110210094762    1521   9900015   -321")
P8gen.SetParameters("15:addChannel        1  8.12258276183e-05  1521   9900015   -213")
P8gen.SetParameters("15:addChannel        1  0.0106510988466    1531   9900015   11      -12")
P8gen.SetParameters("15:addChannel        1  0.00253246387486   1531   9900015   11      -16")
P8gen.SetParameters("15:addChannel        1  0.00970246605066   1531   9900015   13      -14")
P8gen.SetParameters("15:addChannel        1  0.0369003427671    1531   9900015   13      -16")
P8gen.SetParameters("431:new  D_s+    D_s-        1 3 0 1.96849 0.00000 0.00000 0.00000 1.49900e-01 0 1 0 1 0")
P8gen.SetParameters("431:addChannel       1  0.85774273803      0      -13       9900015")
P8gen.SetParameters("431:addChannel       1  0.0527798844317    0      -11       9900015")
P8gen.SetParameters("431:addChannel       1  0.0894773775387    0      -15       16")
P8gen.SetParameters("4122:new Lambda_c+ Lambda_cbar- 2 3 0 2.28646 0.00000 0.00000 0.00000 5.99000e-02 0 1 0 1 0")
P8gen.SetParameters("4122:addChannel      1  1.81870778719e-05  22     -11       9900015  3122")
P8gen.SetParameters("4122:addChannel      1  0.000103181640095  22     -13       9900015  3122")
P8gen.SetParameters("4122:addChannel      1  0.999878631282     0      22        22")
P8gen.SetParameters("411:new D+       D-          1 3 0 1.86962 0.00000 0.00000 0.00000 3.11800e-01 0 1 0 1 0")
P8gen.SetParameters("411:addChannel       1  0.0504027923939    0      -13       9900015")
P8gen.SetParameters("411:addChannel       1  0.003100751144     0      -11       9900015")
P8gen.SetParameters("411:addChannel       1  0.000564532747042  22     -11       9900015  311")
P8gen.SetParameters("411:addChannel       1  0.00706319918653   22     -13       9900015  311")
P8gen.SetParameters("411:addChannel       1  0.938868724528     0      22        22")
P8gen.List(9900015)
```

This configuration is written out to a file `pythia8_conf.txt` when the flag `debug=True` is set at the beginning of the `configure()` method of `pythia8_conf.py`

The largest contribution at $\mathrm{M_N} = 1$ GeV comes from 2 body decays of the D_s (see Fig. 1). This can be understood as follows [11]. Consider the case of pion decay: the muon channel dominates over the electron channel because the muon is heavier. The charged weak current is chiral, and the mass of the heavier lepton enters the amplitude. If the neutrino is massless, only the charged lepton mass is present, and that is what happens for the SM pion decay. Now if the neutrino is massive, its mass can instead dominate the amplitude (see eqn(1)), and hence the heavier the neutrino the larger the amplitude. Thus the amplitudes with heavy neutrinos get amplified. For the τ decay the neutrino is lighter than the τ, so we observe the standard behavior.

With the 'NEW' procedure, we get a relative yield compared to the 'OLD' procedure (where the original branching ratios were kept and decays other than into HNL were forbidden) of 73%, i.e. we lose about 1/4. Figure 3 shows that the $\mathrm{p_{rmT}}$ of the HNL increases from a mean of 0.53 GeV to 0.72 GeV. This is because now most HNLs come via D_s two body leptonic decays, whereas before, they came mainly from D_0 semileptonic three body decays. The larger $\mathrm{p_{rmT}}$ causes the HNLs to go outside the detector acceptance. A comparision between the 'OLD' and 'NEW' procedure of the origin of the reconstructed signal is shown in Table 16.

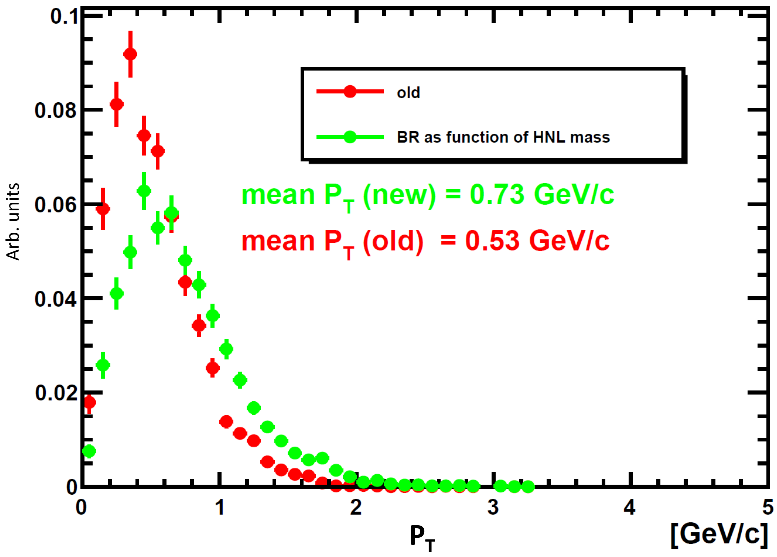


Figure 3: Comparison of P_T distributions between old and new procedure (mass dependent HNL BRs); $M_N =$ 1 GeV.

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