# Charged-current inclusive neutrino cross sections: Superscaling extension to the pion production and realistic spectral function for quasielastic region

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# Abstract

Superscaling approximation (SuSA) predictions to neutrino-induced chargedcurrent  $\pi^+$  production in the  $\Delta$ -resonance region are explored under Mini-BooNE experimental conditions. The results obtained within SuSA for the flux-averaged double-differential cross sections of the pion production for the  $v_{\mu}$  + CH<sub>2</sub> reaction as a function of the muon kinetic energy and of the scattering angle are compared with the corresponding MiniBooNE experimental data. The SuSA charged-current  $\pi^+$  predictions are in good agreement with data on neutrino flux average cross-sections. The SuSA extension to the pion production region and the realistic spectral function  $S(p, \mathscr{E})$  for quasielastic scattering are used for predictions of charged-current inclusive neutrinonucleus cross sections. The results are compared with the T2K experimental data.

# 1. Introduction

The properties of neutrinos, particularly the parameters of their oscillations, are being studied with increasing interest as these may carry important information about the limits of the Standard Model. In most neutrino experiments, the interactions of the neutrinos occur with nucleons bound in nuclei. Model predictions for these reactions involve many different effects such as nuclear correlations, interactions in the final state, possible modification of the nucleon properties inside the nuclear medium, that presently cannot be computed in an unambiguous and precise way. This is particularly true for the channels where neutrino interactions take place by means of excitation of a nucleon resonance and ulterior production of mesons. The data on neutrino-induced charged-current (CC) charged pion production cross sections on mineral oil recently released by the MiniBooNE collaboration [1] provides an unprecedented opportunity to carry out a systematic study of double differential cross section of the processes:  $v_{\mu} p \to \mu^- p \pi^+$  and  $v_{\mu} n \to \mu^- n \pi^+$  averaged over the neutrino flux. Also, new measurements of inclusive CC neutrino-nucleus scattering cross sections, where only the outgoing lepton is detected, have been recently performed by the T2K [2]. For neutrino energies around 1 GeV (T2K) the main contributions to the cross sections are associated with quasielastic (QE) scattering and one pion  $(1\pi)$  production.

The analyses of the world data on inclusive electron-nucleus scattering [3] confirmed the observation of superscaling and thus justified the extraction of a universal nuclear response to be also used for weak interacting probes. However, while there is a number of theoretical models that exhibit superscaling, such as for instance the relativistic Fermi gas (RFG) [4, 5], the nuclear response departs from the one derived from the experimental data. This showed the necessity to consider more complex dy-



Fig. 1: The SuSA scaling function in the  $\Delta$ -region  $f^{\Delta}(\psi_{\Delta})$  (solid line) extracted from the world data on electron scattering [7]. The dotted line shows the scaling functions  $f^{\Delta}(\psi_{\Delta})$  in the RFG model.

namical pictures of finite nuclear systems – beyond the RFG – in order to describe the nuclear response at intermediate energies. SuSA predictions are based on the phenomenological superscaling function extracted from the world data on quasielastic electron scattering [6]. The model has been extended to the ∆-resonance region [7] where the response of the nuclear system proceeds through excitation of internal nucleonic degrees of freedom. Indeed, a non-quasielastic cross section for the excitation region in which nucleon excitations, particularly the  $\Delta$ , play a major role was obtained by subtracting from the data QE-equivalent cross sections given by SuSA [8, 9]. This procedure has been possible due to the large amount of available high-quality data of inelastic electron scattering cross sections on  $^{12}C$ , including also separate information on the longitudinal and transverse responses, the latter containing important contributions introduced by effects beyond the impulse approximation (non-nucleonic).

Here we extend the analysis to CC pion production cross-section measured at MiniBooNe, that from the theoretical point of view can be seen as more challenging. For instance,  $\Delta$  properties in the nuclear medium, as well as both coherent and incoherent pion production for the nucleus should be considered in any theoretical approach, while in the SuSA procedure they are included phenomenologically extracted from the electron scattering data. All what is assumed within SuSA approach is the nuclear response to be factorized into a single-nucleon part and a 'nuclear function' accounting for the overall interaction among nucleons. As mentioned before, the SuSA assumptions have been tested against a great deal of electron-nucleus scattering data with fair success (see Section 2.1). The factorization assumption allows to apply the same nuclear responses derived from electron scattering to neutrino-induced reactions, with a mere use of the adequate single-nucleon terms for this case. To show the importance of nuclear interaction effects as predicted within SuSA, as a reference, we also show results obtained within the RFG, with no interactions among nucleons, for which the scaling function in the  $\Delta$ -domain is simply given as  $f_{RFG}^{\Delta}(\psi_{\Delta}) = \frac{3}{4}(1 - \psi_{\Delta}^2)\theta(1 - \psi_{\Delta}^2)$  with  $\psi_{\Delta}$  the dimensionless scaling variable extracted from the RFG analysis that incorporates the typical momentum scale for the selected nucleus [10, 7]. In Fig. 1 we compare the  $\Delta$ -region SuSA [7] and RFG scaling functions, which we use in our study.

# 2. Theoretical scheme and results

# 2.1 Test versus electron scattering

In Fig. 1 we compare our theoretical predictions with inclusive electron scattering data on  $^{12}$ C. In the QE region we use natural orbitals scaling function including final state interaction (NO+FSI), whereas for the ∆ region we make use *e.g.* the scaling function presented in Fig. 1. Details in how the NO+FSI scaling function is obtained is given in Ref. [11] (see also A.N. Antonov in this Proceedings). Here we only show results for a few representative choices of kinematics, similar to those involved in the neutrino



Fig. 2: Double-differential inclusive electron-carbon cross sections, *d*σ/*d*ω*d*Ω. The panels are labeled according to beam energy, scattering angle, and value of  $q<sub>OE</sub>$  at the quasielastic peak. The results are compared with the experimental data from [12].

experiments that we address in the following sections. As observed, results are in good agreement with the data, while some disagreement remains in the comparison to the data in the "dip" region between the QE and  $\Delta$  peaks. Meson-exchange current (MEC) contribution, that is not accounted for in this work, plays a major role in filling the "dip" region.

#### $2.2$  $\pi^+$  production in the MiniBooNE experiment

In what follows we present the results of applying the SuSA and RFG ∆-scaling function to neutrinoinduced CC charged pion production. We follow the formalism given in [7]. The charged-current neutrino cross section in the target laboratory frame is given in the form

$$
\frac{d^2\sigma}{d\Omega dk'} = \frac{(G\cos\theta_c k')^2}{2\pi^2} \left(1 - \frac{|Q^2|}{4\epsilon \varepsilon'}\right) \mathscr{F}^2,\tag{1}
$$

where  $\Omega$ , k' and  $\varepsilon'$  are the scattering angle, momentum and energy of the outgoing muon, *G* is the Fermi constant and  $\theta_c$  is the Cabibbo angle. The function  $\mathcal{F}^2$  depends on the nuclear structure through the *R* responses and can be written as [7, 13]:

$$
\mathcal{F}^2 = \widehat{V}_{\text{CC}}R_{\text{CC}} + 2\widehat{V}_{\text{CL}}R_{\text{CL}} + \widehat{V}_{\text{LL}}R_{\text{LL}} + \widehat{V}_{\text{T}}R_{\text{T}} + 2\widehat{V}_{\text{T}}R_{\text{T}'}
$$

that is, as a generalized Rosenbluth decomposition having charge-charge (CC), charge-longitudinal  $(CL)$ , longitudinal-longitudinal (LL) and two types of transverse  $(T, T')$  responses  $(R's)$  with the corresponding leptonic kinematical factors  $(V's)$ . The nuclear response functions in  $\Delta$ -region are expressed in terms of the nuclear tensor  $W^{\mu\nu}$  in the corresponding region. The basic expressions used to calculate the single-nucleon cross sections are given in [7]. These involve the leptonic and hadronic tensors as well as the response and structure functions for single nucleons. A convenient parametrization of the single-nucleon  $W^+n \to \Delta^+$  vertex is given in terms of eight form-factors: four vector  $(C_{3,4,5,6}^V)$  and four axial  $(C_{3,4,5,6}^A)$  ones. Vector form factors have been determined from the analysis of photo and



Fig. 3: (Color online) The double-differential cross section averaged over the neutrino energy flux as a function of the muon kinetic energy  $T_{\mu}$  obtained by SuSA and RFG  $\Delta$ -region scaling functions. In each subfigure the results have been averaged over the corresponding angular bin of  $\cos\theta$ . For vector and axial form-factors two parameterizations, "PR1" [14] and "PR2" [15], are used.

electro-production data, mostly on a deuteron target. Among the axial form factors, the most important contribution comes from  $C_5^A$ . The factor  $C_6^A$ , whose contribution to the differential cross section vanishes for massless leptons, can be related to  $C_5^A$  by PCAC. Since there are no other theoretical constraints for  $C_{3,4,5}^{A}(q^2)$ , they have to be fitted to data. We use two different parameterizations: the one given in [14] where deuteron effects were evaluated (authors estimated that the latter reduce the cross section by 10%), denoted as "PR1", and the one from [15], called "PR2".

With these ingredients, we evaluate the cross section for CC  $\Delta^{++}$  and  $\Delta^{+}$  production on proton and neutron, respectively. Once produced, the  $\Delta$  decays into  $\pi N$  pairs. For the amplitudes  $\mathscr A$  of pion production the following isospin decomposition applies:  $\mathscr{A}(v_l p \to l^- p \pi^+) = \mathscr{A}_3$ ,  $\mathscr{A}(v_l n \to l^- n \pi^+) =$  $\frac{1}{3}$   $\mathscr{A}_3 + \frac{2\sqrt{2}}{3} \mathscr{A}_1$ ,  $\mathscr{A}(v_l n \rightarrow l^- p \pi^0) = -\frac{\sqrt{2}}{3} \mathscr{A}_3 + \frac{2}{3} \mathscr{A}_1$ , with  $\mathscr{A}_3$  being the amplitude for the isospin 3/2 state of the  $\pi N$  system, predominantly  $\Delta$ , and  $\mathscr{A}_1$  the amplitude for the isospin 1/2 state that is not considered here.

The double-differential cross section for CC neutrino-induced  $\pi^+$  production averaged over the neutrino energy flux as a function of the muon kinetic energy  $T_{\mu}$  is presented in Fig. 3. Each panel corresponds to a bin of  $\cos\theta$ . PR1 and PR2 parametrizations have been considered. Results with the PR1 parameterization are about 5% higher, that is a measure of the degree of uncertainty that we expect from the choice of the single-nucleon response for this reaction. We compare the predictions of SuSA and RFG with the MiniBooNE data [1]. The nuclear target has been considered as carbon and hydrogen in the mineral oil target. Here we show that SuSA predictions are in good agreement with the MiniBooNE experimental data for  $\pi^+$  cross-section in the case of the flux averaged data.

# 2.3 Charged-current inclusive neutrino cross sections in the T2K experiment

In Fig. 4 we show the CC inclusive  $v_{\mu}$ −<sup>12</sup>C double-differential cross section per nucleon versus the muon momentum,  $p_{\mu}$ , for different angular bins, folded with the T2K flux. The QE curve corresponds to the results obtained using NO+FSI scaling function [11] (see also A.N. Antonov in this Proceedings). The NO+FSI scaling function is obtained using realistic energy dependence of the spectral function  $S(p, \mathscr{E})$  and an account for the effects of short-range nucleon-nucleon correlations when natural orbitals



**Fig. 4:** The CC inclusive T2K flux-folded  $v_{\mu}$ -<sup>12</sup>C double-differential cross section per nucleon evaluated using NO+FSI scaling function in the QE region [QE(NO+FSI)] and SuSA scaling function in the  $\Delta$ -region [1 $\pi$ ] is displayed as a function of the muon momentum for different bins in the muon angle. The separate contributions of the QE and  $1\pi$  are displayed. The data are from [2].

(NOs) from the Jastrow method are included. The NO+FSI scaling function is accounting also for the role of the final-state interactions (FSI). The resonant pion production curve  $(1\pi)$  is derived with the SuSA scaling function in the  $\Delta$ -region  $f^{\Delta}(\psi_{\Delta})$  (Fig. 1). The band corresponds to the two different parametrizations, PR1 and PR2, described in Section 2.2. We observe that the model yields very good agreement with the T2K data.

# 3. Conclusions

We conclude that the idea of the SuSA approach for the ∆-region (extracted from electron scattering experiments) in addition with the use of natural orbitals scaling function and including final state interaction (NO+FSI) for the QE-region, when being extended to neutrino processes, proves to be very successful in describing  $v_{\mu}$  inclusive charged-current cross sections. Our model, after being tested against electron scattering data, has been proved to explain with success neutrino scattering data taken at different kinematics and explaining several regions of great interest, such as the QE and ∆ ones.

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# References

- [1] A.A. Aguilar-Arevalo *et al.* (MiniBooNE Collaboration), Phys. Rev. D 83 (2011) 052007.
- [2] K. Abe *et al.* [T2K Collaboration], Phys. Rev. D 87 (2013) 092003.
- [3] T. W. Donnelly and I. Sick, Phys. Rev. Lett. 82 (1999) 3212; Phys. Rev. C 60 (1999) 065502.
- [4] W. M. Alberico, A. Molinari, T. W. Donnelly, E. L. Kronenberg, and J. W. Van Orden, Phys. Rev. C 38 (1988) 1801.
- [5] M. B. Barbaro, R. Cenni, A. De Pace, T. W. Donnelly, and A. Molinari, Nucl. Phys. A 643 (1998) 137.
- [6] J. Jourdan, Nucl. Phys. A **603** (1996) 117.
- [7] J. E. Amaro, M. B. Barbaro, J. A. Caballero, T. W. Donnelly, A. Molinari and I. Sick, Phys. Rev. C 71 (2005) 015501.
- [8] M. B. Barbaro, J. A. Caballero, T. W. Donnelly, and C. Maieron, Phys. Rev. C 69 (2004) 035502.
- [9] C. Maieron, J. E. Amaro, M. B. Barbaro, J. A. Caballero, T. W. Donnelly, C. F. Williamson, Phys. Rev. C 80 (2009) 035504.
- [10] C. Maieron, T. W. Donnelly, and I. Sick, Phys. Rev. C 65 (2002) 025502.
- [11] M. V. Ivanov, A. N. Antonov, J. A. Caballero, G. D. Megias, M. B. Barbaro, E. Moya de Guerra, J. M. Udias, Phys. Rev. C 89 (2014) 014607; M. V. Ivanov, A. N. Antonov, M. B. Barbaro, C. Giusti, A. Meucci, J. A. Caballero, R. González-Jiménez, E. Moya de Guerra, and J. M. Udías, Phys. Rev. C 91 (2015) 034607.
- [12] O. Benhar, D. Day, I. Sick, Rev. Mod. Phys. 80 (2008) 189; <http://faculty.virginia.edu/qes-archive/>.
- [13] Y. Umino and J. M. Udias, Phys. Rev. C 52 (1995) 3399.
- [14] L. Alvarez-Ruso, S. K. Singh and M. J. Vicente Vacas, Phys. Rev. C 59 (1999) 3386.
- [15] E. A. Paschos, J.-Y. Yu and M. Sakuda, Phys. Rev. D 69 (2004) 014013.