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Amplitude dependent closest tune approach

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Summary

Recent observations in the LHC point to the existence of an amplitude dependent closest tune approach. However this dynamical behavior and its underlying mechanism remain unknown. This effect is highly relevant for the LHC as an unexpectedly closest tune approach varying with amplitude modifies the frequency content of the beam and, hence, the Landau damping. Furthermore the single particle stability would also be affected by this effect as it would modify how particles with varying amplitudes approach and cross resonances. We present analytic derivations that lead to a mechanism generating an amplitude dependent closest tune approach.

1 Introduction

Compelling experimental observations in the LHC [1] suggested the existence of an amplitude dependent closest tune approach. These have been further complemented with computer simulations of the LHC in [2] identifying the key ingredients to reproduce the observations, namely linear coupling and normal octupole fields. Furthermore, [2] also shows the possibility to penetrate the linear coupling stopband for increasing amplitudes thanks to octupoles. This implies a reduction of the closest tune approach with amplitude.

In [3] emittance exchange during resonance crossing is studied in presence of space charge. This phenomena has great similarities with emittance exchange driven by linear coupling [4]. An intensity dependent closest tune approach is proposed and evaluated through multiparticle simulations in [3]. This could be interpreted as the result of the amplitude dependent closest tune approach generated by the non-linear space charge fields acting on the ensemble of particles.

The influence of linear difference coupling resonance in the long-term particle stability has been thoroughly studied [5, 6, 7, 8]. The mechanisms are twofold, linear coupling directly modifies the excitation of lattice resonances but it also affects how resonances are approached and crossed via the transverse emittance exchange and the closest tune approach. For example, in [9, 10] a skew octupolar Hamiltonian term (h_{1012}), which can be generated via linear coupling and octupoles, is identified as particularly relevant for the long-term particle stability. An amplitude dependent closest tune approach would further contribute to the previous mechanisms.

Landau damping is generated in the LHC via strong octupoles [11] and is fundamental to suppress instabilities from collective effects. It has been observed that linear coupling can destabilize the beam in HERA [12] and possibly also in LHC [13]. An amplitude dependent closest tune approach would modify the frequency content of the beam altering the effective Landau damping.

The structure of the paper is as follows. Section 2 introduces the linear coupling theory and the nomenclature. Section 3 illustrates that an amplitude detuning closest tune approach is not generated in a trivial manner. Section 4 identifies a mechanism for the appearance of an amplitude dependent closest tune approach based on the interplay between linear coupling and cross amplitude detuning (h_{1111}). Section 5 explores the interplay between linear coupling and the Hamiltonian term h_{2002} but the complexity of the equations avoids the identification of any amplitude dependent closest tune approach. In the last Section 6 LHC simulation results with linear coupling and octupoles show qualitative agreement with the predictions from Section 4, this is, assuming the interplay between linear coupling and the h_{1111} amplitude detuning term as the main mechanism for amplitude dependent closest tune approach.

2 Linear coupling theory

In [14] the Hamiltonian of the linear coupled motion is given as

$$H = \frac{1}{2}\left[K_1 x^2 + K_2 z^2 + p_x^2 + p_z^2\right] + Kxz \tag{1}$$

including only skew quadrupoles as perturbing Hamiltonian, $H_1 = Kxz$. $x,\ z,\ p_x,\ p_z$ are the canonical variables and $K_{1,2}$ represent the linear uncoupled gradients. The general solution of the unperturbed Hamiltonian is given by

$$\begin{aligned} x &= a_1 u(\theta) e^{iQ_H\theta} + \overline{a}_1 \overline{u}(\theta) e^{-iQ_H\theta} \ , \\ z &= a_2 v(\theta) e^{iQ_V\theta} + \overline{a}_2 \overline{v}(\theta) e^{-iQ_V\theta} \ , \end{aligned} \tag{2}$$

where $u,\ v$ and $\overline{u},\ \overline{v}$ are the Floquet functions and their complex conjugates, respectively. $a_1,\ a_2,\ \overline{a}_1,\ \overline{a}_2$ are the constants of motion. Always following [14] the equations of motion in presence of the perturbing Hamiltonian are derived using the former constants of motion as new variables,

$$\begin{aligned} \frac{\mathrm{d}a_1}{\mathrm{d}\theta} &= i\frac{\partial U}{\partial \overline{a}_1} \\ \frac{\mathrm{d}\overline{a}_1}{\mathrm{d}\theta} &= -i\frac{\partial U}{\partial a_1} \\ \frac{\mathrm{d}a_2}{\mathrm{d}\theta} &= i\frac{\partial U}{\partial \overline{a}_2} \\ \frac{\mathrm{d}\overline{a}_2}{\mathrm{d}\theta} &= -i\frac{\partial U}{\partial a_2} \end{aligned} \tag{3}$$

where U is the perturbing Hamiltonian as function of the new variables and admits the following Fourier expansion,

$$U = \sum_{jklm} \sum_{q=-\infty}^{\infty} h_{jklmq} a_1^j \overline{a}_1^k a_2^l \overline{a}_2^m e^{i[(j-k)Q_H + (l-m)Q_V + q]\theta} \ , \tag{4}$$

where

$$h_{jklmq} = \frac{1}{2\pi} \int_0^{2\pi} h_{jklm} e^{-iq\theta}\, \mathrm{d}\theta \ , \tag{5}$$

and h_{jklm} are the Hamiltonian terms.

Considering only the slowly varying U term close to the difference coupling resonance ($Q_H - Q_V = p$) the equations of motion are approximated by

$$\begin{aligned} \frac{\mathrm{d}a_1}{\mathrm{d}\theta} &= i\overline{\kappa} a_2 e^{-i\Delta\theta} \\ \frac{\mathrm{d}a_2}{\mathrm{d}\theta} &= i\kappa a_1 e^{i\Delta\theta} \end{aligned} \tag{6}$$

with $\kappa = h_{1001-p}$ and $\Delta = Q_H - Q_V - p$. The general solution of these coupled differential equations follows,

$$\begin{aligned} a_1 &= \overline{\kappa}\left(\frac{A_+}{w_+}e^{iw_+\theta} + \frac{A_-}{w_-}e^{iw_-\theta}\right) , \\ a_2 &= \left(A_+e^{iw_+\theta} + A_-e^{iw_-\theta}\right)e^{i\theta\Delta} , \end{aligned} \tag{7}$$

where $A_\pm$ are complex constants of motion and $w_\pm$ are the frequencies given by

$$w_\pm = -\frac{\Delta}{2} \pm \sqrt{\left(\frac{\Delta}{2}\right)^2 + |\kappa|^2} . \tag{8}$$

$2|\kappa|$ is therefore the minimum separation between the two frequencies, i.e. the closest tune approach. Some relevant quantities follow,

$$\begin{aligned} a_1\overline{a}_1 &= |\kappa|^2\left[\frac{|A_+|^2}{w_+^2} + \frac{|A_-|^2}{w_-^2} + 2\frac{|A_+||A_-|}{w_+w_-}\cos((w_+ - w_-)\theta + \phi)\right] , \\ a_2\overline{a}_2 &= |A_+|^2 + |A_-|^2 + 2|A_+||A_-|\cos((w_+ - w_-)\theta + \phi) , \\ a_1\overline{a}_2 &= \overline{\kappa}e^{-i\Delta\theta}\left[\frac{|A_+|^2}{w_+} + \frac{|A_-|^2}{w_-} + \frac{A_+A_-}{w_+}e^{i(w_+-w_-)\theta} + \frac{A_+A_-}{w_-}e^{-i(w_+-w_-)\theta}\right] \end{aligned} \tag{9}$$

The reader can check that the quantities

$$\mathcal{C} = a_1\overline{a}_1 + a_2\overline{a}_2 \tag{10}$$

$$\mathcal{Q} = \Re\left\{a_1\overline{a}_2e^{i\Delta\theta}\right\} - \frac{\Delta}{4\kappa}(a_2\overline{a}_2 - a_1\overline{a}_1) \tag{11}$$

are invariants of the motion.

3 A first exploration

Finding mechanisms that truly generate an amplitude dependent closest tune approach is not trivial. In this section we try to find a Hamiltonian that would lead to the following equations of motion

$$\begin{aligned} \frac{\mathrm{d}a_1}{\mathrm{d}\theta} &= i\overline{\kappa(\mathcal{C})}a_2e^{-i\Delta\theta} \\ \frac{\mathrm{d}a_2}{\mathrm{d}\theta} &= i\kappa(\mathcal{C})a_1e^{i\Delta\theta} \end{aligned} \tag{12}$$

which are identical to Eqs. (6) but with an amplitude dependent κ via the only action-like invariant of the motion $\mathcal{C}$. This change is transparent to the differential equations and yields the same solutions as in Eqs. (7) and (8) but keeping the $\mathcal{C}$ dependency in κ, implying a *pure* amplitude dependent closest tune approach. Now it only remains to identify which kind of Hamiltonian terms could produce such differential equations with $\kappa(\mathcal{C})$. From Eqs. (3),

$$i\frac{\partial^2 U}{\partial\overline{a}_1\partial\overline{a}_2} = \frac{\partial}{\partial\overline{a}_2}\left(\frac{\mathrm{d}a_1}{\mathrm{d}\theta}\right) = \frac{\partial}{\partial\overline{a}_1}\left(\frac{\mathrm{d}a_2}{\mathrm{d}\theta}\right) . \tag{13}$$

and using Eqs. (12) yields,

$$\frac{\partial\overline{\kappa(\mathcal{C})}}{\partial\overline{a_2}}a_2e^{-i\Delta\theta} = \frac{\partial\kappa(\mathcal{C})}{\partial\overline{a_1}}a_1e^{i\Delta\theta} . \tag{14}$$

This condition imposes severe constraints in the possible $\kappa(\mathcal{C})$. From Eq. (10)

$$\frac{\partial \mathcal{C}}{\partial \overline{a_j}} = a_j \quad , \text{with} j \in \{1,2\} \; , \tag{15}$$

which turns Eq. (14) into

$$\frac{\partial \overline{\kappa(\mathcal{C})}}{\partial \mathcal{C}} a_2^2 e^{-i\Delta\theta} = \frac{\partial \kappa(\mathcal{C})}{\partial \mathcal{C}} a_1^2 e^{i\Delta\theta} \tag{16}$$

taking the absolute value yields,

$$\left|\frac{\partial \kappa(\mathcal{C})}{\partial \mathcal{C}}\right| \left(|a_2|^2 - |a_1|^2\right) = 0 \; , \tag{17}$$

and as a_1 and a_2 are independent in general, this leaves as only possible solution

$$\frac{\partial \kappa}{\partial \mathcal{C}} = 0 \tag{18}$$

which implies the exact opposite of our initial quest. Therefore there is no Hamiltonian that leads to Eqs. (12) with κ being a function of $\mathcal{C}$, which would have automatically led to an amplitude dependent closest tune approach. Of course, this does not exclude other mechanisms for the appearance of an amplitude dependent closest tune approach, such as the one identified in the following section.

4 Amplitude dependent closest tune approach via linear coupling and h_{1111}

The Hamiltonian term h_{1111} generates cross amplitude detuning. The differential equations in presence of linear coupling and this Hamiltonian term follow ($h_1 = h_{1111}$),

$$\begin{aligned} \frac{\mathrm{d}a_1}{\mathrm{d}\theta} &= i\overline{\kappa} a_2 e^{-i\Delta\theta} + i h_1 a_1 a_2 \overline{a_2} \\ \frac{\mathrm{d}a_2}{\mathrm{d}\theta} &= i\kappa a_1 e^{i\Delta\theta} + i h_1 a_2 a_1 \overline{a_1} \; . \end{aligned} \tag{19}$$

We assume κ to be a real positive number without any loss of generality as its phase can be evenly split between a_2 and a_1 in Eqs. (19). In the absence of coupling $a_1\overline{a_1}$ corresponds to the action invariant of the motion J_x. According to Eqs. (19) its derivative versus θ is expressed as

$$\begin{aligned} \frac{\mathrm{d}(a_1\overline{a_1})}{\mathrm{d}\theta} &= -2\Im\left\{\kappa a_2 \overline{a_1} e^{-i\Delta\theta}\right\} \\ \frac{\mathrm{d}(a_2\overline{a_2})}{\mathrm{d}\theta} &= -2\Im\left\{\kappa a_1 \overline{a_2} e^{i\Delta\theta}\right\} \; , \end{aligned} \tag{20}$$

from these equations it can be seen that $\mathcal{C} = a_1\overline{a_1} + a_2\overline{a_2}$ is a constant of the motion. Defining

$$F = \mathcal{C} - 2a_1\overline{a_1} = a_2\overline{a_2} - a_1\overline{a_1} \tag{21}$$

$$S = a_1\overline{a_2} e^{i\Delta\theta} \tag{22}$$

$$\tag{23}$$

we obtain

$$\frac{\mathrm{d}F}{\mathrm{d}\theta} = -4\kappa\Im\{S\} \tag{24}$$

$$\frac{\mathrm{d}S}{\mathrm{d}\theta} = iF[\kappa + h_1 S] + \Delta i S \; . \tag{25}$$

Inspecting the real and imaginary parts of Eq. (25) the following expression is obtained,

$$\frac{\mathrm{d}(\Re\{S\})}{\mathrm{d}\theta} = -\Im\{S\}(Fh_1 + \Delta) = \frac{1}{4\kappa}\frac{\mathrm{d}F}{\mathrm{d}\theta}(Fh_1 + \Delta)\ . \tag{26}$$

This equation can actually be integrated, resulting in

$$\Re\{S\} = \frac{h_1}{8\kappa}F^2 + \frac{\Delta}{4\kappa}F + \mathcal{Q}\ , \tag{27}$$

where $\mathcal{Q}$ is another constant of the motion yielding to the invariant

$$\mathcal{Q} = \Re\{S\} - \frac{h_1}{8\kappa}F^2 - \frac{\Delta}{4\kappa}F\ . \tag{28}$$

Note that these equations require $|\kappa| > 0$. For $|\kappa| = 0$ the motion is simply an amplitude dependent betatron oscillation. Computing the second derivative of $a_1\overline{a_1}$,

$$\frac{\mathrm{d}^2(a_1\overline{a_1})}{\mathrm{d}\theta^2} = 2\kappa^2 F + 2\kappa(Fh_1 + \Delta)\Re\{S\}\ \ , \tag{29}$$

and therefore using $F = \mathcal{C} - 2a_1\overline{a_1}$ and Eq. (27)

$$\frac{\mathrm{d}^2 F}{\mathrm{d}\theta^2} = -F\left(4\kappa^2 + \frac{h_1^2}{2}F^2 + \frac{3\Delta h_1}{2}F + 4h_1\kappa\mathcal{Q} + \Delta^2\right) - 4\kappa\Delta\mathcal{Q}\ , \tag{30}$$

For $h_1 = 0$ the linear motion is retrieved. To find the closest tune approach we are interested in $\Delta = 0$,

$$\frac{\mathrm{d}^2 F}{\mathrm{d}\theta^2} = -F\left(4\kappa^2 + \frac{h_1^2}{2}F^2 + 4h_1\kappa\mathcal{Q}\right)\ . \tag{31}$$

This equation can be transformed into the $\mathrm{cn}(x,k)$ Jacobi elliptic differential equation, however a perturbative approach is enough to illustrate the appearance of the amplitude dependent closest tune approach. By assuming $F = A\cos(2\hat{\kappa}\theta) + B\cos(6\hat{\kappa}\theta)$, and neglecting terms of order above h_1^2 we obtain a new amplitude dependent $\hat{\kappa}$ given by

$$\hat{\kappa} = \sqrt{\kappa^2 + h_1\kappa\mathcal{Q} + \frac{3h_1^2}{32}A^2} \tag{32}$$

The choice of F implies that the initial $a_1\overline{a_2}$ is real and $A = a_2\overline{a_2} - a_1\overline{a_1}$, therefore

$$\hat{\kappa} = \sqrt{\kappa^2 + h_1\kappa a_1\overline{a_2} - \frac{h_1^2}{32}(a_2\overline{a_2} - a_1\overline{a_1})^2} \tag{33}$$

This equation shows the appearance of a non-linear closest tune approach when both κ and h_1 are different than zero. An important feature of this equation is that the closest tune approach can increase or decrease with amplitude depending on the phase space initial conditions and the sign of the Hamiltonian terms.

5 Does h_{2002} generate amplitude dependent closest tune approach?

The Hamiltonian term h_{2002} is investigated in this section. The differential equations in presence of linear coupling and this Hamiltonian term follow ($h_2 = h_{2002}$),

$$\begin{aligned}\frac{\mathrm{d}a_1}{\mathrm{d}\theta} &= i\overline{\kappa}a_2e^{-i\Delta\theta} + ih_2\overline{a_1}a_2^2e^{-i2\Delta\theta}\\ \frac{\mathrm{d}a_2}{\mathrm{d}\theta} &= i\kappa a_1e^{i\Delta\theta} + ih_2a_1^2\overline{a_2}e^{i2\Delta\theta}\ .\end{aligned} \tag{34}$$

After some algebra similar to previous section we find

$$\frac{\mathrm{d}F}{\mathrm{d}\theta} = -4\Im\left\{S(\kappa+\overline{h_2}S)\right\} \tag{35}$$

$$\frac{\mathrm{d}S}{\mathrm{d}\theta} = iF[\overline{\kappa}+h_2\overline{S}]+i\Delta S \tag{36}$$

For simplicity we assume that both h_2 and κ are real numbers. Separating S into its real and imaginary parts $S=R_S+iI_S$ Eq. (36) is rewritten as

$$\frac{\mathrm{d}F}{\mathrm{d}\theta} = -4I_S(\kappa+2h_2R_S) \tag{37}$$

$$\frac{\mathrm{d}I_S}{\mathrm{d}\theta} = F[\kappa+h_2R_S]+\Delta R_S \tag{38}$$

$$\frac{\mathrm{d}R_S}{\mathrm{d}\theta} = I_S(h_2F-\Delta) \tag{39}$$

The first and the third equations can be combined to reach the following integrable equation,

$$\frac{\mathrm{d}F}{\mathrm{d}\theta}(h_2F-\Delta) = -4\frac{\mathrm{d}R_S}{\mathrm{d}\theta}(\kappa+2h_2R_S)\ , \tag{40}$$

resulting, after integration, in

$$\frac{1}{2}h_2F^2-F\Delta+\mathcal{X} = -4(\kappa R_S+h_2R_S^2)\ , \tag{41}$$

where $\mathcal{X}$ is a constant of the motion. This equation allows to express F as a function of R_S. Taking the second derivative of R_S in Eqs. (36) and operating an equation including only R_S is obtained,

$$\frac{\mathrm{d}^2\,R_S}{\mathrm{d}\theta^2} = (\kappa F+R_S(h_2F+\Delta))(h_2F-\Delta)+\frac{\mathrm{d}R_S}{\mathrm{d}\theta}\frac{h_2}{h_2F-\Delta}\frac{\mathrm{d}F}{\mathrm{d}\theta} \tag{42}$$

This equation is highly complicated and no amplitude dependent closest tune approach can be easily identified, contrary to the previous case with h_{1111}. Nevertheless it cannot be discarded that h_{2002} generates amplitude dependent closest tune approach.

6 Observations from Simulations

Simulations are presented in this section supporting the existence of an amplitude dependent closest tune approach and in qualitative agreement with predictions from Section 4. A model of the LHC beam 2 at injection is tracked using MADX and PTC. The LHC is equipped with 2 families of Landau damping octupoles (MOF and MOD) and their nominal settings corresponding to the first part of 2012 is -3 m^{-4}. The vertical tune is matched to values ranging from 59.28 to 59.30 and for each of these settings kicks between 0.1 mm to 4.5 mm were performed (at β_x=175 m and β_y=179 m). The actions are reconstructed using the amplitude and the beta functions for each BPM, as described in [1].

Using the nominal model without any skew quadrupolar components the detuning behaves linearly, as seen in figure 1. We observe that none of the points are on the diagonal which could indicate a small closest tune approach without a clear trend with amplitude.

Coupling is introduced using the skew quadrupoles placed in the LHC arcs to generate a closest tune approach of 0.015. Figure 2 shows the tunes for different initial tune splits demonstrating that there is a mechanism pushing the tunes away from each other already far away from the linear closest tune approach (light red area).

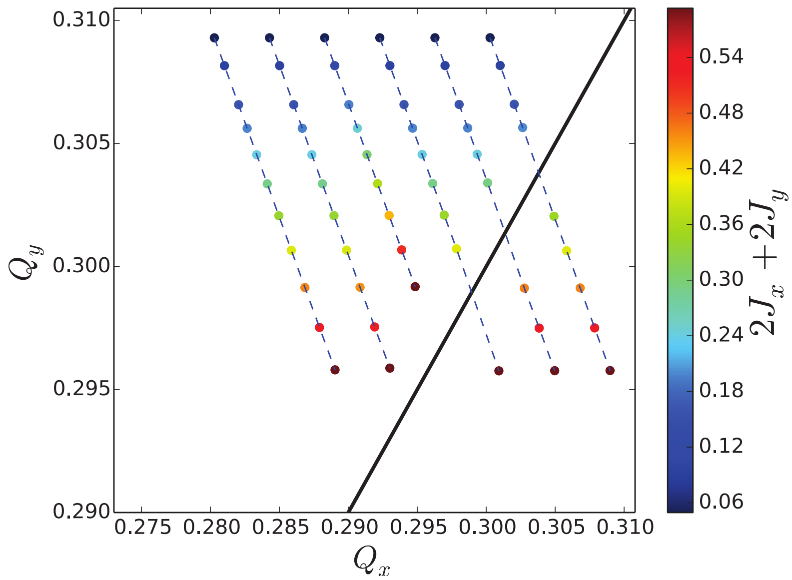

Figure 1: Tunes from particle tracking for different initial tunes and for increasing amplitudes of the vertical kicks. The color code represents the sum of the horizontal and vertical actions. The black diagonal line indicates the resonance $Q_x = Q_y$.

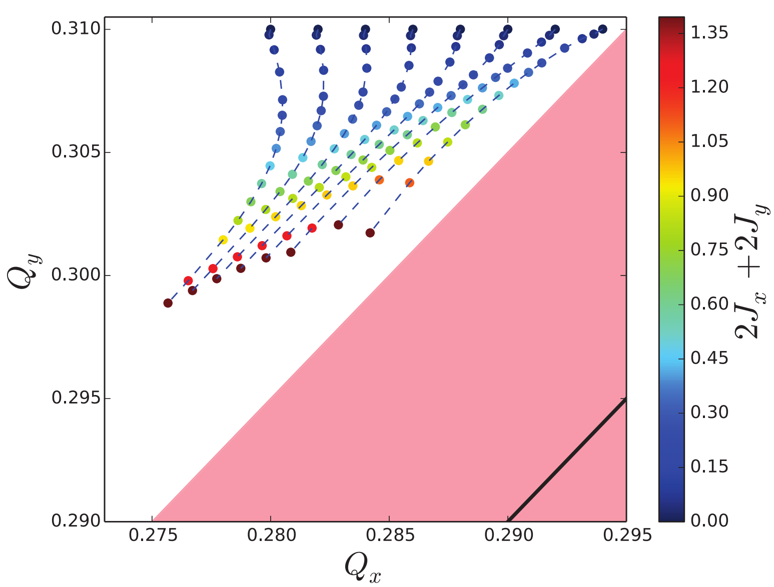

Figure 2: Tunes from tracking with linear closest tune approach of 0.015 and nominal octupoles. The vertical kicks ranged from 0.5 mm to 4.5 mm while the horizontal were kept at 0.5 mm. The light red area delimits the linear closest tune approach of 0.015.

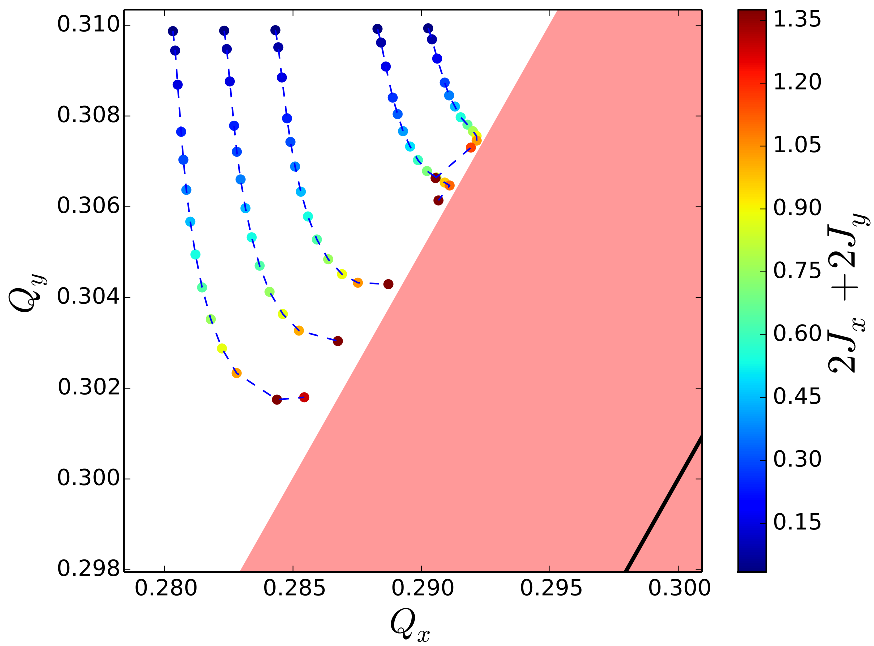

Figure 3: Tunes from tracking with linear closest tune approach of 0.015 and h_{1111}=0 by powering MOF with opposite polarity than MOD. The vertical kicks range from 0.5 mm to 4.5 mm while the horizontal were kept at 0.5 mm. The light red area delimits the linear coupling stopband of 0.015.

Figure 3 shows the situation where the focusing octupoles (MOF) are powered with the opposite strength compared to the defocusing octupoles (MOD) but with same absolute value as in the previous case. This configuration causes the h_{1111} Hamiltonian term to be very close to zero. In this case we observe that tunes reach the linear coupling stopband independently of the amplitude. This is consistent with the fact that the amplitude dependent closest tune approach requires both κ and h_{1111} to be different than zero. This configuration with opposite MOF and MOD polarities might be interesting for the LHC operation to feature Landau damping but with fully suppressed amplitude dependent closest tune approach even in the presence of linear coupling.

The same tracking procedure is repeated for kicks in the horizontal plane and nominal octupole powering. The horizontal tunes are changed but the linear coupling and vertical tune are kept the same and the magnitude of the horizontal kicks were increased. It is a remarkable observation that for some of the kicks, starting close to the linear closest tune approach, the particles penetrate the stopband. This means that the tunes can approach each other closer than what is possible in linear coupling theory at larger amplitudes.

Similar tracking simulations with different non-linear sources have been performed to discard other possible sources of amplitude dependent closest tune approach. In particular skew octupoles alone do not feature any amplitude dependent closest tune approach.

7 Conclusion

Motivated by LHC experimental observations and simulations we have found analytically a mechanism to generate an amplitude dependent closest tune approach based on the interplay between linear coupling and the cross term amplitude detuning h_{1111} given by

$$\Delta Q_{min} = 2|\hat{\kappa}| = 2\sqrt{\kappa^2 + h_{1111}\kappa a_1 \overline{a_2} - \frac{h_{1111}^2}{32}(a_2\overline{a_2} - a_1\overline{a_1})^2} \tag{43}$$

This equation predicts that the amplitude dependence of the closest tune approach can be positive or negative depending on the phase space coordinates $a_{1,2}$, the linear coupling κ and the octupolar term h_{1111}. Simulations have confirmed that both linear coupling and h_{1111} are fundamental to generate a sizable amplitude dependent closest tune approach, and that this can be both positive and negative depending on the excitation plane. The linear stopband has been penetrated in simulations with octupoles at large oscillation amplitudes. A particularly interesting case for the LHC consists in canceling the h_{1111} term while keeping a large Landau damping by using opposite strengths in one of the LHC octupole families. In this configuration, as expected, there is no amplitude dependent closest tune approach.

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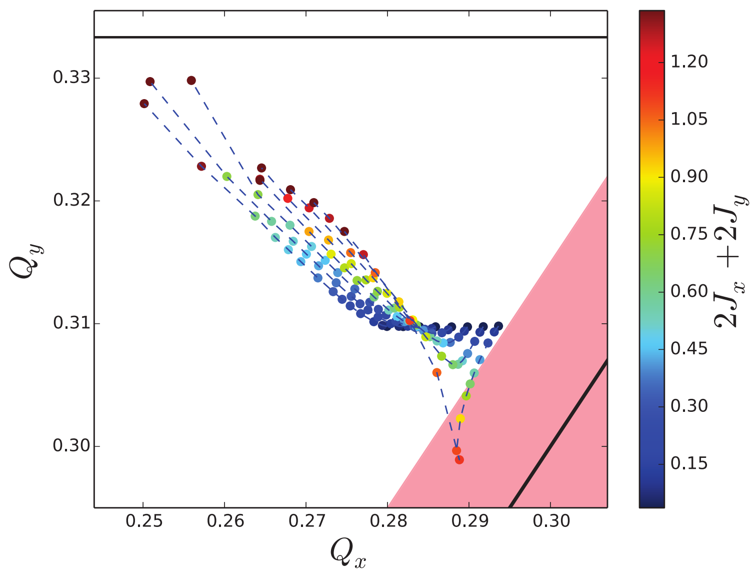

Figure 4: Tunes from tracking with linear closest tune approach of 0.015 and nominal octupoles. The kicks in the horizontal plane ranged from 0.5 mm to 4.5 mm while the kicks in the vertical plane were kept at 0.5 mm. The light red area indicates the linear stopband of 0.015 for all cases. The two black lines show the resonances $Q_x = Q_y$ and $Q_y = 1/3$ respectively.